Package ‘GaSP’

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Author  William J. Welch [aut, cre, cph]
        (<https://orcid.org/0000-0002-4575-3124>),
        Yilin Yang [aut] (<https://orcid.org/0000-0003-0885-6017>)
Maintainer  William J. Welch <will@stat.ubc.ca>
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**borehole**

---

### borehole

*Data for the borehole function*

---

**Description**

Training and test data for the borehole function; see source for background.

**Usage**

```
borehole
```

**Format**

A list with the following four data frames:

- `x` 8-dimensional input for 40 training runs.
- `y` Output (the flow) for the 40 training runs in `x`.
- `x_pred` 8-dimensional input for 1000 test runs at which to predict `y`.
- `y_true` Output for the 1000 runs in `x_pred`.

**Source**

[https://www.sfu.ca/~ssurjano/borehole.html](https://www.sfu.ca/~ssurjano/borehole.html)
CrossValidate  Cross-validated predictions for a GaSPModel object.

Description
Compute leave-one-out cross-validated predictions for a GaSPModel object.

Usage
CrossValidate(GaSP_model)

Arguments
GaSP_model Object of class GaSPModel.

Value
A data frame with two columns: the cross-validated predictions Pred and their standard errors SE.

Note
RMSE computes the root mean squared error of the predictions. PlotPredictions and PlotResiduals plot the predictions or their residuals; PlotStdResiduals and PlotQQ plot the standardized residuals.

Examples
borehole_cv <- CrossValidate(borehole_fit)

DescribeX  Describe the input variables.

Description
Describe the input variables to set up integration or summation ranges for Visualize.

Usage
DescribeX(
  x_names,
  x_min,
  x_max,
  support = NULL,
  num_levels = NULL,
  distribution = NULL
)
Arguments

\textbf{x_names} \hspace{1cm} \text{A vector of character strings containing the names of the input variables.}

\textbf{x_min, x_max} \hspace{1cm} \text{Vectors of the same length as \textbf{x_names} containing the minima and maxima, respectively, of the input variables.}

\textbf{support} \hspace{1cm} \text{Optional vector of character strings of the same length as \textbf{x_names}. Valid strings for a variable are: "Continuous" (continuous between the input's \textbf{x_min} and \textbf{x_max}); "Fixed" (the input's \textbf{x_min} must equal its \textbf{x_max}); and "Grid" (which requires the next argument).}

\textbf{num_levels} \hspace{1cm} \text{An optional vector of integers for the number of levels of each input; must be present if the support argument includes "Grid". An input's number of levels is 0 if it is "Continuous", 1 if it is "Fixed", or > 1 if it is "Grid" to define an equally spaced grid inclusive of the input's \textbf{x_min} and \textbf{x_max}.}

\textbf{distribution} \hspace{1cm} \text{An optional vector of character strings of the same length as \textbf{x_names} to define the weight distributions of the input variables. Valid strings are "Uniform" or "Normal" (ignored for "Fixed" inputs).}

Value

A data frame with the following columns: \textbf{Variable} (containing \textbf{x_names}), \textbf{Min} (containing \textbf{x_min}), and \textbf{Max} (containing \textbf{x_max}), plus the optional columns \textbf{Support} (from \textbf{support}), \textbf{NumberLevels} (from \textbf{num_levels}), and \textbf{Distribution} (from \textbf{distribution}).

Note

Does not check against \texttt{GaSPModel} and all characters are CASE SENSITIVE.

Examples

\begin{verbatim}
  borehole_x_names <- colnames(borehole$x)
  borehole_min <- c(0.05, 100.00, 63070.00, 990.00, 63.10, 700.00, 1120.00, 9855.00)
  borehole_max <- c(0.15, 50000.00, 115600.00, 1110.00, 116.00, 820.00, 1680.00, 12045.00)
  borehole_x_desc <- DescribeX(borehole_x_names, borehole_min, borehole_max)
\end{verbatim}

\begin{verbatim}
  Fit <- Fit(a GaSP model.
\end{verbatim}

Description

Fit (train) a GaSP model.
Usage

Fit(
  x,
  y,
  reg_model,
  sp_model = NULL,
  cor_family = c("PowerExponential", "Matern"),
  cor_par = data.frame(0),
  random_error = c(FALSE, TRUE),
  sp_var = -1,
  error_var = -1,
  nugget = 1e-09,
  tries = 10,
  seed = 500,
  fit_objective = c("Likelihood", "Posterior"),
  theta_standardized_min = 0,
  theta_standardized_max = .Machine$double.xmax,
  alpha_min = 0,
  alpha_max = 1,
  derivatives_min = 0,
  derivatives_max = 3,
  log_obj_tol = 1e-05,
  log_obj_diff = 0,
  lambda_prior = 0.1,
  model_comparison = c("Objective", "CV")
)

Arguments

x          A data frame containing the input (explanatory variable) training data.
y          A vector or a data frame with one column containing the output (response) training data.
reg_model   The regression model, specified as a formula, but note the left-hand side of the formula is unused; see example.
sp_model   An optional stochastic process model, specified as a formula, but note the left-hand side of the formula and the intercept are unused. The default NULL uses all column names in x.
cor_family A character string specifying the (product, anisoptropic) correlation-function family: "PowerExponential" for the power-exponential family or "Matern" for the Matern family.
cor_par    An optional data frame containing the correlation parameters with one row per sp_model term and two columns set up as described in GaSPModel Details; only used to start the first objective maximization (see Details).
random_error A boolean for the presence or not of a random (measurement, white-noise) error term.
sp_var, error_var
The stochastic process and error variances; legal values are only used if random_error = TRUE to start the first objective maximization (see Details).

nugget
For numerical stability the proportion of the total variance due to random error is fixed at this value (random_error = FALSE) or bounded below by it (random_error = TRUE).

tries
Number of optimizations of the objective from different random starting points.

seed
The random-number seed to generate starting points.

fit_objective
The objective that Fit attempts to maximize: "Likelihood" (maximum likelihood estimation) or "Posterior" (Bayesian maximum a posteriori estimation).

theta_standardized_min, theta_standardized_max
The minimum and maximum of the standardized θ parameter (see Details).

alpha_min, alpha_max
The minimum and maximum of the α parameter of power-exponential.

derivatives_min, derivatives_max
The minimum and maximum of the δ parameter of Matern.

log_obj_tol
An absolute tolerance for terminating the maximization of the log of the objective.

log_obj_diff
The critical value for the change in the log objective for informal tests during optimization of correlation parameters. No testing is done with the default of 0; a larger critical value such as 2 may give a more parsimonious model.

lambda_prior
The rate parameter of an exponential prior for each θ parameter; used only if fit_objective = "Posterior".

model_comparison
The criterion used to select from multiple solutions when tries > 1: the objective function ("Objective") or leave-one-out cross validation ("CV").

Details
Fit numerically maximizes the profile objective function with respect to the correlation parameters; the mean and overall variance parameters are estimated in closed form given the correlation parameters.

A cor_par data frame supplied by the user is the starting point for the first optimization try. If random_error = TRUE, then sp_var / (sp_var + error_var) is another correlation parameter to be optimized; sp_var and error_var values supplied by the user will initialize this parameter for the first try.

Set random_error = TRUE to estimate the variance of the random (measurement, white-noise) error; a small nugget error variance is for numerical stability.

For term j in the stochastic-process model, the estimate of θ_j is constrained between theta_standardized_min / r_j^2 and theta_standardized_max / r_j^2, where r_j is the range of term j. Note that Fit returns unscaled estimates relating to the original, unscaled inputs.

Value
A GaSPModel object, which is a list with the following components:
GaSPModel

Create a GaSPModel object.

Description

Return a template for a GaSPModel object.

Usage

GaSPModel(
  x,
  y,
  reg_model,
  sp_model = NULL,
  cor_family = c("PowerExponential", "Matern"),
  cor_par = NULL,
  random_error = FALSE,
  nugget = 0,
  fit_objective = "Posterior",
  sp_var = NULL,
  error_var = NULL,
  beta = NULL,
  objective = NULL,
  cond_num = NULL,
  CVRMSE = NULL)
GaSPModel

cor_par, random_error = c(FALSE, TRUE),
sp_var, error_var = 0

Arguments

x A data frame containing the input (explanatory variable) training data.
y A vector or a data frame with one column containing the output (response) training data.
reg_model The regression model, specified as a formula, but note the left-hand side of the formula is unused; see example.
sp_model An optional stochastic process model, specified as a formula, but note the left-hand side of the formula and the intercept are unused. The default NULL uses all column names in x.
cor_family A character string specifying the (product, anisotropic) correlation-function family: "PowerExponential" for the power-exponential family or "Matern" for the Matern family.
cor_par A data frame containing the correlation parameters with one row per sp_model term and two columns (see Details).
random_error A boolean for the presence or not of a random (measurement, white-noise) error term.
sp_var The stochastic process variance.
error_var The random error variance, with default 0.

Details

The data frame cor_par contains one row for each term in the stochastic process model. There are two columns. The first is named Theta, and the second is either Alpha (power-exponential) or Derivatives (Matern). Let \( h_j \) be a distance between points for term \( j \) in the stochastic-process model. For power-exponential, the contribution to the product correlation from term \( j \) depends on a distance-scale parameter \( \theta_j \) from the Theta column and a smoothness parameter \( \alpha_j \) from the Alpha column; the contribution is \( \exp(-\theta_j h_j^{2-\alpha_j}) \). For example, \( \alpha_j = 0 \) gives the squared-exponential (Gaussian) correlation. The contribution to the product correlation for Matern also depends on \( \theta_j \), and the second parameter is the number of derivatives \( \delta_j = 0, 1, 2, 3 \) from the Derivatives column. The contribution is \( \exp(-\theta_j h_j) \) for \( \delta_j = 0 \) (the exponential correlation), \( \exp(-\theta_j h_j)((\theta_j h_j)^2/3 + \theta_j h_j + 1) \) for \( \delta_j = 1 \), \( \exp(-\theta_j h_j)((\theta_j h_j)^2/3 + \theta_j h_j + 1) \) for \( \delta_j = 2 \), and \( \exp(-\theta_j h_j^3) \) for \( \delta_j = 3 \) (the squared-exponential correlation). Note that \( \delta_j = 3 \) codes for a limiting infinite number of derivatives. This is not the usual parameterization of the Matern, but it is consistent with power-exponential for the exponential and squared-exponential special cases common to both.

A value should be given to error_var if the model has a random-error term (random_error = TRUE), and a small "nugget" such as \( 10^{-9} \) may be needed for improved numerical conditioning.
Value

A GaSPModel object, which is a list with the following components:

- **x**: The data frame containing the input training data.
- **y**: The training output data, now as a vector.
- **reg_model**: The regression model, now in the form of a data frame.
- **sp_model**: The stochastic process model, now in the form of a data frame.
- **cor_family**: The correlation family.
- **cor_par**: The data frame containing the correlation parameters.
- **random_error**: The boolean for the presence or not of a random error term.
- **sp_var**: The stochastic process variance.
- **error_var**: The random error variance.
- **beta**: A placeholder for a data frame to hold the regression-model parameters.
- **objective**: A placeholder for the maximum fit objective.
- **cond_num**: A placeholder for the condition number.
- **CVRMSE**: A placeholder for the model’s cross-validated root mean squared error.

Note

This function does not execute **Fit** and is intended for **CrossValidate**, **Predict** and **Visualize** with models trained otherwise by the user. Placeholders do not need to be specified to execute these further functions, as they are always recomputed as needed.

References


Examples

```r
x <- borehole$x
y <- borehole$y
theta <- c(
  5.767699e+01, 0.000000e+00, 0.000000e+00, 1.433571e-06,
  0.000000e+00, 2.366557e-06, 1.695619e-07, 2.454376e-09
)
alpha <- c(
  1.110223e-16, 0.000000e+00, 0.000000e+00, 0.000000e+00,
  0.000000e+00, 0.000000e+00, 2.494862e-03, 0.000000e+00
)
cor_par <- data.frame(Theta = theta, Alpha = alpha)
rownames(cor_par) <- colnames(borehole$x)
sp_var <- 38783.7
borehole_gasp <- GaSPModel(
  x = borehole$x, y = borehole$y,
  reg_model = ~1, cor_family = "PowerExponential",
  cor_par = cor_par, random_error = FALSE,
)```
sp_var = sp_var
)

---

PlotAll  

Description

Execute PlotPredictions, PlotResiduals, PlotStdResiduals, PlotMainEffects, and PlotJointEffects.

Usage

PlotAll(
  GaSP_model,  
cross_validation,  
visualization,  
y_name = "y",  
y_units = "",  
x_units = NULL,  
se_plot = TRUE,  
y_values = NULL,  
se_values = NULL,  
pch = 1
)

Arguments

GaSP_model  
  Object of class GaSPModel, the entire model will be verified but only x and y will be used.

cross_validation  
  A data frame returned by CrossValidate.

visualization  
  A list object returned by Visualize.

y_name  
  An optional character string containing the output variable name (for labels).

y_units  
  An optional character string containing the units of the output variable (for labels).

x_units  
  An optional vector of character strings containing the units of the input variables (for labels).

se_plot  
  An optional boolean indicating whether to make standard-error contour plots.

y_values  
  An optional vector of contour values for the estimated joint effects.

se_values  
  An optional vector of contour values for the standard errors.

pch  
  Plotting symbol for plot; default is open circle.
**Value**

No return value, generates plots.

**Examples**

```
PlotAll(borehole_fit, borehole_cv, borehole_vis)
```

---

**PlotJointEffects**  
*Plot the estimated joint effects.*

**Description**

Plot the estimated joint effects.

**Usage**

```
PlotJointEffects(
  joint_effect,  
anova_percent,  
x_units = NULL,  
y_name = "y",  
y_units = "",  
se_plot = TRUE,  
y_values = NULL,  
se_values = NULL
)
```

**Arguments**

- `joint_effect`  
  A data frame from `Visualize` with plotting coordinates for the estimated joint effects.

- `anova_percent`  
  A data frame from `Visualize` of ANOVA percentages.

- `x_units`  
  An optional vector of character strings containing the units of the input variables (for labels).

- `y_name`  
  An optional character string containing the output variable name (for labels).

- `y_units`  
  An optional character string containing the units of the output variable (for labels).

- `se_plot`  
  An optional boolean indicating whether to make standard-error contour plots.

- `y_values`  
  An optional vector of contour values for the estimated joint effects.

- `se_values`  
  An optional vector of contour values for the standard errors.

**Details**

Plots are sent to the active device.
PlotMainEffects

Value
No return value, generates plots.

Examples

PlotJointEffects(borehole_vis$joint_effect, borehole_vis$anova_percent)

Description
Plot the estimated main effects.

Usage
PlotMainEffects(
  main_effect,  
anova_percent,  
x_units = NULL,  
y_name = "y",  
y_units = ""
)

Arguments

main_effect     A data frame from Visualize with plotting coordinates for the estimated main effects.
anova_percent  A data frame from Visualize of ANOVA percentages.
x_units        An optional vector of character strings containing the units of the input variables (for labels).
y_name          An optional character string containing the output variable name (for labels).
y_units         An optional character string containing the units of the output variable (for labels).

Details
Plots are sent to the active device. Each plot shows an estimated main effect (red solid line) and pointwise approximate 95% confidence limits (green dashed line).

Value
No return value, generates plots.
PlotPredictions

Examples

PlotMainEffects(borehole_vis$main_effect, borehole_vis$anova_percent)

PlotPredictions Plot true versus predicted output.

Description

Plot true versus predicted output (response) made by Predict or CrossValidate.

Usage

PlotPredictions(
  y_pred, y,
  y_name = "y",
  y_units = "",
  title = c("Predict", "CrossValidate"),
  pch = 1
)

Arguments

y_pred A data frame of predicted output values made by Predict or CrossValidate.
y A vector of length equal to the number of rows in y_pred containing the true output values.
y_name An optional character string containing the output variable name (for labels).
y_units An optional character string containing the units of the output variable (for labels).
title A character string for the name of the function generating the predictions (for an appropriate title): "Predict" from Predict or "CrossValidate" from CrossValidate; "" for no title.
pch Plotting symbol for plot; default is open circle.

Value

No return value, generates plots.

Examples

PlotPredictions(borehole_cv, y, title = "CrossValidate")

PlotPredictions(borehole_pred$y_pred, borehole$y_true, title = "Predict")
PlotQQ

Normal Q-Q plot.

Description

Normal Q-Q plot of the standardized residuals of predictions from Predict or CrossValidate.

Usage

PlotQQ(y_pred, y, y_name = "y")

Arguments

y_pred A data frame of predicted output values made by Predict or CrossValidate.
y A vector of length equal to the number of rows in y_pred containing the true output values.
y_name An optional character string containing the output variable name (for labels).

Value

No return value, generates plots.

Examples

PlotQQ(borehole_cv, y)

PlotResiduals

Plot residuals versus each input variable.

Description

Plot residuals versus each input variable.

Usage

PlotResiduals(
  x,
  y_pred,
  y,
  x_units = NULL,
  y_name = "y",
  y_units = "",
  pch = 1
)
PlotStdResiduals

Arguments

x  A data frame with number of rows equal to the number of rows in y_pred containing the input (explanatory) variables.
y_pred  A data frame of predicted output values made by Predict or CrossValidate.
y  A vector of length equal to the number of rows in y_pred containing the true output values.
x_units  An optional vector of character strings containing the units of the input variables in x (for labels).
y_name  An optional character string containing the output variable name (for labels).
y_units  An optional character string containing the units of the output variable (for labels).
pch  Plotting symbol for plot; default is open circle.

Value

No return value, generates plots.

Examples

PlotResiduals(x, borehole_cv, y)

PlotStdResiduals  Plot standardized residuals versus predictions.

Description

Plot standardized residuals versus predictions made by Predict or CrossValidate.

Usage

PlotStdResiduals(
  y_pred,  
y,  
y_name = "y",  
y_units = "",  
title = c("Predict", "CrossValidate"),  
pch = 1
)
Arguments

- `y_pred`: A data frame of predicted output values made by `Predict` or `CrossValidate`.
- `y`: A vector of length equal to the number of rows in `y_pred` containing the true output values.
- `y_name`: An optional character string containing the output variable name (for labels).
- `y_units`: An optional character string containing the units of the output variable (for labels).
- `title`: A character string for the name of the function generating the predictions (for an appropriate title): "Predict" from `Predict` or "CrossValidate" from `CrossValidate`; "" for no title.
- `pch`: Plotting symbol for plot; default is open circle.

Value

No return value, generates plots.

Examples

```r
PlotStdResiduals(borehole_cv, y, title = "CrossValidate")
```

---

**Predict**

*Predict from a GaSPModel object.*

Description

Predict from a `GaSPModel` object.

Usage

```r
Predict(GaSP_model, x_pred, generate_coefficients = c(FALSE, TRUE))
```

Arguments

- `GaSP_model`: Object of class `GaSPModel`.
- `x_pred`: A data frame containing the values of the input variables at which to predict the output.
- `generate_coefficients`: A boolean indicating whether coefficients for further external predictions are generated.
**Value**

A list with the following elements:

- **y_pred**: A data frame with two columns: the predictions `Pred` and their standard errors `SE`.
- **pred_coeffs**: A vector of coefficients for further predictions; `NULL` if `generate_coefficients` is `FALSE`.

**Note**

The vector of prediction coefficients in `pred_coeffs` can be used as follows. Let $c$ denote the coefficients and let $r$ denote a vector with element $i$ containing the correlation between the output at a given new point and the output at training point $i$. Then the prediction for the output at the new point is the dot product of $c$ and $r$.

**RMSE** computes the root mean squared error of the predictions. `PlotPredictions` and `PlotResiduals` plot the predictions or their residuals; `PlotStdResiduals` and `PlotQQ` plot the standardized residuals.

**Examples**

```r
borehole_pred <- Predict(
  GaSP_model = borehole_fit,
  x_pred = borehole$x_pred,
  generate_coefficients = TRUE
)
```

---

**RMSE**

*Calculate the root mean squared error (RMSE) of prediction*

**Description**

Calculate the root mean squared error (RMSE) of prediction

**Usage**

```r
RMSE(y_pred, y_true, normalized = FALSE)
```

**Arguments**

- **y_pred**: A vector of predicted output values.
- **y_true**: A vector of true output values.
- **normalized**: An optional boolean: if `TRUE`, the RMSE is normalized by dividing it by the standard deviation of `y_true`. 
Visualize

Value

The RMSE or normalized RMSE.

Examples

\[
\text{RMSE(borehole\_pred\$y\_pred\$Pred, borehole\$y\_true)}
\]

\[
\text{RMSE(borehole\_cv\$Pred, y)}
\]

Description

Carry out a functional analysis of variance (ANOVA) of a GaSPModel object and generate plotting coordinates for its estimated main and 2-input joint effects.

Usage

Visualize(GaSP\_model, x\_description, main\_percent = 0, interaction\_percent = 0)

Arguments

GaSP\_model Object of class GaSPModel.

x\_description A data frame describing the input variables. See DescribeX.

main\_percent An optional minimum percentage of variation explained by an input’s main effect to return the effect’s plotting coordinates; the default of zero gives plotting coordinates for all inputs.

interaction\_percent An optional minimum percentage of variation explained by the interaction effect of a pair of inputs to return the plotting coordinates for their joint effect (main effects plus interaction effect); the default of zero gives plotting coordinates for all pairs of inputs.

Details

If there are many inputs, to avoid excessive plotting of many trivial joint effects set interaction\_percent = 1 say.

Value

A list with the following elements:

anova\_percent A data frame containing the ANOVA percentages for all main effects and 2-input interaction effects.

main\_effect A data frame with plotting coordinates for the estimated main effects.
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<th>Description</th>
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<td>joint_effect</td>
<td>A data frame with plotting coordinates for the estimated 2-input joint effects.</td>
</tr>
<tr>
<td>total_percent</td>
<td>Total percentage of the prediction variation accounted for by all main effects and 2-input interaction effects.</td>
</tr>
<tr>
<td>average</td>
<td>Overall average of the prediction function, averaged with respect to all inputs.</td>
</tr>
<tr>
<td>SE_average</td>
<td>Standard error of the overall average.</td>
</tr>
</tbody>
</table>

References


Examples

```r
borehole_vis <- Visualize(borehole_fit, borehole_x_desc)
```
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