Package ‘MixedPoisson’

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Title Mixed Poisson Models

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Description The estimation of the parameters in mixed Poisson models.

License GPL-2

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MixedPoisson-package *Mixed Poisson Models*

Description

The package provides functions, which support to fit parameters of different mixed Poisson models using the Expectation-Maximization (EM) algorithm of estimation, cf. (Ghitany et al., 2012, pp. 6848). In the model the assumptions are: conditional $N|\theta$ is of distribution $N|\theta \sim POIS(\lambda\theta)$, parameter θ is a random variable distributed according to the density function $f_\theta(\cdot)$, $E[\theta] = 1$ and $\lambda = \exp(\mathbf{x}_i'\boldsymbol{\beta})$ – the regression component. The E-step is carried out through the numerical integration using Laquerre quadrature. The M-step estimates the parameters β using GLM Poisson with pseudo values from E-step and mixing parameters using optimize function.

Details

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Author(s)

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References

Karlis, D. (2005). EM algorithm for mixed Poisson and other discrete distributions. Astin Bulletin, 35(01), 3-24. Ghitany, M. E., Karlis, D., Al-Mutairi, D. K., & Al-Awadhi, F. A. (2012). An EM algorithm for multivariate mixed Poisson regression models and its application. Applied Mathematical Sciences, 6(137), 6843-6856.

est.delta *Estimation of delta parameter of inverse-Gaussian distribution*

Description

The function estimates the value of the parameter delta using `optimize`.

Usage

```
est.delta(t)
```

Arguments

`t`	the vector of values

Details

The form of the distribution is as in the function `ll.invGauss`

Value

`nu`	the estimates of ν
`ll.delta.max`	the value of loglikehood

Author(s)

Michal Trzesiok

Examples

```
est.delta(t=c(3,8))
```

`est.gamma` *Estimation of gamma parameter of Gamma distribution*

Description

The function estimates the value of the parameter gamma using `optimize`.

Usage

```
est.gamma(t)
```

Arguments

`t`	the vector of values

Details

The form of the distribution is as in the function `ll.gamma`

Value

`gamma`	the estimates of γ
`ll.gamma.max`	the value of loglikehood

Author(s)

Michal Trzesiok

Examples

```
est.gamma(t=c(3,8))
```

`est.nu` *Estimation of nu parameter of log-normal distribution*

Description

The function estimates the value of the parameter nu using `optimize`.

Usage

```
est.nu(t)
```

Arguments

`t`	the vector of values

Details

The form of the distribution is as in the function `ll.lognorm`

Value

`nu`	the estimates of ν
`ll.nu.max`	the value of loglikehood

Author(s)

Michal Trzesiok

Examples

```
est.nu(t=c(3,8))
```

`Gamma.density` *Gamma density*

Description

The function returns the vector of values of density function for of Gamma distribution with one parameter γ.

Usage

```
Gamma.density(theta, gamma.par)
```

Arguments

`theta`	the vector of values
`gamma.par`	the parameter of Gamma distribution

Details

The pdf of Gamma is of the form $f_\theta(\theta) = \frac{\gamma^\gamma}{\Gamma(\gamma)}\theta^{\gamma-1}\exp(-\gamma\theta)$

Value

`Gamma.density(theta, nu)`
the density – the vector of values

Author(s)

Michal Trzesiok

Examples

```
Gamma.density(c(2,3,5,4,6,7,4), 5)
```

`invGauss.density` *inverse-Gaussian Density*

Description

The function returns the vector of values of density function for of inverse-Gaussian distribution with one parameter δ.

Usage

```
invGauss.density(theta, delta)
```

Arguments

theta	the vector of values
delta	the parameter of inverse-Gaussian distribution

Details

The pdf of inverse-Gaussian is of the form $f_\theta(\theta) = \frac{\delta}{2\pi}\exp(\delta^2)\theta^{-\frac{3}{2}}\exp(-\frac{\delta^2}{2}(\frac{1}{\theta}+\theta))$

Value

`invGauss.density(theta, delta)`
the density – the vector of values

Author(s)

Michal Trzesiok

Examples

```
invGauss.density(c(2,3,5,4,6,7,6), 5)
```

`lambda_m_step` *Estimation of Lambda in M-step – Expectation-Maximization (EM) algorithm*

Description

The function fits the GLM Poisson with given offset.

Usage

```
lambda_m_step(variable, X, offset)
```

Arguments

variable	the vector of numbers
X	model matrix of the form $X = model.matrix(\ regressor)$. In the model without regressor the X sould be defined as $X = as.matrix(rep(1, length(variable)))$
offset	offset in GLM Poisson

Details

It fits the GLM Poisson, where $variable \sim 1$ and the ofsset is given as the vector of the variable's length. The results are used in M-step of EM algorithm, cf. [Karlis, 2012] pp. 6850.

Value

`lambda`	$\hat{\lambda} = \hat{\beta} X$
`beta`	regressor parameters
`glm`	output of `glm`

Author(s)

Alicja Wolny–Dominiak, Michal Trzesiok

Examples

```
set.seed(1234)
variable=rpois(50,4)
X=as.matrix(rep(1, length(variable)))
t=pseudo_values(variable, mixing=c("invGauss"), lambda=4, delta=1, n=100)
lambda_m_step(variable, X, offset=t$pseudo_values)
```

`lambda_start` *Estimation of starting lambda in Expectation-Maximization (EM) algorithm*

Description

The function fits the GLM Poisson without regressors.

Usage

```
lambda_start(variable, X)
```

Arguments

`variable`	the vector of numbers
`X`	model matrix of the form $X = model.matrix(regressor)$. In the model without regressor the X sould be defined as $X = as.matrix(rep(1, length(variable)))$

Details

It fits the GLM Poisson, where $variable \sim 1$. The results are taken as the starting value of EM algorithm.

Value

`lambda`	$\hat{\lambda} = \hat{\beta} X$
`beta`	regressor parameters
`glm`	output of `glm`

Author(s)

Alicja Wolny–Dominiak, Michal Trzesiok

Examples

```
set.seed(1234)
variable=rpois(50,4)
X=as.matrix(rep(1, length(variable)))
t=pseudo_values(variable, mixing=c("invGauss"), lambda=4, delta=1, n=100)
lambda_m_step(variable, X, offset=t$pseudo_values)
```

`ll.gamma` *Gamma Log-likelihood*

Description

The function returns the value of log-likelihood function for of Gamma distribution with one parameter γ.

Usage

```
ll.gamma(gamma.par, t)
```

Arguments

`gamma.par`	γ parameter
`t`	the vector of values

Details

The pdf of Gamma is of the form $f_\theta(\theta) = \frac{\gamma^\gamma}{\Gamma(\gamma)}\theta^{\gamma-1}\exp(-\gamma\theta)$

Value

`ll.gamma`	the value

Author(s)

Michal Trzesiok

Examples

```
ll.gamma(1, c(3,8))
```

`ll.invGauss` *Inverse-Gaussian Log-likelihood*

Description

The function returns the value of log-likelihood function for of inverse-Gaussian distribution with one parameter δ.

Usage

```
ll.invGauss(delta, t)
```

Arguments

`delta`	δ parameter
`t`	the vector of values

Details

The pdf of inverse-Gaussian is of the form $f_\theta(\theta) = \frac{\delta}{2\pi}\exp(\delta^2)\theta^{-\frac{3}{2}}\exp(-\frac{\delta^2}{2}(\frac{1}{\theta}+\theta))$

Value

`ll.invGauss`	the value

Author(s)

Michal Trzesiok

Examples

```
ll.invGauss(1, c(3,8))
```

`ll.lognorm` *Log-normal Log-likelihood*

Description

The function returns the value of log-likelihood function of log-normal distribution with one parameter ν.

Usage

```
ll.lognorm(nu, t)
```

Arguments

`nu`	ν parameter
`t`	the vector of values

Details

The pdf of log-normal is of the form $f_\theta(\theta) = \frac{1}{\sqrt{2\pi\nu\theta}} \exp[-\frac{(\log(\theta)+\frac{\nu^2}{2})^2}{2\nu^2}]$

Value

`ll.lognorm`	the value

Author(s)

Michal Trzesiok

Examples

```
ll.lognorm(1, c(3,8))
```

`lognorm.density` *Log-normal Density*

Description

The function returns the vector of values of density function for of log-normal distribution with one parameter ν.

Usage

```
lognorm.density(theta, nu)
```

Arguments

`theta`	the vector of values
`nu`	the parameter of log-normal distribution

Details

The pdf of log-normal is of the form $f_\theta(\theta) = \frac{1}{\sqrt{2\pi\nu\theta}} \exp[-\frac{(\log(\theta)+\frac{\nu^2}{2})^2}{2\nu^2}]$

Value

`lognorm.density(theta, nu)`
the density – the vector of values

Author(s)

Michal Trzesiok

Examples

```
lognorm.density(c(2,3,5,4,6,7,6), 5)
```

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pg.dist *Poisson-Gamma Distribution (Negative-Binomial)*

Description

The function fits a mixed Poisson distribution, in which the random parameter follows Gamma distribution (the negative-binomial distribution). As teh method of estimation Expectation-maximization algorithm is used. In M-step the analytical formulas taken from [Karlis, 2005] are applied.

Usage

```
pg.dist(variable, alpha.start, beta.start, epsylon)
```

Arguments

`variable`	The count variable.
`alpha.start`	The starting value of the parameter alpha. Default to 1.
`beta.start`	The starting value of the parameter beta. Default to 0.3
`epsylon`	Default to epsylon = 10^(-8)

Details

This function provides estimated parameters of the model $N|\lambda \sim Poisson(\lambda)$ where λ parameter is also a random variable follows Gamma distribution with hiperparameters α, β. The pdf of Gamma is of the form $f_\lambda(\lambda) = \frac{\lambda^{\alpha-1}\exp(-\beta\lambda)\beta^\lambda}{\Gamma(\alpha)}$.

Value

`alpha`	the parameter of mixing Gamma distribution
`beta`	the parameter of mixing Gamma distribution
`theta`	the value 1/beta
`n.iter`	the number of steps in EM algorithm

References

Karlis, D. (2005). EM algorithm for mixed Poisson and other discrete distributions. Astin bulletin, 35(01), 3-24.

Examples

```
library(MASS)
pGamma1 = pg.dist(variable=quine$Days)
print(pGamma1)
```

`pl.dist` *Poisson-Lindley Distribution*

Description

The function fits a mixed Poisson distribution, in which the random parameter follows Lindley distribution. As teh method of estimation Expectation-maximization algorithm is used.

Usage

```
pl.dist(variable, p.start, epsylon)
```

Arguments

`variable`	The count variable.
`p.start`	The starting value of p parameter. Default to 0.1.
`epsylon`	Default to epsylon = 10^(-8)

Details

This function provides estimated parameters of the model $N|\lambda \sim Poisson(\lambda)$ where λ parameter is also a random variable follows Lindley distribution with hiperparameter p. The pdf of Lindley is of the form $f_\lambda(\lambda) = \frac{p^2}{p+1}(\lambda + 1)\exp(-\lambda p)$.

Value

`p`	the parameter of mixing Lindley distribution
`n.iter`	the number of steps in EM algorithm

References

Karlis, D. (2005). EM algorithm for mixed Poisson and other discrete distributions. Astin bulletin, 35(01), 3-24.

Examples

```
library(MASS)
pLindley = pl.dist(variable=quine$Days)
print(pLindley)
```

`pseudo_values` *Pseudo values – Expectation-Maximization (EM) algorithm*

Description

The function returns the pseudo values t_i defined as the conditional expectation $E[\theta_i|k_1, ..., k_n]$, where $k_1, ..., k_n$ are realizations of the count variable N.

Usage

```
pseudo_values(variable, mixing, lambda, gamma.par, nu, delta, n)
```

Arguments

`variable`	the vector of numbers
`mixing`	the name of mixing distribution – "Gamma", "lognorm", "invGauss"
`lambda`	λ parameter in mixed Poisson model
`gamma.par`	γ parameter in Gamma mixing distribution
`nu`	ν parameter in log-normal mixing distribution
`delta`	δ parameter in inverse-Gaussian mixing distribution
`n`	The integer value for the Laguerre quadrature. Default to 100

Details

The function calculates the vector of pseudo values $t_i = E[\theta_i|k_1, ..., k_n]$ in E-step of EM algorithm. It applies the numerical integration using $laguerre.quadrature$ in the nominator and the denominator of the formula

The proper parameter γ, ν, δ should be chosen according to the mixing distribution.

Value

`pseudo_values`	pseudo values $t_1, ..., t_n$
`nominator`	nominator in the formula
`denominator`	denominator in the formula

Author(s)

Alicja Wolny–Dominiak, Michal Trzesiok

Examples

```
variable=rpois(30,4)
pseudo_values(variable, mixing="Gamma", lambda=4, gamma.par=0.7, n=100)
```

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