Package ‘QPot’

April 6, 2019

Version 1.5

Date 2019-03-24

Title Quasi-Potential Analysis for Stochastic Differential Equations

Depends R (>= 3.0.2)

Imports MASS

Suggests R.rsp, R.devices,phaseR,plot3D,viridis,markdown

VignetteBuilder R.rsp

Maintainer Christopher Stieha <stieha@hotmail.com>

Description Tools to 1) simulate and visualize stochastic differential equations and 2) determine stability of equilibria using the ordered-upwind method to compute the quasi-potential.

License GPL-2

URL https://www.r-project.org, https://github.com/bmarkslash7/QPot

BugReports https://github.com/bmarkslash7/QPot/issues

RoxygenNote 5.0.0

NeedsCompilation yes

Author Christopher Moore [aut],
Christopher Stieha [aut, cre],
Ben Nolting [aut],
Maria Cameron [aut],
Karen Abbott [aut],
James Gregson [cph] (author of expression_parser library: https://github.com/jamesgregson/expression_parser)

Repository CRAN

Date/Publication 2019-04-06 14:42:43 UTC

R topics documented:

Model2String 2
QPContour 3

QPGlobal 6
QPInterp 7
QPotential 8
TSDensity 10
TSPlot 11
TSTraj 12
VecDecomAll 14
VecDecomGrad 15
VecDecomPlot 16
VecDecomRem 18
VecDecomVec 19

Index **21**

`Model2String` *Inserts parameter values into equations*

Description

Converts differential equations from string-format (or function-format) with parameters (e.g. "a*x+b) to string-format with parameter values (e.g. 2*x+3). Specifically, Model2String reads in the equations, searches for the differential equations within the function (if required), and replaces the parameters with numerical values given by the user. Returns an array of strings containing the differential equations. This code is specifically given so that any problems can be found in R as opposed to receiving terse errors from the C code if this was built into the QPotential() function.

Usage

```
Model2String(model = NULL, parms = NULL, deSolve.form = FALSE,
  x.lhs.term = "dx", y.lhs.term = "dy", supress.print = FALSE,
  width.cutoff = 500)
```

Arguments

`model`	contains the differential equations as given to `TSTraj`. Can either be a string or a function used by the package `deSolve` (see third example).
`parms`	a named vector of paramters and their respective values for the deterministic equations. If inputing a function and parms is empty, Model2String will return the equation as a string.
`deSolve.form`	if FALSE (default) interprets model as a string containing the rhs of the equation. If TRUE, treats model as the function input into the package `deSolve`.
`x.lhs.term`	string containing the left hand side of the first equation to search for if deSolve.form is TRUE. Default is 'dx'.
`y.lhs.term`	string containing the left hand side of the second equation to search for if deSolve.form is TRUE. Default is 'dy'.
`supress.print`	if FALSE (default), suppress output. TRUE prints out equations converted to strings with parameters replaced with values.

`width.cutoff` parameter width.cutoff from `deparse` in package:base. Determines the cutoff in bytes at which line breaking is tried. Default is 500 with possible range of 2 to 500.

Value

equations: an array with strings of the equations with substituted parameter values. If deSolve.form is TRUE, the first is the x equation, the second is the y equation.

Examples

```
# First example with the right hand side of an equation
test.eqn.x = "(alpha*x)*(1-(x/beta)) - ((delta*(x^2)*y)/(kappa + (x^2)))"
model.parms <- c(alpha=1.54, beta=10.14, delta=1, kappa=1)
equations.as.strings.x <- Model2String(test.eqn.x, parms = model.parms)

# Second example with individual strings with left and right hand sides
# Note the use deSolve.form = TRUE to remove the dx and dy
test.eqn.x = "dx = (alpha*x)*(1-(x/beta)) - ((delta*(x^2)*y)/(kappa + (x^2)))"
test.eqn.y = "dy = ((gamma*(x^2)*y)/(kappa + (x^2))) - mu*(y^2)"
model.parms <- c(alpha=1.54, beta=10.14, delta=1, kappa=1, gamma=0.476, mu=0.112509)
equations.as.strings.x <- Model2String(test.eqn.x, parms = model.parms,
 deSolve.form = TRUE, x.lhs.term = 'dx', y.lhs.term = 'dy')
equations.as.strings.y <- Model2String(test.eqn.y, parms = model.parms,
 deSolve.form = TRUE, x.lhs.term = 'dx', y.lhs.term = 'dy')

# Third example with deSolve-style function call:
model.parms <- c(alpha=1.54, beta=10.14, delta=1, kappa=1, gamma=0.476, mu=0.112509)
ModelEquations <- function(t, state, parms) {
with(as.list(c(state, parms)), {
dx <- (alpha*x)*(1-(x/beta)) - ((delta*(x^2)*y)/(kappa + (x^2)))
dy <- ((gamma*(x^2)*y)/(kappa + (x^2))) - mu*(y^2)
list(c(dx,dy))
})
}

Model2String(ModelEquations, parms = model.parms, deSolve.form = TRUE,
 x.lhs.term = 'dx', y.lhs.term = 'dy')
```

`QPContour` *Contour plot of quasi-potential surfaces*

Description

This function allows users to create a contour plot of quasi-potential surfaces from `QPGlobal`

Usage

```
QPContour(surface, dens, x.bound, y.bound, xlim = NULL, ylim = NULL,
  n.filled.contour = 25, n.contour.lines = 25, c.parm = 1, col.contour,
  contour.lines = TRUE, xlab = "X", ylab = "Y", contour.lwd = 1, ...)
```

Arguments

`surface`	the surface to be plotted, from `QPGlobal`.
`dens`	vector respectively for the number of x and y points to be plotted.
`x.bound`	a two-element vector with the minimum and maximum x values used for computing the quasi-potential.
`y.bound`	a two-element vector with the minimum and maximum y values used for computing the quasi-potential.
`xlim`	numeric vectors of length 2, giving the x coordinate range. Default = `NULL` automatically sizes plot window.
`ylim`	numeric vectors of length 2, giving the y coordinate range. Default = `NULL` automatically sizes plot window.
`n.filled.contour`	numeric value for the nubmber of breaks in the filled contour.
`n.contour.lines`	numeric value for the number of breaks in the contour lines.
`c.parm`	contour line adjustment (see details).
`col.contour`	colors to interpolate; must be a valid argument to `col2rgb`.
`contour.lines`	if `TRUE`, then contour lines plotted over filled contour; vice versa if `FALSE`.
`xlab`	a title for the x axis. Default is 'X'
`ylab`	a title for the y axis. Default is 'Y'
`contour.lwd`	line width of contour lines.
`...`	passes arguments to `plot`.

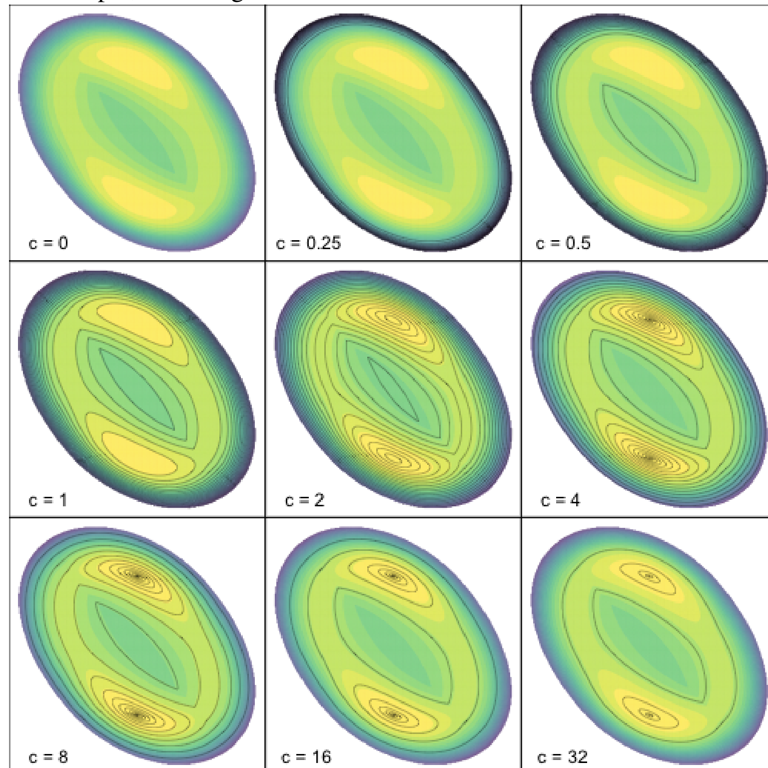
Details

Because, in general, capturing the topological features of a surface can be subtle, we implemented a feature in `QPContour` to keep the filled contour region while changing the contour lines. Specifically, `filled.contour` takes the range of the surface values (ϕ), divides by the number of the specified contours (i.e., `n.filled.contour`), and creates a contour at each break, which happenes to be equal across the range. But because visualizing some topology may (i) require looking between contour breaks and (ii) adding contour lines would overload the plot with lines, we use an equation to modify the distribution of contour lines. Namely, adjusting the `c` argument in the `QPContour` function adjusts the c paramter in the following equation:

$$max_\phi \times \left(\frac{x}{n-1}\right)^c.$$

This allows the user to keep the same number of contour lines (i.e., specified with `n.contour.lines`), but focus them toward the troughs or peaks of the surfaces. At $c = 1$, the contour lines correspond to the filled.contour breaks. If $c > 1$, then the contour lines become more concentrated towards the trough. Similarly, if $c < 1$, then the contour lines are more focused at the peaks of the surface. As

an example, we change c :



Examples

```
# First, System of equations
equationx <- "1.54*x*(1.0-(x/10.14)) - (y*x*x)/(1.0+x*x)"
equationy <- "((0.476*x*x*y)/(1+x*x)) - 0.112590*y*y"

# Second, shared parameters for each quasi-potential run
xbounds <- c(-0.5, 10.0)
ybounds <- c(-0.5, 10.0)
xstepnumber <- 150
ystepnumber <- 150

# Third, first local quasi-potential run
xinit1 <- 1.40491
yinit1 <- 2.80808
storage.eq1 <- QPotential(x.rhs = equationx, x.start = xinit1,
x.bound = xbounds, x.num.steps = xstepnumber, y.rhs = equationy,
y.start = yinit1, y.bound = ybounds, y.num.steps = ystepnumber)

# Fourth, second local quasi-potential run
xinit2 <- 4.9040
yinit2 <- 4.06187
storage.eq2 <- QPotential(x.rhs = equationx, x.start = xinit2,
x.bound = xbounds, x.num.steps = xstepnumber, y.rhs = equationy,
```

```
y.start = yinit2, y.bound = ybounds, y.num.steps = ystepnumber)

# Fifth, determine global quasi-potential
unst.x <- c(0, 4.2008)
unst.y <- c(0, 4.0039)
ex1.global <- QPGlobal(local.surfaces = list(storage.eq1, storage.eq2),
unstable.eq.x = unst.x, unstable.eq.y = unst.y, x.bound = xbounds,
y.bound = ybounds)

# Sixth, contour of the quasi-potential
QPContour(ex1.global, dens = c(100,100), x.bound = xbounds,
y.bound = ybounds, c.parm = 5)
```

QPGlobal *Finding the global quasi-potential*

Description

This function allows you to find the global quasi-potential values for several local quasi-potential surfaces

Usage

```
QPGlobal(local.surfaces, unstable.eq.x, unstable.eq.y, x.bound, y.bound)
```

Arguments

`local.surfaces`	a list of local quasi-potential surfaces, each of which is stored in discretized form as a matrix.
`unstable.eq.x`	a vector of the x-coordinates of the unstable equilibria. Must be in the same order as unstable.eq.y.
`unstable.eq.y`	a vector of the y-coordinates of the unstable equilibria. Must be in the same order as unstable.eq.x.
`x.bound`	a two-element vector with the minimum and maximum x values used for computing the quasi-potential.
`y.bound`	a two-element vector with the minimum and maximum y values used for computing the quasi-potential.

Examples

```
# First, System of equations
equationx <- "1.54*x*(1.0-(x/10.14)) - (y*x*x)/(1.0+x*x)"
equationy <- "((0.476*x*x*y)/(1+x*x)) - 0.112590*y*y"

# Second, shared parameters for each quasi-potential run
xbounds <- c(-0.5, 10.0)
ybounds <- c(-0.5, 10.0)
xstepnumber <- 100
```

```
ystepnumber <- 100

# Third, first local quasi-potential run
xinit1 <- 1.40491
yinit1 <- 2.80808
storage.eq1 <- QPotential(x.rhs = equationx, x.start = xinit1,
x.bound = xbounds, x.num.steps = xstepnumber, y.rhs = equationy,
y.start = yinit1, y.bound = ybounds, y.num.steps = ystepnumber)

# Fourth, second local quasi-potential run
xinit2 <- 4.9040
yinit2 <- 4.06187
storage.eq2 <- QPotential(x.rhs = equationx, x.start = xinit2,
x.bound = xbounds, x.num.steps = xstepnumber, y.rhs = equationy,
y.start = yinit2, y.bound = ybounds, y.num.steps = ystepnumber)

# Fifth, determine global quasi-potential
unst.x <- c(0, 4.2008)
unst.y <- c(0, 4.0039)
ex1.global <- QPGlobal(local.surfaces = list(storage.eq1, storage.eq2),
unstable.eq.x = unst.x, unstable.eq.y = unst.y, x.bound = xbounds,
y.bound = ybounds)
```

`QPInterp` *Quasi-potential interpolation*

Description

This function estimates the quasi-potential value for any x- and y-values

Usage

```
QPInterp(X, Y, x.bound, y.bound, surface)
```

Arguments

`X`	the value of the x to interpolate.
`Y`	the value of the y to interpolate.
`x.bound`	a two-element vector with the minimum and maximum x values used for computing the quasi-potential.
`y.bound`	a two-element vector with the minimum and maximum y values used for computing the quasi-potential.
`surface`	the surface to interpolated, from `QPGlobal`.

Details

this function uses bilinear interpolation for estimattion of any x- and y-value.

`QPotential` *Computes the quasi-potential for a system of stochastic differential equations using the ordered upwind method.*

Description

Computes the quasi-potential for a system of stochastic differential equations using the ordered upwind method.

Usage

```
QPotential(x.rhs = NULL, x.start = NULL, x.bound = NULL,
  x.num.steps = NULL, y.rhs = NULL, y.start = NULL, y.bound = NULL,
  y.num.steps = NULL, filename = NULL, save.to.R = TRUE,
  save.to.HD = FALSE, bounce = "d", bounce.edge = 0.01,
  verboseR = FALSE, verboseC = TRUE, debugC = FALSE, k.x = 20,
  k.y = 20, INFINITY = 1e+06)
```

Arguments

`x.rhs`	a string containing the right hand side of the equation for x.
`x.start`	the starting value of x, usually the x value of the current equilibrium.
`x.bound`	the x boundaries denoted as c(minimum, maximum).
`x.num.steps`	the number of steps between the minimum and maximum x value defined in x range.
`y.rhs`	a string containing the right hand side of the equation for y.
`y.start`	the starting value of y, usually the y value of the current equilibrium.
`y.bound`	the y boundaries denoted as c(minimum, maximum).
`y.num.steps`	the number of steps between the minimum and maximum y value defined in y range.
`filename`	string for the name of the file saved to the hard drive. If save.to.HD=TRUE and filename is left blank, output file saved as defaultname-xX.STARTyY.START.txt, where X.START and Y.START are values in x.start and y.start, respectively. Matrix stored as a tab-delimited file.
`save.to.R`	boolean to output the matrix of results for the ordered upwind method to the current R session. The default is to write the matrix to the R session. save.to.R=FALSE prevents the output from being written to the R session.
`save.to.HD`	boolean to write the matrix of results for the ordered upwind method to the hard drive in a file named filename. Default is FALSE.
`bounce`	by default, the ordered upwind method stops when the boundaries are reached (x.bound and y.bound). The bounce parameter allows the default action (bounce = 'd'), only positive values to be tested (bounce = 'p'), or reflection near the boundaries (bounce = 'b').

bounce.edge	if bounce = 'b', then to prevent the ordered upwind method from reaching the boundaries, temporary boundaries are created inside the boundaries defined by x.bound and y.bound. The boundary edge is bounce.edge of the total range. Default is 0.01
verboseR	NOT IMPLEMENTED: Flag (default = FALSE) for printing out information in QPotential R wrapper.
verboseC	flag (default = TRUE) for printing out useful-for-everyone information from C code implementing the upwind-ordered method (quasipotential.C).
debugC	NOT IMPLEMENTED: Flag (default = FALSE) for printing out debugging C code
k.x	integer anisotropic factor for x. See journal article. Default is 20.
k.y	integer anisotropic factor for y. See journal article. Default is 20.
INFINITY	largest possible quasi-potential value. If computed quasi-potential is ever greater than this number, program will stop. Especially useful when bounce = 'b'. Default is 1,000,000 with a tolerance of 1e-12

Value

filetoHD if save.to.HD enabled, then saves a file in the current directory as either filename or as defaultname-xXSTARTyYSTART.txt

filetoR if save.to.R enabled, then the function QPotential returns a matrix containing the upwind-ordered results to be used for plotting. Requires a variable to catch the returned matrix, i.e. storage <- QPotential(parameters...)

Examples

```
# First, System of equations
equationx <- "1.54*x*(1.0-(x/10.14)) - (y*x*x)/(1.0+x*x)"
equationy <- "((0.476*x*x*y)/(1+x*x)) - 0.112590*y*y"

# Second, shared parameters for each quasi-potential run
xbounds <- c(-0.5, 8.0)
ybounds <- c(-0.5, 8.0)
xstepnumber <- 200
ystepnumber <- 200

# Third, a local quasi-potential run
xinit1 <- 1.40491
yinit1 <- 2.80808
storage.eq1 <- QPotential(x.rhs = equationx, x.start = xinit1,
x.bound = xbounds, x.num.steps = xstepnumber, y.rhs = equationy,
y.start = yinit1, y.bound = ybounds, y.num.steps = ystepnumber)
# Visualize the quasi-potential
QPContour(storage.eq1, dens = c(xstepnumber, ystepnumber),
x.bound = xbounds, y.bound = ybounds, c.parm = 5)
```

TSDensity *Density plot from simulation of two-dimensional stochastic differential equations*

Description

This function creates density plots for the simulation of two-dimensional stochastic differential equations from TSTraj

Usage

```
TSDensity(mat, dim = 1, xlim = NULL, ylim = NULL, contour.levels = 15,
  col2d = c("blue", "yellow", "orange", "red"), contour.lwd = 0.5,
  contour.lines = TRUE, kde2d.n = 100, xlab = "X", ylab = "Y", ...)
```

Arguments

mat	a matrix output from TSTraj.
dim	dimensions of the plot; dim = 1 plots simple density histogram or dim = 2 plots the density in state space (i.e., X and Y respectively on the abscissa and ordinate axes).
xlim	numeric vectors of length 2, giving the x coordinate range. Default = NULL automatically sizes plot window.
ylim	numeric vectors of length 2, giving the y coordinate range. Default = NULL automatically sizes plot window.
contour.levels	the number of contour levels for the two-dimensional plots (i.e., when dim = 2).
col2d	vector of colors to be used in the plot.
contour.lwd	line width of contour lines if contour.lines = TRUE.
contour.lines	if TRUE, then black countour lines added to the graph.
kde2d.n	number of grid points in each direction. Can be scalar or a length-2 integer vector. Passes to argument n in kde2d.
xlab	label for x axis when dim = 2. Default is 'X'
ylab	label for y axis when dim = 2. Default is 'Y'
...	passes arguments to plot.

Examples

```
## Not run:
# First, the parameter values, as found in TSTraj
model.state <- c(x = 3, y = 3)
model.sigma <- 0.2
model.deltat <- 0.005
model.time <- 100
```

```
# Second, write out the deterministic skeleton of the equations to be simulated,
# as found in TSTraj
#Example 1 from article
equationx <- "1.54*x*(1.0-(x/10.14)) - (y*x*x)/(1.0 + x*x)"
equationy <- "((0.476*x*x*y)/(1 + x*x)) - 0.112590*y*y"

# Third, run it, as found in TSTraj
ModelOut <- TSTraj(y0 = model.state, time = model.time, deltat = model.deltat,
x.rhs = equationx, y.rhs = equationy, sigma = model.sigma)
# Fourth, plot it:
# in 1D
TSDensity(ModelOut, dim = 1)
# in 2D
TSDensity(ModelOut, dim = 2, kde2d.n = 20, xlab = "")

## End(Not run)
```

TSPlot *Plot simulation of two-dimensional stochastic differential equations*

Description

This function plots the simulation of two-dimensional stochastic differential equations from TSTraj

Usage

```
TSPlot(mat, deltat, dim = 1, xlim = NULL, ylim = NULL, xaxt.1D = "time",
  dens = TRUE, lwd = 2, line.alpha = 130, zero.axes = TRUE,
  xlab.2D = "X", ylab.2D = "Y", ...)
```

Arguments

mat	a matrix output from TSTraj.
deltat	numeric value indicating the frequency of stochastic perturbation, as Δt, used in the function to recaluculate axes if applicable.
dim	dimensions of the plot; dim = 1 to plot a timeseries with X and Y on the ordinate axis or dim = 2 to plot the trjectories in state space (i.e., X and Y respectively on the abscissa and ordinate axes).
xlim	numeric vectors of length 2, giving the x coordinate range. Default = NULL automatically sizes plot window.
ylim	numeric vectors of length 2, giving the y coordinate range. Default = NULL automatically sizes plot window.
xaxt.1D	for dim = 1, allows user to specify the axis as "time" or "steps," with steps being $time \times \Delta t$
dens	if dens = TRUE, plots a horizontal one-dimensional density plot adjacent to the timerseries.

`lwd`	line width. Defaults to 1.
`line.alpha`	transparency of lines from 0–255.
`zero.axes`	if TRUE, then axes plotted at `X = 0` and `Y = 0`.
`xlab.2D`	a title for the x axis when `dim = 2`.
`ylab.2D`	a title for the y axis when `dim = 2`.
`...`	passes arguments to `plot`.

Examples

```
# First, the parameter values, as found in TSTraj
model.state <- c(x = 3, y = 3)
model.sigma <- 0.2
model.deltat <- 0.05
model.time <- 100

# Second, write out the deterministic skeleton of the equations to be simulated,
# as found in TSTraj
#Example 1 from article
equationx <- "1.54*x*(1.0-(x/10.14)) - (y*x*x)/(1.0 + x*x)"
equationy <- "((0.476*x*x*y)/(1 + x*x)) - 0.112590*y*y"

# Third, run it, as found in TSTraj
ModelOut <- TSTraj(y0 = model.state, time = model.time, deltat = model.deltat,
x.rhs = equationx, y.rhs = equationy, sigma = model.sigma)
# Fourth, plot it:
# in 1D
TSPlot(ModelOut, deltat = model.deltat, dim = 1)
# in 2D
TSPlot(ModelOut, deltat = model.deltat, dim = 2)
```

`TSTraj` *Simulate two-dimensional stochastic differential equations*

Description

This function allows you to simulate two-dimensional stochastic differential equations.

Usage

```
TSTraj(init.x = NULL, init.y = NULL, y0 = NULL, time, deltat, x.rhs,
  y.rhs, parms = NA, sigma, lower.bound = NA, upper.bound = NA)
```

Arguments

`init.x`	initial condition for the x state variable
`init.y`	initial condition for the y state variable

`y0`	instead of init.x and init.y, can assign a two-element vector of the initial conditions for the state variables. Elements must be assigned as objects (see Example below).
`time`	numeric value indicating the total time over which the simulation is to be run.
`deltat`	numeric value indicating the frequency of stochastic perturbation, as Δt.
`x.rhs`	a string containing the right hand side of the equation for x.
`y.rhs`	a string containing the right hand side of the equation for y.
`parms`	n-element vector of objects representing unvalued parameters in the equation. If parameter values are values in the equation, then default is `parms = NA`.
`sigma`	numeric value specifying the noise intensity.
`lower.bound`	numeric value specifying a lower bound in the simulation.
`upper.bound`	numeric value specifying an upper bound in the simulation.

Value

returns a matrix with three columns (timestep, x values, and y values) with a length of `time/deltat` (2*e4 in the examples below).

Examples

```
# First, the parameter values
model.state <- c(x = 3, y = 3)
model.sigma <- 0.2
model.deltat <- 0.1
model.time <- 100

# Second, write out the deterministic skeleton of the equations to be simulated
equationx <- "1.54*x*(1.0-(x/10.14)) - (y*x*x)/(1.0 + x*x)"
equationy <- "((0.476*x*x*y)/(1 + x*x)) - 0.112590*y*y"

# Third, Run it
ModelOut <- TSTraj(y0 = model.state, time = model.time, deltat = model.deltat,
x.rhs = equationx, y.rhs = equationy, sigma = model.sigma)

# Can also input x.rhs and y.rhs as strings that contain parameter names
# and include parms with names and values of parameters
model.state <- c(x = 1, y = 2)
model.parms <- c(alpha = 1.54, beta = 10.14, delta = 1, kappa = 1, gamma = 0.476, mu = 0.112509)
model.sigma <- 0.2
model.time <- 100
model.deltat <- 0.1

test.eqn.x = "(alpha*x)*(1-(x/beta)) - ((delta*(x^2)*y)/(kappa + (x^2)))"
test.eqn.y = "((gamma*(x^2)*y)/(kappa + (x^2))) - mu*(y^2)"

ModelOut.parms <- TSTraj(y0 = model.state, time = model.time, deltat = model.deltat,
x.rhs = test.eqn.x, y.rhs = test.eqn.y, parms = model.parms, sigma = model.sigma)
```

VecDecomAll — *Vector decomposition and remainder fields*

Description

This function calculates the vector, gradient, and remainder fields.

Usage

```
VecDecomAll(surface, x.rhs, y.rhs, x.bound, y.bound)
```

Arguments

surface	matrix output from QPGlobal or QPotential.
x.rhs	a string containing the right hand side of the equation for x.
y.rhs	a string containing the right hand side of the equation for y.
x.bound	the x boundaries denoted at c(minimum, maximum).
y.bound	the y boundaries denoted at c(minimum, maximum).

Value

returns an array of all three vector fields: the deterministic skeleton, the negative gradient of the quasi-potential, and the remainder. The array has three dimensions with the respective lengths of xstepnumber, ystepnumber, and 6. The six are the x- and y-values for each of the three vector fields, as x-deterministic skeleton, y-deterministic skeleton, x-negative gradient of the quasi-potential, y-negative gradient of the quasi-potential, x-remainder, and y-remainder.

Examples

```
# First, the system of equations
equationx <- "1.54*x*(1.0-(x/10.14)) - (y*x*x)/(1.0+x*x)"
equationy <- "((0.476*x*x*y)/(1+x*x)) - 0.112590*y*y"

# Second, shared parameters for each quasi-potential run
xbounds <- c(-0.5, 10.0)
ybounds <- c(-0.5, 10.0)
xstepnumber <- 100
ystepnumber <- 100

# Third, first local quasi-potential run
xinit1 <- 1.40491
yinit1 <- 2.80808
storage.eq1 <- QPotential(x.rhs = equationx, x.start = xinit1,
x.bound = xbounds, x.num.steps = xstepnumber, y.rhs = equationy,
y.start = yinit1, y.bound = ybounds, y.num.steps = ystepnumber)

# Fourth, second local quasi-potential run
xinit2 <- 4.9040
```

```
yinit2 <- 4.06187
storage.eq2 <- QPotential(x.rhs = equationx, x.start = xinit2,
x.bound = xbounds, x.num.steps = xstepnumber, y.rhs = equationy,
y.start = yinit2, y.bound = ybounds, y.num.steps = ystepnumber)

# Fifth, determine global quasi-potential
unst.x <- c(0, 4.2008)
unst.y <- c(0, 4.0039)
ex1.global <- QPGlobal(local.surfaces = list(storage.eq1, storage.eq2),
unstable.eq.x = unst.x, unstable.eq.y = unst.y, x.bound = xbounds,
y.bound = ybounds)

# Sixth, decompose the global quasi-potential into the
# deterministic skeleton, gradient, and remainder vector fields
VDAll <- VecDecomAll(surface = ex1.global, x.rhs = equationx, y.rhs = equationy,
x.bound = xbounds, y.bound = ybounds)
```

VecDecomGrad *Vector decomposition and remainder fields*

Description

This function calculates the gradient field.

Usage

```
VecDecomGrad(surface)
```

Arguments

surface	matrix output from QPGlobal or QPotential.

Value

returns an array of the gradient vector field. The array has three dimensions with the respective lengths of the columns of the surface, the rows of the sruface, and the number of variables (always 2). The two variables are the x-negative gradient of the quasi-potential surface and the y-negative gradient of the quasi-potential surface.

Examples

```
# First, the system of equations
equationx <- "1.54*x*(1.0-(x/10.14)) - (y*x*x)/(1.0+x*x)"
equationy <- "((0.476*x*x*y)/(1+x*x)) - 0.112590*y*y"

# Second, shared parameters for each quasi-potential run
xbounds <- c(-0.5, 10.0)
ybounds <- c(-0.5, 10.0)
xstepnumber <- 100
ystepnumber <- 100
```

```
# Third, first local quasi-potential run
xinit1 <- 1.40491
yinit1 <- 2.80808
storage.eq1 <- QPotential(x.rhs = equationx, x.start = xinit1,
x.bound = xbounds, x.num.steps = xstepnumber, y.rhs = equationy,
y.start = yinit1, y.bound = ybounds, y.num.steps = ystepnumber)

# Fourth, second local quasi-potential run
xinit2 <- 4.9040
yinit2 <- 4.06187
storage.eq2 <- QPotential(x.rhs = equationx, x.start = xinit2,
x.bound = xbounds, x.num.steps = xstepnumber, y.rhs = equationy,
y.start = yinit2, y.bound = ybounds, y.num.steps = ystepnumber)

# Fifth, determine global quasi-potential
unst.x <- c(0, 4.2008)
unst.y <- c(0, 4.0039)
ex1.global <- QPGlobal(local.surfaces = list(storage.eq1, storage.eq2),
unstable.eq.x = unst.x, unstable.eq.y = unst.y, x.bound = xbounds,
y.bound = ybounds)

# Sixth, creat the gradient vector field
VDG <- VecDecomGrad(surface = ex1.global)
```

`VecDecomPlot` *Plotting function for vector decomposition and remainder fields*

Description

This function plots various vector fields

Usage

```
VecDecomPlot(x.field, y.field, dens, x.bound, y.bound, xlim = NULL,
  ylim = NULL, arrow.type = "equal", tail.length = 0.25,
  head.length = 0.25, xlab = "X", ylab = "Y", ...)
```

Arguments

`x.field`	a two-dimensional array containing the x-values for the vector field, generated from `VecDecomAll`, `VecDecomVec`, `VecDecomGrad`, or `VecDecomRem`.
`y.field`	a two-dimensional array containing the y-values for the vector field, generated from `VecDecomAll`, `VecDecomVec`, `VecDecomGrad`, or `VecDecomRem`.
`dens`	two-element vector respectively specifying the number of respective arrows in the x and y directions.
`x.bound`	the x boundaries denoted at c(minimum, maximum) for the quasi-potential simulation.

`y.bound`	the y boundaries denoted at c(minimum, maximum) for the quasi-potential simulation.
`xlim`	numeric vectors of length 2, giving the x coordinate range. Default = `NULL` automatically sizes plot window.
`ylim`	numeric vectors of length 2, giving the y coordinate range. Default = `NULL` automatically sizes plot window.
`arrow.type`	sets the type of line segments plotted. If set to "proportional" the length of the line segments reflects the magnitude of the derivative. If set to "equal" the line segments take equal lengths, simply reflecting the gradient of the derivative(s). Defaults to "equal".
`tail.length`	multiplies the current length of the tail (both proportional and equal arrow.types) by the specified factor. The argument defaults to 1, which is length of the longest vector within the domain boundaries (i.e., the entire field).
`head.length`	length of the edges of the arrow head (in inches).
`xlab`	label for x axis. Default is 'X'
`ylab`	label for y axis. Default is 'Y'
`...`	passes arguments to both `plot`.

Details

If `arrow.type = "proportional"`, a common warning, passed from `arrows`, will appear: "The direction of a zero-length arrow is indeterminate, and hence so is the direction of the arrowheads. To allow for rounding error, arrowheads are omitted (with a warning) on any arrow of length less than 1/1000 inch." Either increase `tail.length` or increase the plot window to avoid this warning.

Examples

```
# First, system of equations
equationx <- "1.54*x*(1.0-(x/10.14)) - (y*x*x)/(1.0+x*x)"
equationy <- "((0.476*x*x*y)/(1+x*x)) - 0.112590*y*y"

# Second, shared parameters for each quasi-potential run
xbounds <- c(-0.5, 10.0)
ybounds <- c(-0.5, 10.0)
xstepnumber <- 100
ystepnumber <- 100

# Third, first local quasi-potential run
xinit1 <- 1.40491
yinit1 <- 2.80808
storage.eq1 <- QPotential(x.rhs = equationx, x.start = xinit1,
x.bound = xbounds, x.num.steps = xstepnumber, y.rhs = equationy,
y.start = yinit1, y.bound = ybounds, y.num.steps = ystepnumber)

# Fourth, second local quasi-potential run
xinit2 <- 4.9040
yinit2 <- 4.06187
storage.eq2 <- QPotential(x.rhs = equationx, x.start = xinit2,
```

```
x.bound = xbounds, x.num.steps = xstepnumber, y.rhs = equationy,
y.start = yinit2, y.bound = ybounds, y.num.steps = ystepnumber)

# Fifth, determine global quasi-potential
unst.x <- c(0, 4.2008)
unst.y <- c(0, 4.0039)
ex1.global <- QPGlobal(local.surfaces = list(storage.eq1, storage.eq2),
unstable.eq.x = unst.x, unstable.eq.y = unst.y, x.bound = xbounds,
y.bound = ybounds)

# Sixth, decompose the global quasi-potential into the
# deterministic skeleton, gradient, and remainder vector fields
VDAll <- VecDecomAll(surface = ex1.global, x.rhs = equationx, y.rhs = equationy,
x.bound = xbounds, y.bound = ybounds)

# Seventh, plot all three vector fields
# The deterministic skeleton vector field
VecDecomPlot(x.field = VDAll[,,1], y.field = VDAll[,,2], dens = c(25,25),
x.bound = xbounds, y.bound = ybounds, tail.length = 0.25, head.length = 0.05)
# The gradient vector field
VecDecomPlot(x.field = VDAll[,,3], y.field = VDAll[,,4], dens = c(25,25),
x.bound = xbounds, y.bound = ybounds, tail.length = 0.15, head.length = 0.05)
# The remainder vector field
VecDecomPlot(x.field = VDAll[,,5], y.field = VDAll[,,6], dens = c(25,25),
x.bound = xbounds, y.bound = ybounds, tail.length = 0.15, head.length = 0.05)
```

VecDecomRem *Vector decomposition and remainder fields*

Description

This function calculates the remainder field.

Usage

```
VecDecomRem(surface, x.rhs, y.rhs, x.bound, y.bound)
```

Arguments

surface	matrix output from QPGlobal or QPotential.
x.rhs	a string containing the right hand side of the equation for x.
y.rhs	a string containing the right hand side of the equation for y.
x.bound	the x boundaries denoted at c(minimum, maximum).
y.bound	the y boundaries denoted at c(minimum, maximum).

Value

returns an array of the remainder vector field. The array has three dimensions with the respective lengths of the columns of the surface, the rows of the sruface, and the number of variables (always 2). The two variables are the x-remainder and y-remainder.

Examples

```
# First, the system of equations
equationx <- "1.54*x*(1.0-(x/10.14)) - (y*x*x)/(1.0+x*x)"
equationy <- "((0.476*x*x*y)/(1+x*x)) - 0.112590*y*y"

# Second, shared parameters for each quasi-potential run
xbounds <- c(-0.5, 10.0)
ybounds <- c(-0.5, 10.0)
xstepnumber <- 150
ystepnumber <- 150

# Third, first local quasi-potential run
xinit1 <- 1.40491
yinit1 <- 2.80808
storage.eq1 <- QPotential(x.rhs = equationx, x.start = xinit1,
x.bound = xbounds, x.num.steps = xstepnumber, y.rhs = equationy,
y.start = yinit1, y.bound = ybounds, y.num.steps = ystepnumber)

# Fourth, second local quasi-potential run
xinit2 <- 4.9040
yinit2 <- 4.06187
storage.eq2 <- QPotential(x.rhs = equationx, x.start = xinit2,
x.bound = xbounds, x.num.steps = xstepnumber, y.rhs = equationy,
y.start = yinit2, y.bound = ybounds, y.num.steps = ystepnumber)

# Fifth, determine global quasi-potential
unst.x <- c(0, 4.2008)
unst.y <- c(0, 4.0039)
ex1.global <- QPGlobal(local.surfaces = list(storage.eq1, storage.eq2),
unstable.eq.x = unst.x, unstable.eq.y = unst.y, x.bound = xbounds,
y.bound = ybounds)

# Sixth, create remainder field
VDR <- VecDecomRem(surface = ex1.global, x.rhs = equationx, y.rhs = equationy,
x.bound = xbounds, y.bound = ybounds)
```

`VecDecomVec` *Vector decomposition and remainder fields*

Description

This function calculates the vector field.

Usage

```
VecDecomVec(x.num.steps, y.num.steps, x.rhs, y.rhs, x.bound, y.bound)
```

Arguments

x.num.steps	the number of steps between the minimum and maximum x value defined in x range.
y.num.steps	the number of steps between the minimum and maximum y value defined in y range.
x.rhs	a string containing the right hand side of the equation for x.
y.rhs	a string containing the right hand side of the equation for y.
x.bound	the x boundaries denoted at c(minimum, maximum).
y.bound	the y boundaries denoted at c(minimum, maximum).

Value

returns an array of the deterministic skeleton vector field. The array has three dimensions with the respective lengths of x.num.steps, y.num.steps, and the number of variables (always 2). The two variables are the x-deterministic skeleton and the y-deterministic skeleton.

Examples

```
# First, the system of equations
equationx <- "1.54*x*(1.0-(x/10.14)) - (y*x*x)/(1.0+x*x)"
equationy <- "((0.476*x*x*y)/(1+x*x)) - 0.112590*y*y"

# Second, shared parameters for each quasi-potential run
xbounds <- c(-0.5, 20.0)
ybounds <- c(-0.5, 20.0)
xstepnumber <- 1000
ystepnumber <- 1000

# Third, create the deterministic skeleton vector field
VDV <- VecDecomVec(x.num.steps = xstepnumber, y.num.steps = ystepnumber, x.rhs = equationx,
y.rhs = equationy, x.bound = xbounds, y.bound = ybounds)
```

Index

*Topic **Global**
- QPGlobal, 6

*Topic **Stochastic**
- TSTraj, 12

*Topic **decompoosition,**
- VecDecomRem, 18

*Topic **decomposition,**
- VecDecomGrad, 15

*Topic **detrministic**
- VecDecomPlot, 16

*Topic **field**
- VecDecomGrad, 15
- VecDecomPlot, 16
- VecDecomRem, 18

*Topic **gradient**
- VecDecomGrad, 15
- VecDecomPlot, 16

*Topic **interpolation**
- QPInterp, 7

*Topic **plot,**
- VecDecomPlot, 16

*Topic **plot**
- TSDensity, 10
- TSPlot, 11
- VecDecomPlot, 16

*Topic **quasi-potential**
- QPGlobal, 6

*Topic **remainder**
- VecDecomPlot, 16
- VecDecomRem, 18

*Topic **simulations**
- TSDensity, 10
- TSPlot, 11

*Topic **simulation**
- TSTraj, 12

*Topic **skeleton**
- VecDecomPlot, 16

*Topic **stochastic**
- TSDensity, 10
- TSPlot, 11

*Topic **vector**
- VecDecomGrad, 15
- VecDecomPlot, 16
- VecDecomRem, 18

arrows, *17*

col2rgb, *4*

filled.contour, *4*

kde2d, *10*

Model2String, 2

plot, *4*, *10*, *12*, *17*

QPContour, 3, *4*
QPGlobal, *3*, *4*, 6, *7*, *14*, *15*, *18*
QPInterp, 7
QPotential, 8, *14*, *15*, *18*

TSDensity, 10
TSPlot, 11
TSTraj, 2, *10*, *11*, 12

VecDecomAll, 14, *16*
VecDecomGrad, 15, *16*
VecDecomPlot, 16
VecDecomRem, *16*, 18
VecDecomVec, *16*, 19