

# Package ‘Sim.DiffProc’

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**Title** Simulation of Diffusion Processes

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rmarkdown (>= 0.8), scatterplot3d (>= 0.3-36), sm (>= 2.2-5.3)

**BugReports** <https://github.com/acguidoum/Sim.DiffProc/issues>

**URL** <https://github.com/acguidoum/Sim.DiffProc>

**VignetteBuilder** knitr

**Encoding** UTF-8

**Description** It provides users with a wide range of tools to simulate, estimate, analyze, and visualize the dynamics of stochastic differential systems in both forms Ito and Stratonovich. Statistical analysis with parallel Monte Carlo and moment equations methods of SDE's. Enabled many searchers in different domains to use these equations to modeling practical problems in financial and actuarial modeling and other areas of application, e.g., modeling and simulate of first passage time problem in shallow water using the attractive center (Boukhetala K, 1996) ISBN:1-56252-342-2.

**License** GPL (>= 2)

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Sim.DiffProc-package    *Simulation of Diffusion Processes*

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**Description**

A package for symbolic and numerical computations on scalar and multivariate systems of stochastic differential equations. It provides users with a wide range of tools to simulate, estimate, analyze, and visualize the dynamics of these systems in both forms Itô and Stratonovich. Statistical analysis with parallel Monte-Carlo and moment equations methods of SDE's. Enabled many searchers in different domains to use these equations to modeling practical problems in financial and actuarial modeling and other areas of application, e.g., modeling and simulate of first passage time problem in shallow water using the attractive center (Boukhetala K, 1996) ISBN:1-56252-342-2.

**Details**

|          |              |
|----------|--------------|
| Package: | Sim.DiffProc |
| Type:    | Package      |
| Version: | 4.3          |
| Date:    | 2018-11-28   |
| License: | GPL (>= 2)   |

Depends: R (>= 2.15.1)  
 Imports: Deriv (>= 3.8.0), MASS (>= 7.3-30), parallel  
 Suggests: deSolve (>= 1.11), knitr (>= 1.10), rgl (>= 0.93.991), rmarkdown (>= 0.8), scatterplot3d (>= 0.3-36), s  
 Classification/MSC: 37H10, 37M10, 60H05, 60H10, 60H35, 60J60, 65C05, 68N15, 68Q10

There are main types of functions in this package:

1. Simulation of solution to 1,2 and 3-dim stochastic differential equations of Itô and Stratonovich types, with different methods.
2. Simulation of solution to 1,2 and 3-dim diffusion bridge of Itô and Stratonovich types, with different methods.
3. Simulation the first-passage-time (f.p.t) in 1,2 and 3-dim sde of Itô and Stratonovich types.
4. Calculate symbolic ODE's of moment equations (means and variances-covariance) for 1,2 and 3-dim SDE's.
5. Monte-Carlo replicates of a statistic applied to 1,2 and 3-dim SDE's at any time t.
6. Computing the basic statistics (mean, var, median, ...) of the processes at any time t using the Monte Carlo method.
7. Random number generators (RN's) to generate 1,2 and 3-dim sde of Itô and Stratonovich types.
8. Approximate the transition density 1,2 and 3-dim of the processes at any time t.
9. Approximate the density of first-passage-time in 1,2 and 3-dim SDE's.
10. Computing the stochastic integrals of Itô and Stratonovich types.
11. Estimate drift and diffusion parameters by the method of maximum pseudo-likelihood of the 1-dim stochastic differential equation.
12. Converting Sim.DiffProc objects to LaTeX.
13. Displaying an object inheriting from class "sde" (1,2 and 3 dim).

## Main Features

### Stochastic integrals:

We consider a simple example to simulation Itô integral, used `st.int` function:

$$\int_{t_0}^t W_s^n dW_s = \frac{1}{n+1} [W_t^{n+1} - W_{t_0}^{n+1}] - \frac{n}{2} \int_{t_0}^t W_s^{n-1} ds$$

And the Stratonovich integral

$$\int_{t_0}^t W_s^n \circ dW_s = \frac{1}{n+1} [W_t^{n+1} - W_{t_0}^{n+1}]$$

```
R> f <- expression( w )
R> Itô <- st.int(f,type="Ito",M=500,lower=0,upper=1)
R> Itô
```

Itô integral:

|  $X(t) = \text{integral}(f(s,w) * dw(s))$

|  $f(t,w) = w$

Summary:

| Number of subinterval |  $N = 1001$ .

| Number of simulation |  $M = 500$ .

| Limits of integration |  $t$  in  $[0,1]$ .

R> summary(Itô)

Monte-Carlo Statistics for  $\text{integral}(f(s,w) * dw(s))$  at time  $t = 1$

|  $f(t,w) = w$

|                  |          |
|------------------|----------|
| Mean             | 0.01330  |
| Variance         | 0.51102  |
| Median           | -0.28645 |
| Mode             | -0.42772 |
| First quartile   | -0.44666 |
| Third quartile   | 0.22534  |
| Minimum          | -0.55198 |
| Maximum          | 4.38802  |
| Skewness         | 2.27133  |
| Kurtosis         | 9.27393  |
| Coef-variation   | 53.75783 |
| 3th-order moment | 0.82972  |
| 4th-order moment | 2.42178  |
| 5th-order moment | 7.60355  |
| 6th-order moment | 26.72897 |

R> str <- st.int(f,type="str",M=500,lower=0,upper=1)

R> str

Stratonovich integral:

|  $X(t) = \text{integral}(f(s,w) \circ dw(s))$

|  $f(t,w) = w$

Summary:

| Number of subinterval |  $N = 1001$ .

| Number of simulation |  $M = 500$ .

| Limits of integration |  $t$  in  $[0,1]$ .

R> summary(str)

Monte-Carlo Statistics for  $\text{integral}(f(s,w) \circ dw(s))$  at time  $t = 1$

|  $f(t,w) = w$

|                |         |
|----------------|---------|
| Mean           | 0.55655 |
| Variance       | 0.66663 |
| Median         | 0.21223 |
| Mode           | 0.08249 |
| First quartile | 0.04269 |
| Third quartile | 0.79322 |
| Minimum        | 0.00000 |
| Maximum        | 6.70508 |

|                  |           |
|------------------|-----------|
| Skewness         | 2.72896   |
| Kurtosis         | 13.34205  |
| Coef-variation   | 1.46702   |
| 3th-order moment | 1.48532   |
| 4th-order moment | 5.92908   |
| 5th-order moment | 26.87087  |
| 6th-order moment | 138.27901 |

### SDE's 1,2 and 3-dim:

There are thus two widely used types of stochastic calculus, Stratonovich and Itô, differing in respect of the stochastic integral used. Modelling issues typically dictate which version is appropriate, but once one has been chosen a corresponding equation of the other type with the same solutions can be determined. Thus it is possible to switch between the two stochastic calculi. Specifically, the processes  $\{X_t, t \geq 0\}$  solution to the Itô SDE:

$$dX_t = f(t, X_t)dt + g(t, X_t)dW_t$$

where  $\{W_t, t \geq 0\}$  is the standard Wiener process or standard Brownian motion, the drift  $f(t, X_t)$  and diffusion  $g(t, X_t)$  are known functions that are assumed to be sufficiently regular (Lipschitz, bounded growth) for existence and uniqueness of solution; has the same solutions as the Stratonovich SDE:

$$dX_t = \underline{f}(t, X_t)dt + g(t, X_t) \circ dW_t$$

with the modified drift coefficient

$$\underline{f}(t, X_t) = f(t, X_t) - \frac{1}{2}g(t, X_t)\frac{\partial g}{\partial x}(t, X_t)$$

The following examples for different methods of simulation of SDEs (1,2 and 3-dim) use the [snssde1d](#), [snssde2d](#) and [snssde3d](#) functions.

```
R> ## 1-dim sde
R> f <- expression(2*(3-x) )
R> g <- expression(2*x)
R> res1 <- snssde1d(drift=f,diffusion=g,M=1000,x0=1)
R> res1
Itô Sde 1D:
| dX(t) = 2 * (3 - X(t)) * dt + 2 * X(t) * dW(t)
Method:
| Euler scheme with order 0.5
Summary:
| Size of process          | N = 1001.
| Number of simulation     | M = 1000.
| Initial value           | x0 = 1.
| Time of process         | t in [0,1].
| Discretization          | Dt = 0.001.
R> res2 <- snssde1d(drift=f,diffusion=g,M=1000,x0=1,type="str")
R> res2
Stratonovich Sde 1D:
```

```

      |  $dX(t) = 2 * (3 - X(t)) * dt + 2 * X(t) \circ dW(t)$ 
Method:
      | Euler scheme with order 0.5
Summary:
      | Size of process          | N = 1001.
      | Number of simulation     | M = 1000.
      | Initial value            |  $x_0 = 1$ .
      | Time of process          | t in [0,1].
      | Discretization           | Dt = 0.001.

R> ## 2-dim sde
R> fx <- expression(x-y, y-x)
R> gx <- expression(2*y, 2*x)
R> res2d <- snssde2d(drift=fx,diffusion=gx,x0=c(1,1))
R> res2d
Itô Sde 2D:
      |  $dX(t) = X(t) - Y(t) * dt + 2 * Y(t) * dW1(t)$ 
      |  $dY(t) = Y(t) - X(t) * dt + 2 * X(t) * dW2(t)$ 
Method:
      | Euler scheme with order 0.5
Summary:
      | Size of process          | N = 1001.
      | Number of simulation     | M = 1.
      | Initial values          |  $(x_0,y_0) = (1,1)$ .
      | Time of process          | t in [0,1].
      | Discretization           | Dt = 0.001.
R> plot2d(res2d)

R> ## 3-dim sde
R> fx <- expression(y, 0, 0)
R> gx <- expression(z, 1, 1)
R> res3d <- snssde3d(drift=fx,diffusion=gx,M=1000)
R> res3d
Itô Sde 3D:
      |  $dX(t) = Y(t) * dt + Z(t) * dW1(t)$ 
      |  $dY(t) = 0 * dt + 1 * dW2(t)$ 
      |  $dZ(t) = 0 * dt + 1 * dW3(t)$ 
Method:
      | Euler scheme with order 0.5
Summary:
      | Size of process          | N = 1001.
      | Number of simulation     | M = 1000.
      | Initial values          |  $(x_0,y_0,z_0) = (0,0,0)$ .
      | Time of process          | t in [0,1].
      | Discretization           | Dt = 0.001.
plot3D(res3d)

```

**Bridge SDE's 1,2 and 3-dim:**

Simulation of bridge SDEs (1,2 and 3-dim) with `bridgesde1d`, `bridgesde2d` and `bridgesde3d` functions.

```
R> ## 1-dim bridge sde
R> f <- expression( 2*(1-x) )
R> g <- expression( 1 )
R> mod1 <- bridgesde1d(drift=f,diffusion=g,x0=2,y=1,M=1000)
R> mod1
Itô Bridges Sde 1D:
  | dX(t) = 2 * (1 - X(t)) * dt + 1 * dW(t)
Method:
  | Euler scheme with order 0.5
Summary:
  | Size of process          | N = 1001.
  | Crossing realized        | C = 843 among 1000.
  | Initial value            | x0 = 2.
  | Ending value             | y = 1.
  | Time of process          | t in [0,1].
  | Discretization           | Dt = 0.001.
R> summary(mod1) ## Monte-Carlo statistics at T/2=0.5
```

Monte-Carlo Statistics for  $X(t)$  at time  $t = 0.5$

Crossing realized 843 among 1000

|                  |          |
|------------------|----------|
| Mean             | 1.31263  |
| Variance         | 0.18352  |
| Median           | 1.30504  |
| Mode             | 1.46713  |
| First quartile   | 1.02722  |
| Third quartile   | 1.60984  |
| Minimum          | -0.22080 |
| Maximum          | 2.83339  |
| Skewness         | 0.01722  |
| Kurtosis         | 3.19888  |
| Coef-variation   | 0.32636  |
| 3th-order moment | 0.00135  |
| 4th-order moment | 0.10773  |
| 5th-order moment | 0.00645  |
| 6th-order moment | 0.11233  |

```
R> plot(mod1)
R> den <- dsde1d(mod1)
Density of  $X(t-t_0)|X(t_0)=x_0$  at time  $t = 1$ 
```

Data: x (843 obs.); Bandwidth 'bw' = 0.2339

|          | x        | f(x)            |
|----------|----------|-----------------|
| Min.     | :0.29822 | Min. :0.01913   |
| 1st Qu.: | 0.64911  | 1st Qu.:0.13600 |

```

Median :1.00000  Median :0.55258
Mean   :1.00000  Mean    :0.70988
3rd Qu.:1.35089  3rd Qu.:1.28369
Max.   :1.70178  Max.    :1.70511
R> plot(den)

R> ## 2 and 3-dim Bridge sde
R> example(bridgesde2d)
R> example(bridgesde3d)

```

### Estimate the parameters of 1-dim sde:

Consider a process solution of the general stochastic differential equation:

$$dX_t = f(t, X_t, \underline{\theta})dt + g(t, X_t, \underline{\theta})dW_t$$

The package Sim.DiffProc implements the function `fitsde` of estimate drift and diffusion parameters  $\underline{\theta} = (\theta_1, \theta_2, \dots, \theta_p)$  with different methods of maximum pseudo-likelihood of the 1-dim stochastic differential equation.

An example we use a real data, fit with the CKLS model:

$$dX_t = (\theta_1 + \theta_2 X_t)dt + \theta_3 X_t^{\theta_4} dW_t$$

we estimate the vector of parameters  $\underline{\theta} = (\theta_1, \theta_2, \theta_3, \theta_4)$ , using Euler pseudo-likelihood.

```

R> ## 1-dim fitsde
R> data(Irates)
R> rates <- Irates[,"r1"]
R> rates <- window(rates, start=1964.471, end=1989.333)
R> fx <- expression(theta[1]+theta[2]*x)
R> gx <- expression(theta[3]*x^theta[4])
R> ## theta = (theta1,theta2,theta3,theta4), p=4
R> fitmod <- fitsde(rates,drift=fx,diffusion=gx,pmle="euler",start = list(theta1=1,
                                theta2=1,theta3=1,theta4=1),optim.method = "L-BFGS-B")

R> fitmod
Call:
fitsde(data = rates, drift = fx, diffusion = gx, pmle = "euler",
       start = list(theta1 = 1, theta2 = 1, theta3 = 1, theta4 = 1),
       optim.method = "L-BFGS-B")
Coefficients:
  theta1  theta2  theta3  theta4
 2.0769516 -0.2631871  0.1302158  1.4513173
R> summary(fitmod)
Pseudo maximum likelihood estimation
Method: Euler
Call:
fitsde(data = rates, drift = fx, diffusion = gx, pmle = "euler",
       start = list(theta1 = 1, theta2 = 1, theta3 = 1, theta4 = 1),

```

```

      optim.method = "L-BFGS-B")
Coefficients:
      Estimate Std. Error
theta1  2.0769516 0.98838467
theta2 -0.2631871 0.19544290
theta3  0.1302158 0.02523105
theta4  1.4513173 0.10323740

-2 log L: 475.7572
R> coef(fitmod)
      theta1      theta2      theta3      theta4
2.0769516 -0.2631871  0.1302158  1.4513173
R> logLik(fitmod)
[1] -237.8786
R> AIC(fitmod)
[1] 483.7572
R> BIC(fitmod)
[1] 487.1514
R> vcov(fitmod)
      theta1      theta2      theta3      theta4
theta1  0.9769042534 -1.843596e-01 -2.714334e-04  0.0011374342
theta2 -0.1843595796  3.819793e-02  5.169849e-05 -0.0002165286
theta3 -0.0002714334  5.169849e-05  6.366061e-04 -0.0025457493
theta4  0.0011374342 -2.165286e-04 -2.545749e-03  0.0106579616
R> confint(fitmod,level=0.95)
      2.5 %    97.5 %
theta1  0.13975321 4.0141499
theta2 -0.64624812 0.1198740
theta3  0.08076388 0.1796678
theta4  1.24897569 1.6536589

```

### Transition density and Random number generators (RN's) for 1,2 and 3-dim sde:

Simulation M-sample for the random variable  $X_{at}$  at time  $t = at$  by a simulated 1, 2 and 3-dim sde, using the functions `rsde1d`, `rsde2d` and `rsde3d`. And `dsde1d`, `dsde2d` and `dsde3d` returns a kernel approximate of transitional densities.

```

R> f <- expression(-2*(x<=0)+2*(x>=0))
R> g <- expression(0.5)
R> res1 <- snssde1d(drift=f,diffusion=g,M=50,type="str",T=10)
R> x <- rsde1d(res1,at=10)
R> x
[1] -17.64115  21.67111 -19.00162 -20.21546  20.65829  19.59535
[7] -20.00676 -18.75649 -19.04453 -15.55535 -18.75077  18.89528
[13] -22.99474 -19.66526 -19.75898  22.02310 -19.68301 -19.08581
[19] -19.15081 -19.24476 -22.24332  17.74989  19.88449 -18.17091
[25] -18.65697  19.08473 -17.81218  19.58453  19.27531 -21.88292
[31]  19.03283 -19.29196  21.99163  20.12123  21.09657 -20.20252

```

```

[37] 20.85097 -19.41987 -18.67530 -19.36289 19.50057 16.30538
[43] 19.34247 -17.97358 22.81003 -18.40051 -18.47490 -21.86839
[49] -21.32638 -18.96264
R> summary(x)
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-22.990 -19.350 -18.440  -4.436  19.330  22.810
R> den <- dsde1d(res1,at=10)
R> ## 2 and 3-dim rsde
R> example(dsde2d)
R> example(dsde3d)

```

### First-passage-time (f.p.t) in 1,2 and 3-dim sde

The functions `fptsde1d` (`fptsde2d` and `fptsde3d` for 2 and 3-dim) returns a random variable  $\tau_{(X(t),S(t))}$  "first passage time", is defined as:

$$\tau_{(X(t),S(t))} = \{t \geq 0; X_t \geq S(t)\}, \quad \text{if } X(t_0) < S(t_0)$$

$$\tau_{(X(t),S(t))} = \{t \geq 0; X_t \leq S(t)\}, \quad \text{if } X(t_0) > S(t_0)$$

And `dfptsde1d`, `dfptsde2d` and `dfptsde3d` returns a kernel density approximation for first passage time. with  $S(t)$  is through a continuous boundary (barrier).

```

R> f <- expression( 0.5*x*t )
R> g <- expression( sqrt(1+x^2) )
R> St <- expression(-0.5*sqrt(t)+exp(t^2))
R> mod <- snssde1d(drift=f,diffusion=g,x0=2,M=1000)
R> fptmod <- fptsde1d(mod,boundary=St)
R> fptmod
Itô Sde 1D:
  | dX(t) = 0.5 * X(t) * t * dt + sqrt(1 + X(t)^2) * dW(t)
  | t in [0,1].
Boundary:
  | S(t) = -0.5 * sqrt(t) + exp(t^2)
F.P.T:
  | T(S(t),X(t)) = inf{t >= 0 : X(t) <= -0.5 * sqrt(t) + exp(t^2) }
  | Crossing realized 738 among 1000.
R> summary(fptmod)

```

Monte-Carlo Statistics of F.P.T:

```
|T(S(t),X(t)) = inf{t >= 0 : X(t) <= -0.5 * sqrt(t) + exp(t^2) }
```

```

Mean          0.47742
Variance      0.07348
Median        0.44831
Mode          0.18582
First quartile 0.23746
Third quartile 0.71321
Minimum       0.03002

```

```

Maximum          0.98877
Skewness         0.22793
Kurtosis         1.79959
Coef-variation   0.56778
3th-order moment 0.00454
4th-order moment 0.00972
5th-order moment 0.00134
6th-order moment 0.00158
R> den <- dfptsde1d(mod,boundary=St)
R> den
Kernel density for the F.P.T of X(t)
T(S,X) = inf{t >= 0 : X(t) <= -0.5 * sqrt(t) + exp(t^2)}

Data: fpt (738 obs.); Bandwidth 'bw' = 0.0828

      x          f(x)
Min. :-0.2095 Min. :0.0019
1st Qu.: 0.1458 1st Qu.:0.2163
Median : 0.5010 Median :0.5307
Mean   : 0.5010 Mean   :0.7029
3rd Qu.: 0.8563 3rd Qu.:1.1943
Max.   : 1.2116 Max.   :1.8548
R> ## fpt in 2 and 3-dim sde
R> example(dfptsde2d)
R> example(dfptsde3d)

```

For other examples see `demo(Sim.DiffProc)`, and for an overview of this package, see `browseVignettes('Sim.DiffProc')` for more informations.

## Requirements

R version  $\geq 3.0.0$

## Licence

This package and its documentation are usable under the terms of the "GNU General Public License", a copy of which is distributed with the package.

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Please send comments, error reports, etc. to the author via the addresses email.

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### See Also

sde, yumia, QPot, DiffusionRgqd, fptdApprox.

---

|    |   |
|----|---|
| BM | <i>Brownian motion, Brownian bridge, geometric Brownian motion, and arithmetic Brownian motion simulators</i> |
|----|---|

---

### Description

The (S3) generic function for simulation of brownian motion, brownian bridge, geometric brownian motion, and arithmetic brownian motion.

### Usage

```
BM(N, ...)
BB(N, ...)
GBM(N, ...)
ABM(N, ...)

## Default S3 method:
BM(N =1000,M=1,x0=0,t0=0,T=1,Dt=NULL, ...)
## Default S3 method:
BB(N =1000,M=1,x0=0,y=0,t0=0,T=1,Dt=NULL, ...)
## Default S3 method:
GBM(N =1000,M=1,x0=1,t0=0,T=1,Dt=NULL,theta=1,sigma=1, ...)
## Default S3 method:
ABM(N =1000,M=1,x0=0,t0=0,T=1,Dt=NULL,theta=1,sigma=1, ...)
```

### Arguments

|       |   |
|-------|---|
| N     | number of simulation steps.   |
| M     | number of trajectories.   |
| x0    | initial value of the process at time $t_0$ .  |
| y     | terminal value of the process at time $T$ of the BB.  |
| t0    | initial time.   |
| T     | final time.   |
| Dt    | time step of the simulation (discretization). If it is <code>NULL</code> a default $\Delta t = \frac{T-t_0}{N}$ . |
| theta | the interest rate of the ABM and GBM.   |
| sigma | the volatility of the ABM and GBM.  |
| ...   | potentially further arguments for (non-default) methods.  |

### Details

The function BM returns a trajectory of the **standard Brownian motion** (Wiener process) in the time interval  $[t_0, T]$ . Indeed, for  $W(dt)$  it holds true that  $W(dt) - W(0) \rightarrow \mathcal{N}(0, dt)$ , where  $\mathcal{N}(0, 1)$  is normal distribution [Normal](#).

The function BB returns a trajectory of the **Brownian bridge** starting at  $x_0$  at time  $t_0$  and ending at  $y$  at time  $T$ ; i.e., the diffusion process solution of stochastic differential equation:

$$dX_t = \frac{y - X_t}{T - t} dt + dW_t$$

The function GBM returns a trajectory of the **geometric Brownian motion** starting at  $x_0$  at time  $t_0$ ; i.e., the diffusion process solution of stochastic differential equation:

$$dX_t = \theta X_t dt + \sigma X_t dW_t$$

The function ABM returns a trajectory of the **arithmetic Brownian motion** starting at  $x_0$  at time  $t_0$ ; i.e.,; the diffusion process solution of stochastic differential equation:

$$dX_t = \theta dt + \sigma dW_t$$

### Value

X an visible ts object.

### Author(s)

A.C. Guidoum, K. Boukhetala.

### References

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- Jedrzejewski, F. (2009). *Modeles aleatoires et physique probabiliste*. Springer-Verlag, New York.
- Henderson, D and Plaschko, P. (2006). *Stochastic differential equations in science and engineering*. World Scientific.

### See Also

This functions [BM](#), [BBridge](#) and [GBM](#) are available in other packages such as "sde".

### Examples

```
op <- par(mfrow = c(2, 2))

## Brownian motion
set.seed(1234)
X <- BM(M = 100)
plot(X, plot.type="single")
lines(as.vector(time(X)), rowMeans(X), col="red")
```

```

## Brownian bridge
set.seed(1234)
X <- BB(M = 100)
plot(X, plot.type = "single")
lines(as.vector(time(X)), rowMeans(X), col = "red")

## Geometric Brownian motion
set.seed(1234)
X <- GBM(M = 100)
plot(X, plot.type = "single")
lines(as.vector(time(X)), rowMeans(X), col = "red")

## Arithmetic Brownian motion
set.seed(1234)
X <- ABM(M = 100)
plot(X, plot.type = "single")
lines(as.vector(time(X)), rowMeans(X), col = "red")

par(op)

```

---

bridgesde1d

*Simulation of 1-D Bridge SDE*


---

## Description

The (S3) generic function `bridgesde1d` for simulation of 1-dim bridge stochastic differential equations, Itô or Stratonovich type, with different methods.

## Usage

```

bridgesde1d(N, ...)
## Default S3 method:
bridgesde1d(N = 1000, M = 1, x0 = 0, y = 0, t0 = 0, T = 1, Dt = NULL,
  drift, diffusion, alpha = 0.5, mu = 0.5, type = c("ito", "str"),
  method = c("euler", "milstein", "predcorr", "smilstein", "taylor",
    "heun", "rk1", "rk2", "rk3"), ...)

## S3 method for class 'bridgesde1d'
summary(object, at, digits = NULL, ...)
## S3 method for class 'bridgesde1d'
time(x, ...)
## S3 method for class 'bridgesde1d'
mean(x, at, ...)
## S3 method for class 'bridgesde1d'
Median(x, at, ...)
## S3 method for class 'bridgesde1d'
Mode(x, at, ...)
## S3 method for class 'bridgesde1d'

```

```

quantile(x, at, ...)
## S3 method for class 'bridgesde1d'
kurtosis(x, at, ...)
## S3 method for class 'bridgesde1d'
skewness(x, at, ...)
## S3 method for class 'bridgesde1d'
min(x, at, ...)
## S3 method for class 'bridgesde1d'
max(x, at, ...)
## S3 method for class 'bridgesde1d'
moment(x, at, ...)
## S3 method for class 'bridgesde1d'
cv(x, at, ...)
## S3 method for class 'bridgesde1d'
bconfint(x, at, ...)

## S3 method for class 'bridgesde1d'
plot(x, ...)
## S3 method for class 'bridgesde1d'
lines(x, ...)
## S3 method for class 'bridgesde1d'
points(x, ...)

```

### Arguments

|           |  |
|-----------|--|
| N         | number of simulation steps.  |
| M         | number of trajectories.  |
| x0        | initial value of the process at time t0.   |
| y         | terminal value of the process at time T.   |
| t0        | initial time.  |
| T         | final time.  |
| Dt        | time step of the simulation (discretization). If it is <code>NULL</code> a default $\Delta t = \frac{T-t_0}{N}$ .                                |
| drift     | drift coefficient: an <a href="#">expression</a> of two variables t and x.   |
| diffusion | diffusion coefficient: an <a href="#">expression</a> of two variables t and x.   |
| alpha, mu | weight of the predictor-corrector scheme; the default alpha = 0.5 and mu = 0.5.  |
| type      | if type="ito" simulation diffusion bridge of Itô type, else type="str" simulation diffusion bridge of Stratonovich type; the default type="ito". |
| method    | numerical methods of simulation, the default method = "euler"; see <a href="#">snssde1d</a> .  |
| x, object | an object inheriting from class "bridgesde1d".   |
| at        | time between t0 and T. Monte-Carlo statistics of the solution $X_t$ at time at. The default at = T/2.  |
| digits    | integer, used for number formatting.   |
| ...       | potentially further arguments for (non-default) methods.   |

**Details**

The function `bridgesde1d` returns a trajectory of the diffusion bridge starting at  $x$  at time  $t_0$  and ending at  $y$  at time  $T$ .

The methods of approximation are classified according to their different properties. Mainly two criteria of optimality are used in the literature: the strong and the weak (orders of) convergence. The method of simulation can be one among: Euler-Maruyama Order 0.5, Milstein Order 1, Milstein Second-Order, Predictor-Corrector method, Itô-Taylor Order 1.5, Heun Order 2 and Runge-Kutta Order 1, 2 and 3.

An overview of this package, see `browseVignettes('Sim.DiffProc')` for more informations.

**Value**

`bridgesde1d` returns an object inheriting from `class "bridgesde1d"`.

|                        |  |
|------------------------|--|
| <code>X</code>         | an invisible <code>ts</code> object.     |
| <code>drift</code>     | drift coefficient.                       |
| <code>diffusion</code> | diffusion coefficient.                   |
| <code>C</code>         | indices of crossing realized of $X(t)$ . |
| <code>type</code>      | type of sde.                             |
| <code>method</code>    | the numerical method used.               |

**Author(s)**

A.C. Guidoum, K. Boukhetala.

**References**

Bladt, M. and Sorensen, M. (2007). Simple simulation of diffusion bridges with application to likelihood inference for diffusions. *Working Paper, University of Copenhagen*.

Iacus, S.M. (2008). *Simulation and inference for stochastic differential equations: with R examples*. Springer-Verlag, New York

**See Also**

`bridgesde2d` and `bridgesde3d` for 2 and 3-dim.

`DBridge` in package "sde".

**Examples**

```
## Example 1: Ito bridge sde
## Ito Bridge sde
## dX(t) = 2*(1-X(t)) *dt + dW(t)
## x0 = 2 at time t0=0 , and y = 1 at time T=1
set.seed(1234)

f <- expression( 2*(1-x) )
g <- expression( 1 )
```

```

mod1 <- bridgesde1d(drift=f,diffusion=g,x0=2,y=1,M=1000)
mod1
summary(mod1) ## Monte-Carlo statistics at T/2=0.5
summary(mod1,at=0.75) ## Monte-Carlo statistics at 0.75
## Not run:
plot(mod1)
lines(time(mod1),apply(mod1$X,1,mean),col=2,lwd=2)
lines(time(mod1),apply(mod1$X,1,bconfint,level=0.95)[1,],col=4,lwd=2)
lines(time(mod1),apply(mod1$X,1,bconfint,level=0.95)[2,],col=4,lwd=2)
legend("topleft",c("mean path",paste("bound of", 95," percent confidence")),
      inset = .01,col=c(2,4),lwd=2,cex=0.8)

## End(Not run)

## Example 2: Stratonovich sde
##  $dX(t) = ((2-X(t))/(2-t)) dt + X(t) \circ dW(t)$ 
##  $x_0 = 2$  at time  $t_0=0$  , and  $y = 2$  at time  $T=1$ 
set.seed(1234)

f <- expression((2-x)/(2-t))
g <- expression(x)
mod2 <- bridgesde1d(type="str",drift=f,diffusion=g,M=1000,x0=2,y=2)
mod2
summary(mod2,at = 0.25) ## Monte-Carlo statistics at 0.25
summary(mod2,at = 0.5) ## Monte-Carlo statistics at 0.5
summary(mod2,at = 0.75) ## Monte-Carlo statistics at 0.75
## Not run:
plot(mod2)
lines(time(mod2),apply(mod2$X,1,mean),col=2,lwd=2)
lines(time(mod2),apply(mod2$X,1,bconfint,level=0.95)[1,],col=4,lwd=2)
lines(time(mod2),apply(mod2$X,1,bconfint,level=0.95)[2,],col=4,lwd=2)
legend("topright",c("mean path",paste("bound of", 95," percent confidence")),
      inset = .01,col=c(2,4),lwd=2,cex=0.8)

## End(Not run)

```

---

bridgesde2d

*Simulation of 2-D Bridge SDE's*


---

## Description

The (S3) generic function `bridgesde2d` for simulation of 2-dim bridge stochastic differential equations, Itô or Stratonovich type, with different methods.

## Usage

```

bridgesde2d(N, ...)
## Default S3 method:
bridgesde2d(N = 1000, M = 1, x0 = c(0, 0),
            y = c(0, 0), t0 = 0, T = 1, Dt=NULL, drift, diffusion,

```

```

alpha = 0.5, mu = 0.5, type = c("ito", "str"), method =
c("euler", "milstein", "predcorr", "smilstein", "taylor",
"heun", "rk1", "rk2", "rk3"), ...)

## S3 method for class 'bridgesde2d'
summary(object, at,
         digits=NULL, ...)
## S3 method for class 'bridgesde2d'
time(x, ...)
## S3 method for class 'bridgesde2d'
mean(x, at, ...)
## S3 method for class 'bridgesde2d'
Median(x, at, ...)
## S3 method for class 'bridgesde2d'
Mode(x, at, ...)
## S3 method for class 'bridgesde2d'
quantile(x, at, ...)
## S3 method for class 'bridgesde2d'
kurtosis(x, at, ...)
## S3 method for class 'bridgesde2d'
skewness(x, at, ...)
## S3 method for class 'bridgesde2d'
min(x, at, ...)
## S3 method for class 'bridgesde2d'
max(x, at, ...)
## S3 method for class 'bridgesde2d'
moment(x, at, ...)
## S3 method for class 'bridgesde2d'
cv(x, at, ...)
## S3 method for class 'bridgesde2d'
bconfint(x, at, ...)

## S3 method for class 'bridgesde2d'
plot(x, ...)
## S3 method for class 'bridgesde2d'
lines(x, ...)
## S3 method for class 'bridgesde2d'
points(x, ...)
## S3 method for class 'bridgesde2d'
plot2d(x, ...)
## S3 method for class 'bridgesde2d'
lines2d(x, ...)
## S3 method for class 'bridgesde2d'
points2d(x, ...)

```

### Arguments

N                    number of simulation steps.

|           |  |
|-----------|--|
| M         | number of trajectories.  |
| x0        | initial value (numeric vector of length 2) of the process $X_t$ and $Y_t$ at time $t_0$ .  |
| y         | terminal value (numeric vector of length 2) of the process $X_t$ and $Y_t$ at time $T$ .   |
| t0        | initial time.  |
| T         | final time.  |
| Dt        | time step of the simulation (discretization). If it is <code>NULL</code> a default $\Delta t = \frac{T-t_0}{N}$ .                                |
| drift     | drift coefficient: an <a href="#">expression</a> of three variables t, x and y for process $X_t$ and $Y_t$ .                                     |
| diffusion | diffusion coefficient: an <a href="#">expression</a> of three variables t, x and y for process $X_t$ and $Y_t$ .                                 |
| alpha, mu | weight of the predictor-corrector scheme; the default alpha = 0.5 and mu = 0.5.  |
| type      | if type="ito" simulation diffusion bridge of Itô type, else type="str" simulation diffusion bridge of Stratonovich type; the default type="ito". |
| method    | numerical methods of simulation, the default method = "euler"; see <a href="#">snsde2d</a> .   |
| x, object | an object inheriting from class "bridgesde2d".   |
| at        | time between t0 and T. Monte-Carlo statistics of the solution $(X_t, Y_t)$ at time at. The default at = T/2.                                     |
| digits    | integer, used for number formatting.   |
| ...       | potentially further arguments for (non-default) methods.   |

## Details

The function `bridgesde2d` returns a `mts` of the diffusion bridge starting at `x` at time `t0` and ending at `y` at time `T`.

The methods of approximation are classified according to their different properties. Mainly two criteria of optimality are used in the literature: the strong and the weak (orders of) convergence. The method of simulation can be one among: Euler-Maruyama Order 0.5, Milstein Order 1, Milstein Second-Order, Predictor-Corrector method, Itô-Taylor Order 1.5, Heun Order 2 and Runge-Kutta Order 1, 2 and 3.

An overview of this package, see [browseVignettes\('Sim.DiffProc'\)](#) for more informations.

## Value

`bridgesde2d` returns an object inheriting from [class](#) "bridgesde2d".

|                |   |
|----------------|---|
| X, Y           | an invisible <code>mts</code> (2-dim) object $(X(t), Y(t))$ . |
| driftx, drifty | drift coefficient of $X(t)$ and $Y(t)$ .                      |
| diffx, diffy   | diffusion coefficient of $X(t)$ and $Y(t)$ .                  |
| Cx, Cy         | indices of crossing realized of $X(t)$ and $Y(t)$ .           |
| type           | type of sde.  |
| method         | the numerical method used.                                    |

**Author(s)**

A.C. Guidoum, K. Boukhetala.

**References**

Bladt, M. and Sorensen, M. (2007). Simple simulation of diffusion bridges with application to likelihood inference for diffusions. *Working Paper, University of Copenhagen*.

Iacus, S.M. (2008). *Simulation and inference for stochastic differential equations: with R examples*. Springer-Verlag, New York

**See Also**

[bridgesde1d](#) for simulation of 1-dim SDE.

[DBridge](#) in package "sde".

**Examples**

```
## dX(t) = 4*(-1-X(t)) dt + 0.2 dW1(t)
## dY(t) = X(t) dt + 0 dW2(t)
## x01 = 0 , y01 = 0
## x02 = 0, y02 = 0
## W1(t) and W2(t) two independent Brownian motion
set.seed(1234)

fx <- expression(4*(-1-x) , x)
gx <- expression(0.2 , 0)
res <- bridgesde2d(drift=fx,diffusion=gx,Dt=0.005,M=500)
res
summary(res) ## Monte-Carlo statistics at time T/2=2.5
summary(res,at=1) ## Monte-Carlo statistics at time 1
summary(res,at=4) ## Monte-Carlo statistics at time 4
##
plot(res,type="n")
lines(time(res),apply(res$X,1,mean),col=3,lwd=2)
lines(time(res),apply(res$Y,1,mean),col=4,lwd=2)
legend("topright",c(expression(E(X[t])),expression(E(Y[t]))),lty=1,inset = .7,col=c(3,4))
##
plot2d(res)
```

---

bridgesde3d

*Simulation of 3-D Bridge SDE's*


---

**Description**

The (S3) generic function `bridgesde3d` for simulation of 3-dim bridge stochastic differential equations, Itô or Stratonovich type, with different methods.

**Usage**

```

bridgesde3d(N, ...)
## Default S3 method:
bridgesde3d(N=1000,M=1, x0=c(0,0,0),
  y=c(0,0,0), t0 = 0, T = 1, Dt=NULL, drift, diffusion,
  alpha = 0.5, mu = 0.5,type = c("ito", "str"), method =
  c("euler", "milstein","predcorr","smilstein", "taylor",
  "heun","rk1", "rk2", "rk3"), ...)

## S3 method for class 'bridgesde3d'
summary(object, at,
  digits=NULL, ...)
## S3 method for class 'bridgesde3d'
time(x, ...)
## S3 method for class 'bridgesde3d'
mean(x, at, ...)
## S3 method for class 'bridgesde3d'
Median(x, at, ...)
## S3 method for class 'bridgesde3d'
Mode(x, at, ...)
## S3 method for class 'bridgesde3d'
quantile(x, at, ...)
## S3 method for class 'bridgesde3d'
kurtosis(x, at, ...)
## S3 method for class 'bridgesde3d'
skewness(x, at, ...)
## S3 method for class 'bridgesde3d'
min(x, at, ...)
## S3 method for class 'bridgesde3d'
max(x, at, ...)
## S3 method for class 'bridgesde3d'
moment(x, at, ...)
## S3 method for class 'bridgesde3d'
cv(x, at, ...)
## S3 method for class 'bridgesde3d'
bconfint(x, at, ...)

## S3 method for class 'bridgesde3d'
plot(x, ...)
## S3 method for class 'bridgesde3d'
lines(x, ...)
## S3 method for class 'bridgesde3d'
points(x, ...)
## S3 method for class 'bridgesde3d'
plot3D(x, display = c("persp","rgl"), ...)

```

**Arguments**

|           |  |
|-----------|--|
| N         | number of simulation steps.  |
| M         | number of trajectories.  |
| x0        | initial value (numeric vector of length 3) of the process $X_t, Y_t$ and $Z_t$ at time $t_0$ .   |
| y         | terminal value (numeric vector of length 3) of the process $X_t, Y_t$ and $Z_t$ at time $T$ .  |
| t0        | initial time.  |
| T         | final time.  |
| dt        | time step of the simulation (discretization). If it is <code>NULL</code> a default $\Delta t = \frac{T-t_0}{N}$ .                                |
| drift     | drift coefficient: an <a href="#">expression</a> of four variables $t, x, y$ and $z$ for process $X_t, Y_t$ and $Z_t$ .                          |
| diffusion | diffusion coefficient: an <a href="#">expression</a> of four variables $t, x, y$ and $z$ for process $X_t, Y_t$ and $Z_t$ .                      |
| alpha     | weight alpha of the predictor-corrector scheme; the default alpha = 0.5.   |
| mu        | weight mu of the predictor-corrector scheme; the default mu = 0.5.   |
| type      | if type="ito" simulation diffusion bridge of Itô type, else type="str" simulation diffusion bridge of Stratonovich type; the default type="ito". |
| method    | numerical methods of simulation, the default method = "euler"; see <a href="#">snsdde3d</a> .  |
| x, object | an object inheriting from class "bridgesde3d".   |
| at        | time between $t_0$ and $T$ . Monte-Carlo statistics of the solution $(X_t, Y_t, Z_t)$ at time at. The default at = $T/2$ .                       |
| digits    | integer, used for number formatting.   |
| display   | "persp" perspective and "rgl" plots.   |
| ...       | potentially further arguments for (non-default) methods.   |

**Details**

The function `bridgesde3d` returns a `mts` of the diffusion bridge starting at  $x$  at time  $t_0$  and ending at  $y$  at time  $T$ .

The methods of approximation are classified according to their different properties. Mainly two criteria of optimality are used in the literature: the strong and the weak (orders of) convergence. The method of simulation can be one among: Euler-Maruyama Order 0.5, Milstein Order 1, Milstein Second-Order, Predictor-Corrector method, Itô-Taylor Order 1.5, Heun Order 2 and Runge-Kutta Order 1, 2 and 3.

An overview of this package, see [browseVignettes\('Sim.DiffProc'\)](#) for more informations.

**Value**

`bridgesde3d` returns an object inheriting from `class` "bridgesde3d".

|                        |   |
|------------------------|---|
| X, Y, Z                | an invisible <code>mts</code> (3-dim) object $(X(t), Y(t), Z(t))$ . |
| driftx, drifty, driftz | drift coefficient of $X(t), Y(t)$ and $Z(t)$ .                      |

diffx, diffy, diffz      diffusion coefficient of X(t), Y(t) and Z(t).  
 Cx, Cy, Cz      indices of crossing realized of X(t), Y(t) and Z(t).  
 type      type of sde.  
 method      the numerical method used.

**Author(s)**

A.C. Guidoum, K. Boukhetala.

**References**

Bladt, M. and Sorensen, M. (2007). Simple simulation of diffusion bridges with application to likelihood inference for diffusions. *Working Paper, University of Copenhagen*.

Iacus, S.M. (2008). *Simulation and inference for stochastic differential equations: with R examples*. Springer-Verlag, New York

**See Also**

[bridgesde1d](#) for simulation of 1-dim SDE. [DBridge](#) in package "sde".

[bridgesde2d](#) for simulation of 2-dim SDE.

**Examples**

```
## dX(t) = 4*(-1-X(t))*Y(t) dt + 0.2 * dW1(t) ; x01 = 0 and y01 = 0
## dY(t) = 4*(1-Y(t)) *X(t) dt + 0.2 * dW2(t) ; x02 = -1 and y02 = -2
## dZ(t) = 4*(1-Z(t)) *Y(t) dt + 0.2 * dW3(t) ; x03 = 0.5 and y03 = 0.5
## W1(t), W2(t) and W3(t) three independent Brownian motion
set.seed(1234)

fx <- expression(4*(-1-x)*y, 4*(1-y)*x, 4*(1-z)*y)
gx <- rep(expression(0.2),3)

res <- bridgesde3d(x0=c(0,-1,0.5),y=c(0,-2,0.5),drift=fx,diffusion=gx,M=200)
res
summary(res) ## Monte-Carlo statistics at time T/2=0.5
summary(res,at=0.25) ## Monte-Carlo statistics at time 0.25
summary(res,at=0.75) ## Monte-Carlo statistics at time 0.75
##
plot(res,type="n")
lines(time(res),apply(res$X,1,mean),col=3,lwd=2)
lines(time(res),apply(res$Y,1,mean),col=4,lwd=2)
lines(time(res),apply(res$Z,1,mean),col=5,lwd=2)
legend("topleft",c(expression(E(X[t])),expression(E(Y[t])),
  expression(E(Z[t]))),lty=1,inset = .01,col=c(3,4,5))
##
plot3D(res,display = "persp",main="3-dim bridge sde")
```

fitsde

*Maximum Pseudo-Likelihood Estimation of 1-D SDE***Description**

The (S3) generic function "fitsde" of estimate drift and diffusion parameters by the method of maximum pseudo-likelihood of the 1-dim stochastic differential equation.

**Usage**

```
fitsde(data, ...)
## Default S3 method:
fitsde(data, drift, diffusion, start = list(), pmle = c("euler", "kessler",
  "ozaki", "shoji"), optim.method = "L-BFGS-B",
  lower = -Inf, upper = Inf, ...)

## S3 method for class 'fitsde'
summary(object, ...)
## S3 method for class 'fitsde'
coef(object, ...)
## S3 method for class 'fitsde'
vcov(object, ...)
## S3 method for class 'fitsde'
logLik(object, ...)
## S3 method for class 'fitsde'
AIC(object, ...)
## S3 method for class 'fitsde'
BIC(object, ...)
## S3 method for class 'fitsde'
confint(object, parm, level=0.95, ...)
```

**Arguments**

|              |  |
|--------------|--|
| data         | a univariate time series ( <code>ts</code> class).   |
| drift        | drift coefficient: an <a href="#">expression</a> of two variables <code>t</code> , <code>x</code> and <code>theta</code> a vector of parameters of sde. See Examples.  |
| diffusion    | diffusion coefficient: an <a href="#">expression</a> of two variables <code>t</code> , <code>x</code> and <code>theta</code> a vector of parameters of sde. See Examples.  |
| start        | named list of starting values for optimizer. See Examples.   |
| pmle         | a <a href="#">character</a> string specifying the method; can be either: "euler" (Euler pseudo-likelihood), "ozaki" (Ozaki pseudo-likelihood), "shoji" (Shoji pseudo-likelihood), and "kessler" (Kessler pseudo-likelihood). |
| optim.method | the method for <a href="#">optim</a> .   |
| lower, upper | bounds on the variables for the "Brent" or "L-BFGS-B" method.  |
| object       | an object inheriting from class "fitsde".  |

|       |  |
|-------|--|
| parm  | a specification of which parameters are to be given confidence intervals, either a vector of names (example parm='theta1'). If missing, all parameters are considered. |
| level | the confidence level required.   |
| ...   | potentially further arguments to pass to <a href="#">optim</a> .   |

### Details

The function `fitsde` returns a pseudo-likelihood estimators of the drift and diffusion parameters in 1-dim stochastic differential equation. The `optim` optimizer is used to find the maximum of the negative log pseudo-likelihood. An approximate covariance matrix for the parameters is obtained by inverting the Hessian matrix at the optimum.

The pmle of pseudo-likelihood can be one among: "euler": Euler pseudo-likelihood), "ozaki": Ozaki pseudo-likelihood, "shoji": Shoji pseudo-likelihood, and "kessler": Kessler pseudo-likelihood.

An overview of this package, see [browseVignettes\('Sim.DiffProc'\)](#) for more informations.

### Value

`fitsde` returns an object inheriting from `class "fitsde"`.

### Author(s)

A.C. Guidoum, K. Boukhetala.

### References

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B.L.S. Prakasa Rao. (1999). *Statistical Inference for Diffusion Type Processes*. Arnold, London and Oxford University press, New York.

Kutoyants, Y.A. (2004). *Statistical Inference for Ergodic Diffusion Processes*. Springer, London.

### See Also

[dcEuler](#), [dcElerian](#), [dcOzaki](#), [dcShoji](#), [dcKessler](#) and [dcSim](#) for approximated conditional law of a diffusion process. [gmm](#) estimator of the generalized method of moments by Hansen, and [HPloglik](#) these functions are useful to calculate approximated maximum likelihood estimators when the transition density of the process is not known, in package "sde".

[qmle](#) in package "yuima" calculate quasi-likelihood and ML estimator of least squares estimator.

### Examples

```
##### Example 1:

## Modele GBM (BS)
## dX(t) = theta1 * X(t) * dt + theta2 * x * dW(t)
## Simulation of data
set.seed(1234)

X <- GBM(N =1000,theta=4,sigma=1)
## Estimation: true theta=c(4,1)
fx <- expression(theta[1]*x)
gx <- expression(theta[2]*x)

fres <- fitsde(data=X,drift=fx,diffusion=gx,start = list(theta1=1,theta2=1),
              lower=c(0,0))

fres
summary(fres)
coef(fres)
logLik(fres)
AIC(fres)
BIC(fres)
vcov(fres)
confint(fres,level=0.95)

##### Example 2:

## Nonlinear mean reversion (Ait-Sahalia) modele
## dX(t) = (theta1 + theta2*x + theta3*x^2) * dt + theta4 * x^theta5 * dW(t)
## Simulation of the process X(t)
set.seed(1234)
```

```

f <- expression(1 - 11*x + 2*x^2)
g <- expression(x^0.5)
res <- snssde1d(drift=f,diffusion=g,M=1,N=1000,Dt=0.001,x0=5)
mydata1 <- res$X

## Estimation
## true param theta= c(1,-11,2,1,0.5)
true <- c(1,-11,2,1,0.5)
pmle <- eval(formals(fitsde.default)$pmle)

fx <- expression(theta[1] + theta[2]*x + theta[3]*x^2)
gx <- expression(theta[4]*x^theta[5])

fres <- lapply(1:4, function(i) fitsde(mydata1,drift=fx,diffusion=gx,
      pmle=pmle[i],start = list(theta1=1,theta2=1,theta3=1,theta4=1,
      theta5=1),optim.method = "L-BFGS-B"))
Coef <- data.frame(true,do.call("cbind",lapply(1:4,function(i) coef(fres[[i]]))))
names(Coef) <- c("True",pmle)
Summary <- data.frame(do.call("rbind",lapply(1:4,function(i) logLik(fres[[i]]))),
      do.call("rbind",lapply(1:4,function(i) AIC(fres[[i]]))),
      do.call("rbind",lapply(1:4,function(i) BIC(fres[[i]]))),
      row.names=pmle)
names(Summary) <- c("logLik","AIC","BIC")
Coef
Summary

##### Example 3:

## dX(t) = (theta1*x*t+theta2*tan(x)) *dt + theta3*t *dW(t)
## Simulation of data
set.seed(1234)

f <- expression(2*x*t-tan(x))
g <- expression(1.25*t)
sim <- snssde1d(drift=f,diffusion=g,M=1,N=1000,Dt=0.001,x0=10)
mydata2 <- sim$X

## Estimation
## true param theta= c(2,-1,1.25)
true <- c(2,-1,1.25)

fx <- expression(theta[1]*x*t+theta[2]*tan(x))
gx <- expression(theta[3]*t)

fres <- lapply(1:4, function(i) fitsde(mydata2,drift=fx,diffusion=gx,
      pmle=pmle[i],start = list(theta1=1,theta2=1,theta3=1),
      optim.method = "L-BFGS-B"))
Coef <- data.frame(true,do.call("cbind",lapply(1:4,function(i) coef(fres[[i]]))))
names(Coef) <- c("True",pmle)
Summary <- data.frame(do.call("rbind",lapply(1:4,function(i) logLik(fres[[i]]))),
      do.call("rbind",lapply(1:4,function(i) AIC(fres[[i]]))),
      do.call("rbind",lapply(1:4,function(i) BIC(fres[[i]]))),

```

```

                                row.names=pmle)
names(Summary) <- c("logLik","AIC","BIC")
Coef
Summary

##### Example 4:

## Application to real data
## CKLS modele vs CIR modele
## CKLS (mod1):  $dX(t) = (\theta_1 + \theta_2 * X(t)) * dt + \theta_3 * X(t)^{\theta_4} * dW(t)$ 
## CIR (mod2):  $dX(t) = (\theta_1 + \theta_2 * X(t)) * dt + \theta_3 * \sqrt{X(t)} * dW(t)$ 
set.seed(1234)

data(Irates)
rates <- Irates[, "r1"]
rates <- window(rates, start=1964.471, end=1989.333)

fx1 <- expression(theta[1]+theta[2]*x)
gx1 <- expression(theta[3]*x^theta[4])
gx2 <- expression(theta[3]*sqrt(x))

fitmod1 <- fitsde(rates,drift=fx1,diffusion=gx1,pmle="euler",start = list(theta1=1,theta2=1,
                                theta3=1,theta4=1),optim.method = "L-BFGS-B")
fitmod2 <- fitsde(rates,drift=fx1,diffusion=gx2,pmle="euler",start = list(theta1=1,theta2=1,
                                theta3=1),optim.method = "L-BFGS-B")

summary(fitmod1)
summary(fitmod2)
coef(fitmod1)
coef(fitmod2)
confint(fitmod1,parm=c('theta2','theta3'))
confint(fitmod2,parm=c('theta2','theta3'))
AIC(fitmod1)
AIC(fitmod2)

## Display
## CKLS Modele
op <- par(mfrow = c(1, 2))
theta <- coef(fitmod1)
N <- length(rates)
res <- snssde1d(drift=fx1,diffusion=gx1,M=200,t0=time(rates)[1],T=time(rates)[N],
               Dt=deltat(rates),x0=rates[1],N)
plot(res,plot.type="single",ylim=c(0,40))
lines(rates,col=2,lwd=2)
legend("topleft",c("real data","CKLS modele"),inset = .01,col=c(2,1),lwd=2,cex=0.8)

## CIR Modele
theta <- coef(fitmod2)
res <- snssde1d(drift=fx1,diffusion=gx2,M=200,t0=time(rates)[1],T=time(rates)[N],
               Dt=deltat(rates),x0=rates[1],N)
plot(res,plot.type="single",ylim=c(0,40))
lines(rates,col=2,lwd=2)
legend("topleft",c("real data","CIR modele"),inset = .01,col=c(2,1),lwd=2,cex=0.8)

```

par(op)

---

|          |  |
|----------|--|
| fptsde1d | <i>Approximate densities and random generation for first passage time in 1-D SDE</i> |
|----------|--|

---

### Description

Kernel density and random generation for first-passage-time (f.p.t) in 1-dim stochastic differential equations.

### Usage

```
fptsde1d(object, ...)
dfptsde1d(object, ...)

## Default S3 method:
fptsde1d(object, boundary, ...)
## S3 method for class 'fptsde1d'
summary(object, digits=NULL, ...)
## S3 method for class 'fptsde1d'
mean(x, ...)
## S3 method for class 'fptsde1d'
Median(x, ...)
## S3 method for class 'fptsde1d'
Mode(x, ...)
## S3 method for class 'fptsde1d'
quantile(x, ...)
## S3 method for class 'fptsde1d'
kurtosis(x, ...)
## S3 method for class 'fptsde1d'
skewness(x, ...)
## S3 method for class 'fptsde1d'
min(x, ...)
## S3 method for class 'fptsde1d'
max(x, ...)
## S3 method for class 'fptsde1d'
moment(x, ...)
## S3 method for class 'fptsde1d'
cv(x, ...)

## Default S3 method:
dfptsde1d(object, ...)
## S3 method for class 'dfptsde1d'
plot(x, hist=FALSE, ...)
```

**Arguments**

|          |   |
|----------|---|
| object   | an object inheriting from class <code>snsde1d</code> for <code>fptsde1d</code> , and <code>fptsde1d</code> for <code>dfptsde1d</code> . |
| boundary | an <a href="#">expression</a> of a constant or time-dependent boundary.   |
| x        | an object inheriting from class <code>dfptsde1d</code> .  |
| hist     | if <code>hist=TRUE</code> plot histogram. Based on <a href="#">truehist</a> function.   |
| digits   | integer, used for number formatting.  |
| ...      | potentially further arguments for (non-default) methods, such as <a href="#">density</a> for <code>dfptsde1d</code> .                   |

**Details**

The function `fptsde1d` returns a random variable  $\tau_{(X(t),S(t))}$  "first passage time", is defined as :

$$\tau_{(X(t),S(t))} = \{t \geq 0; X_t \geq S(t)\}, \quad \text{if } X(t_0) < S(t_0)$$

$$\tau_{(X(t),S(t))} = \{t \geq 0; X_t \leq S(t)\}, \quad \text{if } X(t_0) > S(t_0)$$

And `dfptsde1d` returns a kernel density approximation for  $\tau_{(X(t),S(t))}$  "first passage time". with  $S(t)$  is through a continuous boundary (barrier).

An overview of this package, see [browseVignettes\('Sim.DiffProc'\)](#) for more informations.

**Value**

`dfptsde1d` gives the density estimate of `fpt`. `fptsde1d` generates random of `fpt`.

**Author(s)**

A.C. Guidoum, K. Boukhetala.

**References**

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### See Also

[fptsde2d](#) and [fptsde3d](#) simulation fpt for 2 and 3-dim SDE.

[FPTL](#) for computes values of the first passage time location (FPTL) function, and [Approx.fpt.density](#) for approximate first-passage-time (f.p.t.) density in package "fptdApprox".

[GQD.TIpassage](#) for compute the First Passage Time Density of a GQD With Time Inhomogeneous Coefficients in package "DiffusionRgqd".

### Examples

```
## Example 1: Ito SDE
## dX(t) = -4*X(t) *dt + 0.5*dW(t)
## S(t) = 0 (constant boundary)
set.seed(1234)

# SDE 1d
f <- expression( -4*x )
g <- expression( 0.5 )
mod <- snssde1d(drift=f,diffusion=g,x0=2,M=1000)

# boundary
St <- expression(0)

# random
out <- fptsde1d(mod, boundary=St)
out
summary(out)
# density approximate
den <- dfptsde1d(out)
den
plot(den)

## Example 2: Stratonovich SDE
## dX(t) = 0.5*X(t)*t *dt + sqrt(1+X(t)^2) o dW(t)
## S(t) = -0.5*sqrt(t) + exp(t^2) (time-dependent boundary)
set.seed(1234)

# SDE 1d
f <- expression( 0.5*x*t )
g <- expression( sqrt(1+x^2) )
mod2 <- snssde1d(drift=f,diffusion=g,x0=2,M=1000,type="srt")
```

```

# boundary
St <- expression(-0.5*sqrt(t)+exp(t^2))

# random
out2 <- fptsde1d(mod2,boundary=St)
out2
summary(out2)
# density approximate
plot(dfptsde1d(out2,bw='ucv'))

## Example 3: fptsde1d vs fptdApproximate
## Not run:
f <- expression( -0.5*x+0.5*5 )
g <- expression( 1 )
St <- expression(5+0.25*sin(2*pi*t))
mod <- snssde1d(drift=f,diffusion=g,boundary=St,x0=3,T=10,N=10^4,M =10000)
mod

# random
out3 <- fptsde1d(mod,boundary=St)
out3
summary(out3)
# density approximate:
library("fptdApprox")
# Under `fptdApprox`:
# Define the diffusion process and give its transitional density:
OU <- diffproc(c("alpha*x + beta","sigma^2",
"dnorm((x-(y*exp(alpha*(t-s)) - beta*(1 - exp(alpha*(t-s)))/alpha))/
(sigma*sqrt((exp(2*alpha*(t-s)) - 1)/(2*alpha))),0,1)/
(sigma*sqrt((exp(2*alpha*(t-s)) - 1)/(2*alpha)))",
"pnorm(x, y*exp(alpha*(t-s)) - beta*(1 - exp(alpha*(t-s)))/alpha,
sigma*sqrt((exp(2*alpha*(t-s)) - 1)/(2*alpha)))")
# Approximate the first passage time density for OU, starting in  $X_0 = 3$ 
# passing through  $5+0.25\sin(2\pi t)$  on the time interval  $[0,10]$ :
res <- Approx.fpt.density(OU, 0, 10, 3,"5+0.25*sin(2*pi*t)", list(alpha=-0.5,beta=0.5*5,sigma=1))

##
plot(dfptsde1d(out3,bw='ucv'),main = 'fptsde1d vs fptdApproximate')
lines(res$y~res$x, type = 'l',lwd=2)
legend('topright', lty = c('solid', 'dashed'), col = c(1, 2),
       legend = c('fptdApproximate', 'fptsde1d'), lwd = 2, bty = 'n')

## End(Not run)

```

**Description**

Kernel density and random generation for first-passage-time (f.p.t) in 2-dim stochastic differential equations.

**Usage**

```
fptsde2d(object, ...)
dfptsde2d(object, ...)

## Default S3 method:
fptsde2d(object, boundary, ...)
## S3 method for class 'fptsde2d'
summary(object, digits=NULL, ...)
## S3 method for class 'fptsde2d'
mean(x, ...)
## S3 method for class 'fptsde2d'
Median(x, ...)
## S3 method for class 'fptsde2d'
Mode(x, ...)
## S3 method for class 'fptsde2d'
quantile(x, ...)
## S3 method for class 'fptsde2d'
kurtosis(x, ...)
## S3 method for class 'fptsde2d'
skewness(x, ...)
## S3 method for class 'fptsde2d'
min(x, ...)
## S3 method for class 'fptsde2d'
max(x, ...)
## S3 method for class 'fptsde2d'
moment(x, ...)
## S3 method for class 'fptsde2d'
cv(x, ...)

## Default S3 method:
dfptsde2d(object, pdf=c("Joint","Marginal"), ...)
## S3 method for class 'dfptsde2d'
plot(x,display=c("persp","rgl","image","contour"),
      hist=FALSE, ...)
```

**Arguments**

|          |  |
|----------|--|
| object   | an object inheriting from class <a href="#">snssde2d</a> for fptsde2d, and <a href="#">fptsde2d</a> for dfptsde2d. |
| boundary | an <a href="#">expression</a> of a constant or time-dependent boundary.  |
| pdf      | probability density function Joint or Marginal.  |
| x        | an object inheriting from class fptsde2d.  |
| digits   | integer, used for number formatting.   |

|         |   |
|---------|---|
| display | display plots.  |
| hist    | if hist=TRUE plot histogram. Based on <a href="#">truehist</a> function.  |
| ...     | potentially further arguments for (non-default) methods. arguments to be passed to methods, such as <a href="#">density</a> for marginal density and <a href="#">kde2d</a> for joint density. |

### Details

The function `fptsde1d` returns a random variable  $(\tau_{(X(t),S(t))}, \tau_{(Y(t),S(t))})$  "first passage time", is defined as :

$$\tau_{(X(t),S(t))} = \{t \geq 0; X_t \geq S(t)\}, \quad \text{if } X(t_0) < S(t_0)$$

$$\tau_{(Y(t),S(t))} = \{t \geq 0; Y_t \geq S(t)\}, \quad \text{if } Y(t_0) < S(t_0)$$

and:

$$\tau_{(X(t),S(t))} = \{t \geq 0; X_t \leq S(t)\}, \quad \text{if } X(t_0) > S(t_0)$$

$$\tau_{(Y(t),S(t))} = \{t \geq 0; Y_t \leq S(t)\}, \quad \text{if } Y(t_0) > S(t_0)$$

And `dfptsde2d` returns a kernel density approximation for  $(\tau_{(X(t),S(t))}, \tau_{(Y(t),S(t))})$  "first passage time". with  $S(t)$  is through a continuous boundary (barrier).

An overview of this package, see `browseVignettes('Sim.DiffProc')` for more informations.

### Value

`dfptsde2d` gives the kernel density approximation for `fpt`. `fptsde2d` generates random of `fpt`.

### Author(s)

A.C. Guidoum, K. Boukhetala.

### References

- Argyrakisa, P. and G.H. Weiss (2006). A first-passage time problem for many random walkers. *Physica A*, **363**, 343–347.
- Aytug H., G. J. Koehler (2000). New stopping criterion for genetic algorithms. *European Journal of Operational Research*, **126**, 662–674.
- Boukhetala, K. (1996) Modelling and simulation of a dispersion pollutant with attractive centre. ed by Computational Mechanics Publications, Southampton ,U.K and Computational Mechanics Inc, Boston, USA, 245–252.
- Boukhetala, K. (1998a). Estimation of the first passage time distribution for a simulated diffusion process. *Maghreb Math.Rev*, **7**(1), 1–25.
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- Ding, M. and G. Rangarajan. (2004). First Passage Time Problem: A Fokker-Planck Approach. *New Directions in Statistical Physics*. ed by L. T. Wille. Springer. 31–46.
- Roman, R.P., Serrano, J. J., Torres, F. (2008). First-passage-time location function: Application to determine first-passage-time densities in diffusion processes. *Computational Statistics and Data Analysis*. **52**, 4132–4146.

Roman, R.P., Serrano, J. J., Torres, F. (2012). An R package for an efficient approximation of first-passage-time densities for diffusion processes based on the FPTL function. *Applied Mathematics and Computation*, **218**, 8408–8428.

Gardiner, C. W. (1997). *Handbook of Stochastic Methods*. Springer-Verlag, New York.

### See Also

[fptsde1d](#) for simulation fpt in sde 1-dim. [fptsde3d](#) for simulation fpt in sde 3-dim.

[FPTL](#) for computes values of the first passage time location (FPTL) function, and [Approx.fpt.density](#) for approximate first-passage-time (f.p.t.) density in package "fptdApprox".

[GQD.TIpassage](#) for compute the First Passage Time Density of a GQD With Time Inhomogeneous Coefficients in package "DiffusionRgqd".

### Examples

```
## dX(t) = 5*(-1-Y(t))*X(t) * dt + 0.5 * dW1(t)
## dY(t) = 5*(-1-X(t))*Y(t) * dt + 0.5 * dW2(t)
## x0 = 2, y0 = -2, and barrier -3+5*t.
## W1(t) and W2(t) two independent Brownian motion
set.seed(1234)

# SDE's 2d
fx <- expression(5*(-1-y)*x , 5*(-1-x)*y)
gx <- expression(0.5 , 0.5)
mod2d <- snssde2d(drift=fx,diffusion=gx,x0=c(2,-2),M=100)

# boundary

St <- expression(-1+5*t)

# random fpt

out <- fptsde2d(mod2d,boundary=St)
out
summary(out)

# Marginal density

denM <- dfptsde2d(out,pdf="M")
denM
plot(denM)

# Joint density

denJ <- dfptsde2d(out,pdf="J",n=200,lims=c(0.28,0.4,0.04,0.13))
denJ
plot(denJ)
plot(denJ,display="image")
plot(denJ,display="image",drawpoints=TRUE,cex=0.5,pch=19,col.pt='green')
plot(denJ,display="contour")
```

```
plot(denJ,display="contour",color.palette=colorRampPalette(c('white','green','blue','red')))
```

---

|          |  |
|----------|--|
| fptsde3d | <i>Approximate densities and random generation for first passage time in 3-D SDE's</i> |
|----------|--|

---

## Description

Kernel density and random generation for first-passage-time (f.p.t) in 3-dim stochastic differential equations.

## Usage

```
fptsde3d(object, ...)
dfptsde3d(object, ...)

## Default S3 method:
fptsde3d(object, boundary, ...)
## S3 method for class 'fptsde3d'
summary(object, digits=NULL, ...)
## S3 method for class 'fptsde3d'
mean(x, ...)
## S3 method for class 'fptsde3d'
Median(x, ...)
## S3 method for class 'fptsde3d'
Mode(x, ...)
## S3 method for class 'fptsde3d'
quantile(x, ...)
## S3 method for class 'fptsde3d'
kurtosis(x, ...)
## S3 method for class 'fptsde3d'
skewness(x, ...)
## S3 method for class 'fptsde3d'
min(x, ...)
## S3 method for class 'fptsde3d'
max(x, ...)
## S3 method for class 'fptsde3d'
moment(x, ...)
## S3 method for class 'fptsde3d'
cv(x, ...)

## Default S3 method:
dfptsde3d(object, pdf=c("Joint","Marginal"), ...)
## S3 method for class 'dfptsde3d'
plot(x,display="rgl",hist=FALSE, ...)
```

**Arguments**

|          |   |
|----------|---|
| object   | an object inheriting from class <a href="#">snssde3d</a> for fptsde3d, and <a href="#">fptsde3d</a> for dfptsde3d.                                    |
| boundary | an <a href="#">expression</a> of a constant or time-dependent boundary.   |
| pdf      | probability density function Joint or Marginal.   |
| x        | an object inheriting from class dfptsde3d.  |
| digits   | integer, used for number formatting.  |
| display  | display plots.  |
| hist     | if hist=TRUE plot histogram. Based on <a href="#">truehist</a> function.  |
| ...      | potentially arguments to be passed to methods, such as <a href="#">density</a> for marginal density and <a href="#">sm.density</a> for joint density. |

**Details**

The function fptsde3d returns a random variable  $(\tau_{(X(t),S(t))}, \tau_{(Y(t),S(t))}, \tau_{(Z(t),S(t))})$  "first passage time", is defined as :

$$\tau_{(X(t),S(t))} = \{t \geq 0; X_t \geq S(t)\}, \quad \text{if } X(t_0) < S(t_0)$$

$$\tau_{(Y(t),S(t))} = \{t \geq 0; Y_t \geq S(t)\}, \quad \text{if } Y(t_0) < S(t_0)$$

$$\tau_{(Z(t),S(t))} = \{t \geq 0; Z_t \geq S(t)\}, \quad \text{if } Z(t_0) < S(t_0)$$

and:

$$\tau_{(X(t),S(t))} = \{t \geq 0; X_t \leq S(t)\}, \quad \text{if } X(t_0) > S(t_0)$$

$$\tau_{(Y(t),S(t))} = \{t \geq 0; Y_t \leq S(t)\}, \quad \text{if } Y(t_0) > S(t_0)$$

$$\tau_{(Z(t),S(t))} = \{t \geq 0; Z_t \leq S(t)\}, \quad \text{if } Z(t_0) > S(t_0)$$

And dfptsde3d returns a marginal kernel density approximation for  $(\tau_{(X(t),S(t))}, \tau_{(Y(t),S(t))}, \tau_{(Z(t),S(t))})$  "first passage time". with  $S(t)$  is through a continuous boundary (barrier).

An overview of this package, see [browseVignettes\('Sim.DiffProc'\)](#) for more informations.

**Value**

dfptsde3d gives the marginal kernel density approximation for fpt. fptsde3d generates random of fpt.

**Author(s)**

A.C. Guidoum, K. Boukhetala.

## References

- Argyrakisa, P. and G.H. Weiss (2006). A first-passage time problem for many random walkers. *Physica A*. **363**, 343–347.
- Aytug H., G. J. Koehler (2000). New stopping criterion for genetic algorithms. *European Journal of Operational Research*, **126**, 662–674.
- Boukhetala, K. (1996) Modelling and simulation of a dispersion pollutant with attractive centre. ed by Computational Mechanics Publications, Southampton ,U.K and Computational Mechanics Inc, Boston, USA, 245–252.
- Boukhetala, K. (1998a). Estimation of the first passage time distribution for a simulated diffusion process. *Maghreb Math.Rev*, **7**(1), 1–25.
- Boukhetala, K. (1998b). Kernel density of the exit time in a simulated diffusion. *les Annales Maghrebines De L ingénieur*, **12**, 587–589.
- Ding, M. and G. Rangarajan. (2004). First Passage Time Problem: A Fokker-Planck Approach. *New Directions in Statistical Physics*. ed by L. T. Wille. Springer. 31–46.
- Roman, R.P., Serrano, J. J., Torres, F. (2008). First-passage-time location function: Application to determine first-passage-time densities in diffusion processes. *Computational Statistics and Data Analysis*. **52**, 4132–4146.
- Roman, R.P., Serrano, J. J., Torres, F. (2012). An R package for an efficient approximation of first-passage-time densities for diffusion processes based on the FPTL function. *Applied Mathematics and Computation*, **218**, 8408–8428.
- Gardiner, C. W. (1997). *Handbook of Stochastic Methods*. Springer-Verlag, New York.

## See Also

[fptsde1d](#) for simulation fpt in sde 1-dim. [fptsde2d](#) for simulation fpt in sde 2-dim.

[FPTL](#) for computes values of the first passage time location (FPTL) function, and [Approx.fpt.density](#) for approximate first-passage-time (f.p.t.) density in package "fptdApprox".

[GQD.TIpassage](#) for compute the First Passage Time Density of a GQD With Time Inhomogeneous Coefficients in package "DiffusionRgqd".

## Examples

```
## dX(t) = 4*(-1-X(t))*Y(t) dt + 0.2 * dW1(t)
## dY(t) = 4*(1-Y(t)) *X(t) dt + 0.2 * dW2(t)
## dZ(t) = 4*(1-Z(t)) *Y(t) dt + 0.2 * dW3(t)
## x0 = 0, y0 = -2, z0 = 0, and barrier -3+5*t.
## W1(t), W2(t) and W3(t) three independent Brownian motion
set.seed(1234)

# SDE's 3d

fx <- expression(4*(-1-x)*y, 4*(1-y)*x, 4*(1-z)*y)
gx <- rep(expression(0.2),3)
mod3d <- snssde3d(drift=fx,diffusion=gx,M=500)
```

```

# boundary
St <- expression(-3+5*t)

# random

out <- fptsde3d(mod3d,boundary=St)
out
summary(out)

# Marginal density

denM <- dfptsde3d(out,pdf="M")
denM
plot(denM)

# Multiple isosurfaces
## Not run:
denJ <- dfptsde3d(out,pdf="J")
denJ
plot(denJ,display="rgl")

## End(Not run)

```

---

HWV

*Hull-White/Vasicek, Ornstein-Uhlenbeck process*


---

## Description

The (S3) generic function for simulation of Hull-White/Vasicek or gaussian diffusion models, and Ornstein-Uhlenbeck process.

## Usage

```

HWV(N, ...)
OU(N, ...)

## Default S3 method:
HWV(N = 100, M = 1, x0 = 2, t0 = 0, T = 1, Dt = NULL, mu = 4, theta = 1,
     sigma = 0.1, ...)
## Default S3 method:
OU(N = 100, M = 1, x0 = 2, t0 = 0, T = 1, Dt = NULL, mu = 4, sigma = 0.2, ...)

```

## Arguments

|    |  |
|----|--|
| N  | number of simulation steps.                  |
| M  | number of trajectories.                      |
| x0 | initial value of the process at time $t_0$ . |
| t0 | initial time.                                |

|       |  |
|-------|--|
| T     | final time.  |
| Dt    | time step of the simulation (discretization). If it is <code>missing</code> a default $\Delta t = \frac{T-t_0}{N}$ . |
| mu    | parameter of the HWV and OU; see details.  |
| theta | parameter of the HWV; see details.   |
| sigma | the volatility of the HWV and OU.  |
| ...   | potentially further arguments for (non-default) methods.   |

### Details

The function HWV returns a trajectory of the **Hull-White/Vasicek process** starting at  $x_0$  at time  $t_0$ ; i.e., the diffusion process solution of stochastic differential equation:

$$dX_t = \mu(\theta - X_t)dt + \sigma dW_t$$

The function OU returns a trajectory of the **Ornstein-Uhlenbeck** starting at  $x_0$  at time  $t_0$ ; i.e., the diffusion process solution of stochastic differential equation:

$$dX_t = -\mu X_t dt + \sigma dW_t$$

Constraints:  $\mu, \sigma > 0$ .

Please note that the process is stationary only if  $\mu > 0$ .

### Value

X an visible ts object.

### Author(s)

A.C. Guidoum, K. Boukhetala.

### References

Vasicek, O. (1977). An Equilibrium Characterization of the Term Structure. *Journal of Financial Economics*, 5, 177–188.

### See Also

[rcOU](#) and [rsOU](#) for conditional and stationary law of Vasicek process are available in "sde".

### Examples

```
## Hull-White/Vasicek Models
## dX(t) = 4 * (2.5 - X(t)) * dt + 1 *dW(t), X0=10
set.seed(1234)

X <- HWV(N=1000,M=10,mu = 4, theta = 2.5,sigma = 1,x0=10)
plot(X,plot.type="single")
lines(as.vector(time(X)),rowMeans(X),col="red")
```

```
## Ornstein-Uhlenbeck Process
##  $dX(t) = -4 * X(t) * dt + 1 * dW(t)$  ,  $X_0=2$ 
set.seed(1234)

X <- OU(N=1000,M=10,mu = 4,sigma = 1,x0=10)
plot(X,plot.type="single")
lines(as.vector(time(X)),rowMeans(X),col="red")
```

---

Irates

*Monthly Interest Rates*

---

### Description

monthly observations from 1946–12 to 1991–02  
*number of observations* : 531  
*observation* : country  
*country* : United–States

### Usage

data(Irates)

### Format

A time serie containing :

**r1** interest rate for a maturity of 1 months (% per year).  
**r2** interest rate for a maturity of 2 months (% per year).  
**r3** interest rate for a maturity of 3 months (% per year).  
**r5** interest rate for a maturity of 5 months (% per year).  
**r6** interest rate for a maturity of 6 months (% per year).  
**r11** interest rate for a maturity of 11 months (% per year).  
**r12** interest rate for a maturity of 12 months (% per year).  
**r36** interest rate for a maturity of 36 months (% per year).  
**r60** interest rate for a maturity of 60 months (% per year).  
**r120** interest rate for a maturity of 120 months (% per year).

### Source

McCulloch, J.H. and Kwon, H.C. (1993). U.S. term structure data, 1947–1991, Ohio State Working Paper 93–6, Ohio State University, Columbus  
These datasets [Irates](#) are in package "Ecdat".

## References

Croissant, Y. (2014). Ecdat: Data sets for econometrics. R package version 0.2–5.

## Examples

```
data(Irates)
rates <- Irates[, "r1"]
rates <- window(rates, start=1964.471, end=1989.333)

## CKLS modele vs CIR modele
## CKLS :  $dX(t) = (\theta_1 + \theta_2 * X(t)) * dt + \theta_3 * X(t)^{\theta_4} * dW(t)$ 

fx <- expression(theta[1]+theta[2]*x)
gx <- expression(theta[3]*x^theta[4])
fitmod <- fitsde(rates, drift=fx, diffusion=gx, pmle="euler", start = list(theta1=1, theta2=1,
  theta3=1, theta4=1), optim.method = "L-BFGS-B")
theta <- coef(fitmod)

N <- length(rates)
res <- snssde1d(drift=fx, diffusion=gx, M=1000, t0=time(rates)[1], T=time(rates)[N],
  Dt=deltat(rates), x0=rates[1], N=N)

plot(res, type="n", ylim=c(0, 35))
lines(rates, col=2, lwd=2)
lines(time(res), apply(res$X, 1, mean), col=3, lwd=2)
lines(time(res), apply(res$X, 1, bconfint, level=0.95)[1, ], col=4, lwd=2)
lines(time(res), apply(res$X, 1, bconfint, level=0.95)[2, ], col=4, lwd=2)
legend("topleft", c("real data", "mean path",
  paste("bound of", 95, " confidence")), inset = .01,
  col=2:4, lwd=2, cex=0.8)
```

---

MCM.sde

*Parallel Monte-Carlo Methods for SDE's*

---

## Description

Generate R Monte-Carlo (version parallel) replicates of a statistic applied to SDE's (1,2 and 3 dim) for the two cases Ito and Stratonovich interpretations.

## Usage

```
MCM.sde(model, ...)
```

## Default S3 method:

```
MCM.sde(model, statistic, R = 1000, time, exact = NULL,
  names = NULL, level = 0.95, parallel = c("no", "multicore", "snow"),
  ncpus = getOption("ncpus", 1L), cl = NULL, ...)
```

## S3 method for class 'MCM.sde'

```
plot(x, index = 1, type=c("all", "hist", "qqplot", "boxplot", "CI"), ...)
```

**Arguments**

|           |  |
|-----------|--|
| model     | an object from class <a href="#">snssde1d</a> , <a href="#">snssde2d</a> and <a href="#">snssde3d</a> .                        |
| statistic | a function which when applied to model returns a vector containing the statistic(s) of interest.                               |
| R         | the number of Monte-Carlo replicates. Usually this will be a single positive integer.  |
| time      | the time when estimating the statistic(s) of interest time between $t_0$ and $T$ . The default time = $T$ .                    |
| exact     | a named list giving the exact statistic(s) if it exists otherwise exact = NULL.  |
| names     | named the statistic(s) of interest. The default names=c("mu1", "mu2", ...).  |
| level     | the confidence level(s) of the required interval(s).   |
| parallel  | the type of parallel operation to be used (if any). The default parallel = "no".   |
| ncpus     | integer: number of processes to be used in parallel operation: typically one would chose this to the number of available CPUs. |
| cl        | an optional parallel or snow cluster for use if parallel = "snow".   |
| x         | an object inheriting from class "MCM.sde".   |
| index     | the index of the variable of interest within the output of "MCM.sde".  |
| type      | the type of plot of the Monte-Carlo estimation of the variable of interest. The default type = "all".                          |
| ...       | potentially further arguments for (non-default) methods.   |

**Details**

We have here developed Monte-Carlo methods whose essence is the use of repeated experiments to evaluate a statistic(s) of interest in SDE's. For example estimation of moments as: mean, variance, covariance (and other as median, mode, quantile,...). With the standard error and the confidence interval for these estimators.

An overview of this package, see [browseVignettes\('Sim.DiffProc'\)](#) for more informations.

**Value**

The returned value is an object of class "MCM.sde", containing the following components:

|      |  |
|------|--|
| mod  | The SDE's used (class: <a href="#">snssde1d</a> , <a href="#">snssde2d</a> and <a href="#">snssde3d</a> ).                             |
| dim  | Dimension of the model.  |
| call | The original call to "MCM.sde".  |
| Fn   | The function statistic as passed to "MCM.sde".   |
| ech  | A matrix with sum(R) column each of which is a Monte-Carlo replicate of the result of calling statistic.                               |
| time | The time when estimating the statistic(s) of interest.   |
| name | named of statistic(s) of interest.   |
| MC   | Table contains simulation results of statistic(s) of interest: Estimate, Bias (if exact available), Std.Error and Confidence interval. |

**Note**

When `parallel = "multicore"` is used are not available on Windows, `parallel = "snow"` is primarily intended to be used on multi-core Windows machine where `parallel = "multicore"` is not available. For more details see Q.E.McCallum and S.Weston (2011).

**Author(s)**

A.C. Guidoum, K. Boukhetala.

**References**

- Paul Glasserman (2003). *Monte Carlo Methods in Financial Engineering*. Springer-Verlag New York.
- Jun S. Liu (2004). *Monte Carlo Strategies in Scientific Computing*. Springer-Verlag New York.
- Christian Robert and George Casella (2010). *Introducing Monte Carlo Methods with R*. Springer-Verlag New York.
- Nick T. Thomopoulos (2013). *Essentials of Monte Carlo Simulation: Statistical Methods for Building Simulation Models*. Springer-Verlag New York.
- Q. Ethan McCallum and Stephen Weston (2011). *Parallel R*. O'Reilly Media, Inc.

**See Also**

[MEM.sde](#) moment equations methods for SDE's.

**Examples**

```
## Example 1 : (1 dim)
## dX(t) = 3*(1-X(t)) dt + 0.5 * dW(t), X(0)=5, t in [0,10]
## set the model 1d
f <- expression(3*(1-x));g <- expression(0.5)
mod1d <- snssde1d(drift=f,diffusion=g,x0=5,T=10,M=50)

## function of the statistic(s) of interest.
sde.fun1d <- function(data, i){
  d <- data[i, ]
  return(c(mean(d),Mode(d),var(d)))
}

mc.sde1d = MCM.sde(model=mod1d,statistic=sde.fun1d,R=100,exact=list(Me=1,Mo=1,Va=0.5^2/6),
  names=c("Me(10)","Mo(10)","Va(10)"))

mc.sde1d
plot(mc.sde1d,index=1)
plot(mc.sde1d,index=2)
plot(mc.sde1d,index=3)

## Example 2 : with Parallel computing
## Not run:
mod1d <- snssde1d(drift=f,diffusion=g,x0=5,T=10,M=1000)
## On Windows or Unix
```

```

mc.sde1d = MCM.sde(model=mod1d,statistic=sde.fun1d,R=1000,exact=list(Me=1,Mo=1,Va=0.5^2/6),
  names=c("Me(10)","Mo(10)","Va(10)"),parallel="snow",ncpus=parallel::detectCores())
mc.sde1d
## On Unix only
mc.sde1d = MCM.sde(model=mod1d,statistic=sde.fun1d,R=1000,exact=list(Me=1,Mo=1,Va=0.5^2/6),
  names=c("Me(10)","Mo(10)","Va(10)"),parallel="multicore",ncpus=parallel::detectCores())
mc.sde1d

## End(Not run)

## Example 3: (2 dim)
## dX(t) = 1/mu*(theta-X(t)) dt + sqrt(sigma) * dW1(t),
## dY(t) = X(t) dt + 0 * dW2(t)
## Not run:
## Set the model 2d
mu=0.75;sigma=0.1;theta=2
x0=0;y0=0;init=c(x=0,y=0)
f <- expression(1/mu*(theta-x), x)
g <- expression(sqrt(sigma),0)
OUI <- snssde2d(drift=f,diffusion=g,M=1000,Dt=0.01,x0=init)

## function of the statistic(s) of interest.
sde.fun2d <- function(data, i){
  d <- data[i,]
  return(c(mean(d$x),mean(d$y),var(d$x),var(d$y),cov(d$x,d$y)))
}
## Monte-Carlo at time = 5
mc.sde2d_a = MCM.sde(model=OUI,statistic=sde.fun2d,R=100,time=5,
  parallel="snow",ncpus=parallel::detectCores())
mc.sde2d_a
## Monte-Carlo at time = 10
mc.sde2d_b = MCM.sde(model=OUI,statistic=sde.fun2d,R=100,time=10,
  parallel="snow",ncpus=parallel::detectCores())
mc.sde2d_b

## Compared with exact values at time 5 and 10
E_x <- function(t) theta+(x0-theta)*exp(-t/mu)
V_x <- function(t) 0.5*sigma*mu *(1-exp(-2*(t/mu)))
E_y <- function(t) y0+theta*t+(x0-theta)*mu*(1-exp(-t/mu))
V_y <- function(t) sigma*mu^3*((t/mu)-2*(1-exp(-t/mu))+0.5*(1-exp(-2*(t/mu))))
cov_xy <- function(t) 0.5*sigma*mu^2 *(1-2*exp(-t/mu)+exp(-2*(t/mu)))

## at time=5
mc.sde2d_a = MCM.sde(model=OUI,statistic=sde.fun2d,R=100,time=5,
  exact=list(m1=E_x(5),m2=E_y(5),S1=V_x(5),S2=V_y(5),C12=cov_xy(5)),
  parallel="snow",ncpus=parallel::detectCores())
mc.sde2d_a
plot(mc.sde2d_a,index=1)
plot(mc.sde2d_a,index=2)
## at time=10
mc.sde2d_b = MCM.sde(model=OUI,statistic=sde.fun2d,R=100,time=10,
  exact=list(m1=E_x(10),m2=E_y(10),S1=V_x(10),S2=V_y(10),C12=cov_xy(10)),
  parallel="snow",ncpus=parallel::detectCores())

```

```

mc.sde2d_b
plot(mc.sde2d_b,index=1)
plot(mc.sde2d_b,index=2)

## End(Not run)

## Example 4: (3 dim)
## dX(t) = sigma*(Y(t)-X(t)) dt + 0.1 * dW1(t)
## dY(t) = (rho*X(t)-Y(t)-X(t)*Z(t)) dt + 0.1 * dW2(t)
## dZ(t) = (X(t)*Y(t)-bet*Z(t)) dt + 0.1 * dW3(t)
## Not run:
## Set the model 3d
sigma=10;rho=28; bet=8/3
f <- expression(sigma*(y-x),rho*x-y-x*z,x*y-bet*z)
g <- expression(0.1,0.1,0.1)
mod3d <- snssde3d(x0=rep(0,3),drift=f,diffusion=g,M=1000,Dt=0.01)

## function of the statistic(s) of interest.
sde.fun3d <- function(data, i){
  d <- data[i,]
  return(c(mean(d$x),mean(d$y),mean(d$z)))
}
## Monte-Carlo at time = 10
mc.sde3d = MCM.sde(mod3d,statistic=sde.fun3d,R=100,parallel="snow",ncpus=parallel::detectCores())
mc.sde3d

## End(Not run)

```

MEM.sde

*Moment Equations Methods for SDE's***Description**

Calculate and numerical approximation of moment equations (Symbolic ODE's of means and variances-covariance) at any time for SDE's (1,2 and 3 dim) for the two cases Ito and Stratonovich interpretations.

**Usage**

```

MEM.sde(drift, diffusion, ...)

## Default S3 method:
MEM.sde(drift, diffusion, type = c("ito", "str"), solve = FALSE,
        parms = NULL, init = NULL, time = NULL, ...)

## S3 method for class 'MEM.sde'
summary(object, at , ...)

```

**Arguments**

|            |   |
|------------|---|
| drift      | drift coefficient: an <a href="#">expression</a> 1-dim(t,x), 2-dim(t,x,y) or 3-dim(t,x,y,z).  |
| diffusion  | diffusion coefficient: an <a href="#">expression</a> 1-dim(t,x), 2-dim(t,x,y) or 3-dim(t,x,y,z).  |
| type       | type of process "ito" or "Stratonovich"; the default type="ito".  |
| solve      | if solve=TRUE solves a system of ordinary differential equations.   |
| parms      | parameters passed to drift and diffusion.   |
| init       | the initial (state) values for the ODE system. for 1-dim (m=x0,S=0), 2-dim (m1=x0,m2=y0,S1=0,S2=0,C12=0) and for 3-dim (m1=x0,m2=y0,m3=z0,S1=0,S2=0,S3=0,C12=0,C13=0) see examples. |
| time       | time sequence (vector) for which output is wanted; the first value of time must be the initial time.  |
| object, at | an object inheriting from class "MEM.sde" and summaries at any time at.   |
| ...        | potentially arguments to be passed to methods, such as <a href="#">ode</a> for solver for ODE's.  |

**Details**

The stochastic transition is approximated by the moment equations, and the numerical treatment is required to solve these equations from above with given initial conditions.

An overview of this package, see [browseVignettes\('Sim.DiffProc'\)](#) for more informations.

**Value**

Symbolic ODE's of means and variances-covariance. If solve=TRUE approximate the moment of SDE's at any time.

**Author(s)**

A.C. Guidoum, K. Boukhetala.

**References**

Rodriguez R, Tuckwell H (2000). *A dynamical system for the approximate moments of nonlinear stochastic models of spiking neurons and networks*. *Mathematical and Computer Modelling*, 31(4), 175–180.

Alibrandi U, Ricciardi G (2012). *Stochastic Methods in Nonlinear Structural Dynamics*, 3–60. Springer Vienna, Vienna. ISBN 978-3-7091-1306-6.

**See Also**

[MCM.sde](#) Monte-Carlo methods for SDE's.

## Examples

```

library(deSolve)
## Example 1: 1-dim
##  $dX(t) = \mu * X(t) * dt + \sigma * X(t) * dW(t)$ 
## Symbolic ODE's of mean and variance
f <- expression(mu*x)
g <- expression(sigma*x)
res1 <- MEM.sde(drift=f,diffusion=g)
res2 <- MEM.sde(drift=f,diffusion=g,type="str")
res1
res2
## numerical approximation of mean and variance
para <- c(mu=2,sigma=0.5)
t <- seq(0,1,by=0.001)
init <- c(m=1,S=0)
res1 <- MEM.sde(drift=f,diffusion=g,solve=TRUE,init=init,parms=para,time=t)
res1
matplot.0D(res1$sol.ode,main="Mean and Variance of X(t), type Ito")
plot(res1$sol.ode,select=c("m","S"))
## approximation at time = 0.75
summary(res1,at=0.75)

##
res2 <- MEM.sde(drift=f,diffusion=g,solve=TRUE,init=init,parms=para,time=t,type="str")
res2
matplot.0D(res2$sol.ode,main="Mean and Variance of X(t), type Stratonovich")
plot(res2$sol.ode,select=c("m","S"))
## approximation at time = 0.75
summary(res2,at=0.75)

## Comparison:

plot(res1$sol.ode, res2$sol.ode,ylab = c("m(t)"),select="m",xlab = "Time",
     col = c("red", "blue"))
plot(res1$sol.ode, res2$sol.ode,ylab = c("S(t)"),select="S",xlab = "Time",
     col = c("red", "blue"))

## Example2: 2-dim
##  $dX(t) = 1/\mu*(\theta-X(t)) dt + \sqrt{\sigma} * dW1(t)$ ,
##  $dY(t) = X(t) dt + \theta * dW2(t)$ 
## Not run:
para=c(mu=0.75,sigma=0.1,theta=2)
init=c(m1=0,m2=0,S1=0,S2=0,C12=0)
t <- seq(0,10,by=0.001)
f <- expression(1/mu*(theta-x), x)
g <- expression(sqrt(sigma),0)
res2d <- MEM.sde(drift=f,diffusion=g,solve=TRUE,init=init,parms=para,time=t)
res2d

## Exact moment

mu=0.75;sigma=0.1;theta=2;x0=0;y0=0

```

```

E_x <- function(t) theta+(x0-theta)*exp(-t/mu)
V_x <- function(t) 0.5*sigma*mu *(1-exp(-2*(t/mu)))
E_y <- function(t) y0+theta*t+(x0-theta)*mu*(1-exp(-t/mu))
V_y <- function(t) sigma*mu^3*((t/mu)-2*(1-exp(-t/mu))+0.5*(1-exp(-2*(t/mu))))
cov_xy <- function(t) 0.5*sigma*mu^2 *(1-2*exp(-t/mu)+exp(-2*(t/mu)))

##
summary(res2d,at=5)
E_x(5);E_y(5);V_x(5);V_y(5);cov_xy(5)

matplot.0D(res2d$sol.ode,select=c("m1"))
curve(E_x,add=TRUE,col="red")

## plot

plot(res2d$sol.ode)
matplot.0D(res2d$sol.ode,select=c("S1","S2","C12"))
plot(res2d$sol.ode[, "m1"], res2d$sol.ode[, "m2"], xlab = "m1(t)",
      ylab = "m2(t)", type = "l",lwd = 2)
hist(res2d$sol.ode,select=c("m1","m2"), col = c("darkblue", "red", "orange", "black"))

## Example3: 3-dim
## dX(t) = sigma*(Y(t)-X(t)) dt + 0.1 * dW1(t)
## dY(t) = (rho*X(t)-Y(t)-X(t)*Z(t)) dt + 0.1 * dW2(t)
## dZ(t) = (X(t)*Y(t)-bet*Z(t)) dt + 0.1 * dW3(t)
f <- expression(sigma*(y-x),rho*x-y-x*z,x*y-bet*z)
g <- expression(0.1,0.1,0.1)
## Symbolic moments equations
res3d = MEM.sde(drift=f,diffusion=g)
res3d

## Numerical approximation
para=c(sigma=10,rho=28,bet=8/3)
ini=c(m1=1,m2=1,m3=1,S1=0,S2=0,S3=0,C12=0,C13=0,C23=0)
res3d = MEM.sde(drift=f,diffusion=g,solve=T,parms=para,init=ini,time=seq(0,1,by=0.01))
res3d

summary(res3d,at=0.25)
summary(res3d,at=0.50)
summary(res3d,at=0.75)

plot(res3d$sol.ode)
matplot.0D(res3d$sol.ode,select=c("m1","m2","m3"))
matplot.0D(res3d$sol.ode,select=c("S1","S2","S3"))
matplot.0D(res3d$sol.ode,select=c("C12","C13","C23"))

##
library(rgl)
plot3d(res3d$sol.ode[, "m1"], res3d$sol.ode[, "m2"],res3d$sol.ode[, "m3"], xlab = "m1(t)",
      ylab = "m2(t)",zlab="m3(t)", type = "l",lwd = 2,box=F)

## End(Not run)

```

---

moment

*Monte-Carlo statistics of SDE's*


---

### Description

Generic function for compute the kurtosis, skewness, median, mode and coefficient of variation (relative variability), moment and confidence interval of class "sde".

### Usage

```
## Default S3 method:
bconfint(x, level = 0.95, ...)
## Default S3 method:
kurtosis(x, ...)
## Default S3 method:
moment(x, order = 1, center = TRUE, ...)
## Default S3 method:
cv(x, ...)
## Default S3 method:
skewness(x, ...)
## Default S3 method:
Median(x, ...)
## Default S3 method:
Mode(x, ...)
```

### Arguments

|        |  |
|--------|--|
| x      | an object inheriting from class "sde".                   |
| order  | order of moment.   |
| center | if TRUE is a central moment.                             |
| level  | the confidence level required.                           |
| ...    | potentially further arguments for (non-default) methods. |

### Author(s)

A.C. Guidoum, K. Boukhetala.

### Examples

```
## Example 1:
## dX(t) = 2*(3-X(t)) *dt + dW(t)
set.seed(1234)

f <- expression( 2*(3-x) )
g <- expression( 1 )
mod <- snssde1d(drift=f,diffusion=g,M=10000,T=5)
## Monte-Carlo statistics of 5000 trajectory of X(t) at final time T of 'mod'
```

```
summary(mod)
kurtosis(mod)
skewness(mod)
mean(mod)
Median(mod)
Mode(mod)
moment(mod,order=4)
cv(mod)
bconfint(mod,level = 0.95) ## of mean
```

---

plot2d

*Plotting for Class SDE*

---

## Description

Generic function for plotting.

## Usage

```
## Default S3 method:
plot2d(x, ...)
## Default S3 method:
lines2d(x, ...)
## Default S3 method:
points2d(x, ...)
## Default S3 method:
plot3D(x, display = c("persp", "rgl"), ...)
```

## Arguments

|         |   |
|---------|---|
| x       | an object inheriting from class <a href="#">snssde2d</a> , <a href="#">snssde3d</a> , <a href="#">bridgesde2d</a> and <a href="#">bridgesde3d</a> .                             |
| display | "persp" perspective or "rgl" plots.   |
| ...     | other graphics parameters, see <a href="#">par</a> in package "graphics", <a href="#">scatterplot3d</a> in package "scatterplot3d" and <a href="#">plot3d</a> in package "rgl". |

## Details

The 2 and 3-dim plot of class sde.

## Author(s)

A.C. Guidoum, K. Boukhetala.

**Examples**

```
## Example 1:
set.seed(1234)

fx <- rep(expression(0),2)
gx <- rep(expression(1),2)

res <- snssde2d(drift=fx,diffusion=gx,N=10000)
plot2d(res,type="l")

## Example 2:
set.seed(1234)

fx <- rep(expression(0),3)
gx <- rep(expression(1),3)

res <- snssde3d(drift=fx,diffusion=gx,N=10000)
plot3D(res,display="persp")
plot3D(res,display="rgl")
```

---

|        |   |
|--------|---|
| rsde1d | <i>Approximate transitional densities and random generation for 1-D SDE</i> |
|--------|---|

---

**Description**

Transition density and random generation for  $X(t-s) \mid X(s)=x_0$  of the 1-dim SDE.

**Usage**

```
rsde1d(object, ...)
dsde1d(object, ...)

## Default S3 method:
rsde1d(object, at, ...)

## Default S3 method:
dsde1d(object, at, ...)
## S3 method for class 'dsde1d'
plot(x,hist=FALSE, ...)
```

**Arguments**

|        |  |
|--------|--|
| object | an object inheriting from class <a href="#">snssde1d</a> and <a href="#">bridgesde1d</a> .         |
| at     | time between $s=t_0$ and $t=T$ . The default $at = T$ .  |
| x      | an object inheriting from class <a href="#">dsde1d</a> .   |
| hist   | if $hist=TRUE$ plot histogram. Based on <a href="#">truehist</a> function.                         |
| ...    | potentially arguments to be passed to methods, such as <a href="#">density</a> for kernel density. |

## Details

The function `rsde1d` returns a  $M$  random variable  $x_{t=at}$  realize at time  $t = at$  defined by :

$$x_{t=at} = \{t \geq 0; x = X_{t=at}\}$$

And `dsde1d` returns a transition density approximation for  $X(t-s) \mid X(s)=x_0$ . with  $t = at$  is a fixed time between  $t_0$  and  $T$ .

An overview of this package, see [browseVignettes\('Sim.DiffProc'\)](#) for more informations.

## Value

`dsde1d` gives the transition density estimate of  $X(t-s) \mid X(s)=x_0$ . `rsde1d` generates random of  $X(t-s) \mid X(s)=x_0$ .

## Author(s)

A.C. Guidoum, K. Boukhetala.

## See Also

[density](#) Kernel density estimation in "stats" package.

[kde](#) Kernel density estimate for 1- to 6-dimensional data in "ks" package.

[sm.density](#) Nonparametric density estimation in one, two or three dimensions in "sm" package.

[rng](#) random number generators in "yuima" package.

[dcSim](#) Pedersen's simulated transition density in "sde" package.

[rcBS](#), [rcCIR](#), [rcOU](#) and [rsOU](#) in package "sde".

[dcBS](#), [dcCIR](#), [dcOU](#) and [dsOU](#) in package "sde".

[GQD.density](#) Generate the transition density of a scalar generalized quadratic diffusion.

## Examples

```
## Example 1:
## dX(t) = (-2*(X(t)<=0)+2*(X(t)>=0)) *dt + 0.5 * dW(t)
set.seed(1234)

f <- expression(-2*(x<=0)+2*(x>=0))
g <- expression(0.5)
res1 <- snssde1d(drift=f,diffusion=g,M=5000)
x <- rsde1d(res1, at = 1)
summary(x)
dens1 <- dsde1d(res1, at = 1)
dens1
plot(dens1,main="Transition density of X(t=1)|X(s=0)=0") # kernel estimated
plot(dens1,hist=TRUE) # histogramme

## Example 2:
## Transition density of standard Brownian motion W(t) at time = 0.5
```

```

set.seed(1234)

f <- expression(0)
g <- expression(1)
res2 <- snssde1d(drift=f,diffusion=g,M=5000)
plot(dsde1d(res2, at = 0.5),dens=function(x) dnorm(x,0,sqrt(0.5)))
plot(dsde1d(res2, at = 0.5),dens=function(x) dnorm(x,0,sqrt(0.5)),hist=TRUE)

## Example 3: Transition density of Brownian motion W(t) in [0,1]

## Not run:
for (i in seq(res2$t0,res2$T,by=res2$Dt)){
plot(dsde1d(res2, at = i),main=paste0('Transition Density \n t = ',i))
}

## End(Not run)

## Example 4:
## Transition density of bridge Brownian motion W(t) at time = 0.25 and 0.75
set.seed(1234)
## Not run:
f <- expression(0)
g <- expression(1)
Bd <- bridgesde1d(drift=f,diffusion=g,M=5000)
Bd
plot(dsde1d(Bd, at = 0.25))          ## Transition Density at time=0.25
plot(dsde1d(Bd, at = 0.75),add=TRUE)## Transition Density at time=0.75

## End(Not run)

```

---

rsde2d

*Approximate transitional densities and random generation for 2-D SDE's*

---

## Description

Transition density and random generation for the joint and marginal of  $(X(t-s), Y(t-s) \mid X(s)=x_0, Y(s)=y_0)$  of the SDE's 2-d.

## Usage

```

rsde2d(object, ...)
dsde2d(object, ...)

## Default S3 method:
rsde2d(object, at, ...)

## Default S3 method:
dsde2d(object, pdf=c("Joint","Marginal"), at, ...)
## S3 method for class 'dsde2d'
plot(x,display=c("persp","rgl","image","contour"),hist=FALSE,...)

```

**Arguments**

|         |  |
|---------|--|
| object  | an object inheriting from class <a href="#">snssde2d</a> and <a href="#">bridgesde2d</a> .   |
| at      | time between $s=t_0$ and $t=T$ . The default $at = T$ .  |
| pdf     | probability density function Joint or Marginal.  |
| x       | an object inheriting from class <a href="#">dsde2d</a> .   |
| display | display plots.   |
| hist    | if <code>hist=TRUE</code> plot histogram. Based on <a href="#">truehist</a> function.  |
| ...     | potentially potentially arguments to be passed to methods, such as <a href="#">density</a> for marginal density and <a href="#">kde2d</a> for joint density. |

**Details**

The function `rsde2d` returns a  $M$  random variable  $x_{t=at}, y_{t=at}$  realize at time  $t = at$ .

And `dsde2d` returns a bivariate density approximation for  $(X(t-s), Y(t-s) \mid X(s)=x_0, Y(s)=y_0)$ . with  $t = at$  is a fixed time between  $t_0$  and  $T$ .

An overview of this package, see `browseVignettes('Sim.DiffProc')` for more informations.

**Value**

`dsde2d` gives the bivariate density approximation for  $(X(t-s), Y(t-s) \mid X(s)=x_0, Y(s)=y_0)$ .  
`rsde2d` generates random of the couple  $(X(t-s), Y(t-s) \mid X(s)=x_0, Y(s)=y_0)$ .

**Author(s)**

A.C. Guidoum, K. Boukhetala.

**See Also**

[kde2d](#) Two-dimensional kernel density estimation in "MASS" package.

[kde](#) Kernel density estimate for 1- to 6-dimensional data in "ks" package.

[sm.density](#) Nonparametric density estimation in one, two or three dimensions in "sm" package.

[rng](#) random number generators in "yuima" package.

[BiGQD.density](#) Generate the transition density of a bivariate generalized quadratic diffusion model (2D GQD).

**Examples**

```
## Example:1
set.seed(1234)

# SDE's 2d
fx <- expression(3*(2-y),2*x)
gx <- expression(1,y)
mod2d <- snssde2d(drift=fx,diffusion=gx,x0=c(1,2),M=1000)

# random
```

```

r2d <- rsde2d(mod2d,at=0.5)
summary(r2d)

# Marginal density

denM <- dsde2d(mod2d,pdf="M", at=0.5)
denM
plot(denM)

# Joint density
denJ <- dsde2d(mod2d,pdf="J",n=200, at= 0.5,lims=c(-3,4,0,6))
denJ
plot(denJ)
plot(denJ,display="contour")

## Example 2: Bivariate Transition Density of 2 Brownian motion (W1(t),W2(t)) in [0,1]

## Not run:
B2d <- snssde2d(drift=rep(expression(0),2),diffusion=rep(expression(1),2),
  M=10000)
for (i in seq(B2d$Dt,B2d$T,by=B2d$Dt)){
  plot(dsde2d(B2d, at = i,lims=c(-3,3,-3,3),n=100),
    display="contour",main=paste0('Transition Density \n t = ',i))
}

## End(Not run)

## Example 3:

## Not run:
fx <- expression(4*(-1-x)*y , 4*(1-y)*x )
gx <- expression(0.25*y,0.2*x)
mod2d1 <- snssde2d(drift=fx,diffusion=gx,x0=c(x0=1,y0=-1),
  M=5000,type="str")

# Marginal transition density
for (i in seq(mod2d1$Dt,mod2d1$T,by=mod2d1$Dt)){
  plot(dsde2d(mod2d1,pdf="M", at = i),main=
    paste0('Marginal Transition Density \n t = ',i))
}

# Bivariate transition density
for (i in seq(mod2d1$Dt,mod2d1$T,by=mod2d1$Dt)){
  plot(dsde2d(mod2d1, at = i,lims=c(-1,2,-1,1),n=100),
    display="contour",main=paste0('Transition Density \n t = ',i))
}

## End(Not run)

## Example 4: Bivariate Transition Density of 2 bridge Brownian motion (W1(t),W2(t)) in [0,1]

## Not run:
B2d <- bridgesde2d(drift=rep(expression(0),2),

```

```

diffusion=rep(expression(1),2),M=5000)
for (i in seq(0.01,0.99,by=B2d$Dt)){
plot(dsde2d(B2d, at = i,lims=c(-3,3,-3,3),
n=100),display="contour",main=
paste0('Transition Density \n t = ',i))
}

## End(Not run)

## Example 5: Bivariate Transition Density of bridge
## Ornstein-Uhlenbeck process and its integral in [0,5]
## dX(t) = 4*(-1-X(t)) dt + 0.2 dW1(t)
## dY(t) = X(t) dt + 0 dW2(t)
## x01 = 0 , y01 = 0
## x02 = 0, y02 = 0
## Not run:
fx <- expression(4*(-1-x) , x)
gx <- expression(0.2 , 0)
OUI <- bridgesde2d(drift=fx,diffusion=gx,Dt=0.005,M=1000)
for (i in seq(0.01,4.99,by=OUI$Dt)){
plot(dsde2d(OUI, at = i,lims=c(-1.2,0.2,-2.5,0.2),n=100),
display="contour",main=paste0('Transition Density \n t = ',i))
}

## End(Not run)

```

---

rsde3d

*Approximate transitional densities and random generation for 3-D SDE's*

---

## Description

Transition density and random generation for the joint and marginal of  $(X(t-s), Y(t-s), Z(t-s) \mid X(s)=x_0, Y(s)=y_0, Z(s)=z_0)$  of the SDE's 3-d.

## Usage

```

rsde3d(object, ...)
dsde3d(object, ...)

## Default S3 method:
rsde3d(object, at, ...)

## Default S3 method:
dsde3d(object, pdf=c("Joint","Marginal"), at, ...)
## S3 method for class 'dsde3d'
plot(x,display="rgl",hist=FALSE,...)

```

**Arguments**

|         |   |
|---------|---|
| object  | an object inheriting from class <a href="#">snssde3d</a> and <a href="#">bridgesde3d</a> .  |
| at      | time between $s=t_0$ and $t=T$ . The default $at = T$ .   |
| pdf     | probability density function Joint or Marginal.   |
| x       | an object inheriting from class <a href="#">dsde3d</a> .  |
| display | display plots.  |
| hist    | if <code>hist=TRUE</code> plot histogram. Based on <a href="#">truehist</a> function.   |
| ...     | potentially arguments to be passed to methods, such as <a href="#">density</a> for marginal density and <a href="#">sm.density</a> for joint density. |

**Details**

The function `rsde3d` returns a  $M$  random variable  $x_{t=at}, y_{t=at}, z_{t=at}$  realize at time  $t = at$ .

And `dsde3d` returns a trivariate kernel density approximation for  $(X(t-s), Y(t-s), Z(t-s) \mid X(s)=x_0, Y(s)=y_0, Z(s)=z_0)$  with  $t = at$  is a fixed time between  $t_0$  and  $T$ .

An overview of this package, see [browseVignettes\('Sim.DiffProc'\)](#) for more informations.

**Value**

`dsde3d` gives the trivariate density approximation  $(X(t-s), Y(t-s), Z(t-s) \mid X(s)=x_0, Y(s)=y_0, Z(s)=z_0)$ .  
`rsde3d` generates random of the  $(X(t-s), Y(t-s), Z(t-s) \mid X(s)=x_0, Y(s)=y_0, Z(s)=z_0)$ .

**Author(s)**

A.C. Guidoum, K. Boukhetala.

**See Also**

[kde](#) Kernel density estimate for 1- to 6-dimensional data in "ks" package.

[sm.density](#) Nonparametric density estimation in one, two or three dimensions in "sm" package.

[kde3d](#) Compute a three dimension kernel density estimate in "misc3d" package.

[rng](#) random number generators in "yuima" package.

[rcBS](#), [rcCIR](#), [rcOU](#) and [rsOU](#) in package "sde".

**Examples**

```
## Example 1: Ito sde
## dX(t) = (2*(Y(t)>0)-2*(Z(t)<=0)) dt + 0.2 * dW1(t)
## dY(t) = -2*Y(t) dt + 0.2 * dW2(t)
## dZ(t) = -2*Z(t) dt + 0.2 * dW3(t)
## W1(t), W2(t) and W3(t) three independent Brownian motion
set.seed(1234)
fx <- expression(2*(y>0)-2*(z<=0) , -2*y, -2*z)
gx <- rep(expression(0.2),3)
mod3d1 <- snssde3d(x0=c(0,2,-2),drift=fx,diffusion=gx,M=1000,Dt=0.003)
```

```

# random at t= 0.75
r3d1 <- rsde3d(mod3d1,at=0.75)
summary(r3d1)

# Marginal transition density at t=0.75, t0=0

denM <- dsde3d(mod3d1,pdf="M",at=0.75)
denM
plot(denM)

# for Joint transition density at t=0.75;t0=0
# Multiple isosurfaces
## Not run:
denJ <- dsde3d(mod3d1,pdf="J", at= 0.75)
denJ
plot(denJ,display="rgl")

## End(Not run)

## Example 2: Stratonovich sde
## dX(t) = Y(t)* dt + X(t) o dW1(t)
## dY(t) = (4*( 1-X(t)^2 )* Y(t) - X(t))* dt + 0.2 o dW2(t)
## dZ(t) = (4*( 1-X(t)^2 )* Z(t) - X(t))* dt + 0.2 o dW3(t)
set.seed(1234)

fx <- expression( y , (4*( 1-x^2 )* y - x), (4*( 1-x^2 )* z - x))
gx <- expression( x , 0.2, 0.2)
mod3d2 <- snssde3d(drift=fx,diffusion=gx,M=1000,type="str")

# random
r3d2 <- rsde3d(mod3d2)
summary(r3d2)

# Marginal transition density at t=1, t0=0

denM <- dsde3d(mod3d2,pdf="M")
denM
plot(denM)

# for Joint transition density at t=1;t0=0
# Multiple isosurfaces
## Not run:
denJ <- dsde3d(mod3d2,pdf="J")
denJ
plot(denJ,display="rgl")

## End(Not run)

## Example 3: Tivariate Transition Density of 3 Brownian motion (W1(t),W2(t),W3(t)) in [0,1]

## Not run:
B3d <- snssde3d(drift=rep(expression(0),3),diffusion=rep(expression(1),3),M=500)

```

```

for (i in seq(B3d$Dt,B3d$T,by=B3d$Dt)){
plot(dsde3d(B3d, at = i,pdf="J"),box=F,main=paste0('Transition Density t = ',i))
}

## End(Not run)

```

---

snssde1d

*Simulation of 1-D Stochastic Differential Equation*


---

## Description

The (S3) generic function snssde1d of simulation of solution to 1-dim stochastic differential equation of Itô or Stratonovich type, with different methods.

## Usage

```

snssde1d(N, ...)
## Default S3 method:
snssde1d(N = 1000, M = 1, x0 = 0, t0 = 0, T = 1, Dt = NULL,
  drift, diffusion, alpha = 0.5, mu = 0.5, type = c("ito", "str"),
  method = c("euler", "milstein", "predcorr", "smilstein", "taylor",
    "heun", "rk1", "rk2", "rk3"), ...)

## S3 method for class 'snssde1d'
summary(object, at ,digits=NULL, ...)
## S3 method for class 'snssde1d'
time(x, ...)
## S3 method for class 'snssde1d'
mean(x, at, ...)
## S3 method for class 'snssde1d'
Median(x, at, ...)
## S3 method for class 'snssde1d'
Mode(x, at, ...)
## S3 method for class 'snssde1d'
quantile(x, at, ...)
## S3 method for class 'snssde1d'
kurtosis(x, at, ...)
## S3 method for class 'snssde1d'
min(x, at, ...)
## S3 method for class 'snssde1d'
max(x, at, ...)
## S3 method for class 'snssde1d'
skewness(x, at, ...)
## S3 method for class 'snssde1d'
moment(x, at, ...)
## S3 method for class 'snssde1d'

```

```

cv(x, at, ...)
## S3 method for class 'snssde1d'
bconfint(x, at, ...)

## S3 method for class 'snssde1d'
plot(x, ...)
## S3 method for class 'snssde1d'
lines(x, ...)
## S3 method for class 'snssde1d'
points(x, ...)

```

### Arguments

|           |  |
|-----------|--|
| N         | number of simulation steps.  |
| M         | number of trajectories (Monte-Carlo).  |
| x0        | initial value of the process at time t0.   |
| t0        | initial time.  |
| T         | ending time.   |
| Dt        | time step of the simulation (discretization). If it is <code>NULL</code> a default $\Delta t = \frac{T-t_0}{N}$ .      |
| drift     | drift coefficient: an <a href="#">expression</a> of two variables t and x.   |
| diffusion | diffusion coefficient: an <a href="#">expression</a> of two variables t and x.   |
| alpha, mu | weight of the predictor-corrector scheme; the default alpha = 0.5 and mu = 0.5.  |
| type      | if type="ito" simulation sde of Itô type, else type="str" simulation sde of Stratonovich type; the default type="ito". |
| method    | numerical methods of simulation, the default method = "euler".   |
| x, object | an object inheriting from class "snssde1d".  |
| at        | time between t0 and T. Monte-Carlo statistics of the solution $X_t$ at time at. The default at = T.                    |
| digits    | integer, used for number formatting.   |
| ...       | potentially further arguments for (non-default) methods.   |

### Details

The function `snssde1d` returns a `ts` x of length N+1; i.e. solution of the sde of Ito or Stratonovich types; If Dt is not specified, then the best discretization  $\Delta t = \frac{T-t_0}{N}$ .

The Ito stochastic differential equation is:

$$dX(t) = a(t, X(t))dt + b(t, X(t))dW(t)$$

Stratonovich sde :

$$dX(t) = a(t, X(t))dt + b(t, X(t)) \circ dW(t)$$

The methods of approximation are classified according to their different properties. Mainly two criteria of optimality are used in the literature: the strong and the weak (orders of) convergence. The method of simulation can be one among: Euler-Maruyama Order 0.5, Milstein Order 1, Milstein Second-Order, Predictor-Corrector method, Itô-Taylor Order 1.5, Heun Order 2 and Runge-Kutta Order 1, 2 and 3.

An overview of this package, see [browseVignettes\('Sim.DiffProc'\)](#) for more informations.

**Value**

snssde1d returns an object inheriting from `class "snssde1d"`.

|           |                                      |
|-----------|--------------------------------------|
| X         | an invisible <code>ts</code> object. |
| drift     | drift coefficient.                   |
| diffusion | diffusion coefficient.               |
| type      | type of sde.                         |
| method    | the numerical method used.           |

**Author(s)**

A.C. Guidoum, K. Boukhetala.

**References**

- Friedman, A. (1975). *Stochastic differential equations and applications*. Volume 1, ACADEMIC PRESS.
- Henderson, D. and Plaschko, P. (2006). *Stochastic differential equations in science and engineering*. World Scientific.
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**See Also**

[snssde2d](#) and [snssde3d](#) for 2 and 3-dim sde.  
[sde.sim](#) in package "sde".  
[simulate](#) in package "yuima".

## Examples

```
## Example 1: Ito sde
##  $dX(t) = 2*(3-X(t)) dt + 2*X(t) dW(t)$ 
set.seed(1234)

f <- expression(2*(3-x) )
g <- expression(1)
mod1 <- snssde1d(drift=f,diffusion=g,M=4000,x0=10,Dt=0.01)
mod1
summary(mod1)
## Not run:
plot(mod1)
lines(time(mod1),apply(mod1$X,1,mean),col=2,lwd=2)
lines(time(mod1),apply(mod1$X,1,bconfint,level=0.95)[1,],col=4,lwd=2)
lines(time(mod1),apply(mod1$X,1,bconfint,level=0.95)[2,],col=4,lwd=2)
legend("topright",c("mean path",paste("bound of", 95," percent confidence")),
      inset = .01,col=c(2,4),lwd=2,cex=0.8)

## End(Not run)
## Example 2: Stratonovich sde
##  $dX(t) = ((2-X(t))/(2-t)) dt + X(t) \circ dW(t)$ 
set.seed(1234)

f <- expression((2-x)/(2-t))
g <- expression(x)
mod2 <- snssde1d(type="str",drift=f,diffusion=g,M=4000,x0=1, method="milstein")
mod2
summary(mod2,at = 0.25)
summary(mod2,at = 1)
## Not run:
plot(mod2)
lines(time(mod2),apply(mod2$X,1,mean),col=2,lwd=2)
lines(time(mod2),apply(mod2$X,1,bconfint,level=0.95)[1,],col=4,lwd=2)
lines(time(mod2),apply(mod2$X,1,bconfint,level=0.95)[2,],col=4,lwd=2)
legend("topleft",c("mean path",paste("bound of", 95," percent confidence")),
      inset = .01,col=c(2,4),lwd=2,cex=0.8)

## End(Not run)
```

## Description

The (S3) generic function `snssde2d` of simulation of solutions to 2-dim stochastic differential equations of Itô or Stratonovich type, with different methods.

**Usage**

```

snssde2d(N, ...)
## Default S3 method:
snssde2d(N = 1000, M = 1, x0 = c(0,0),t0 = 0, T = 1, Dt=NULL,
  drift,diffusion, alpha = 0.5, mu = 0.5, type = c("ito", "str"),
  method = c("euler", "milstein", "predcorr", "smilstein", "taylor",
    "heun", "rk1", "rk2","rk3"), ...)

## S3 method for class 'snssde2d'
summary(object, at, digits=NULL,...)
## S3 method for class 'snssde2d'
time(x, ...)
## S3 method for class 'snssde2d'
mean(x, at, ...)
## S3 method for class 'snssde2d'
Median(x, at, ...)
## S3 method for class 'snssde2d'
Mode(x, at, ...)
## S3 method for class 'snssde2d'
quantile(x, at, ...)
## S3 method for class 'snssde2d'
kurtosis(x, at, ...)
## S3 method for class 'snssde2d'
skewness(x, at, ...)
## S3 method for class 'snssde2d'
min(x, at, ...)
## S3 method for class 'snssde2d'
max(x, at, ...)
## S3 method for class 'snssde2d'
moment(x, at, ...)
## S3 method for class 'snssde2d'
cv(x, at, ...)
## S3 method for class 'snssde2d'
bconfint(x, at, ...)

## S3 method for class 'snssde2d'
plot(x, ...)
## S3 method for class 'snssde2d'
lines(x, ...)
## S3 method for class 'snssde2d'
points(x, ...)
## S3 method for class 'snssde2d'
plot2d(x, ...)
## S3 method for class 'snssde2d'
lines2d(x, ...)
## S3 method for class 'snssde2d'
points2d(x, ...)

```

**Arguments**

|           |  |
|-----------|--|
| N         | number of simulation steps.  |
| M         | number of trajectories (Monte-Carlo).  |
| x0        | initial values x0=(x,y) of the process $X_t$ and $Y_t$ at time t0.   |
| t0        | initial time.  |
| T         | ending time.   |
| Dt        | time step of the simulation (discretization). If it is <code>NULL</code> a default $\Delta t = \frac{T-t_0}{N}$ .      |
| drift     | drift coefficient: an <a href="#">expression</a> of three variables t, x and y for process $X_t$ and $Y_t$ .           |
| diffusion | diffusion coefficient: an <a href="#">expression</a> of three variables t, x and y for process $X_t$ and $Y_t$ .       |
| alpha, mu | weight of the predictor-corrector scheme; the default alpha = 0.5 and mu = 0.5.  |
| type      | if type="ito" simulation sde of Itô type, else type="str" simulation sde of Stratonovich type; the default type="ito". |
| method    | numerical methods of simulation, the default method = "euler".   |
| x, object | an object inheriting from class "snssde2d".  |
| at        | time between t0 and T. Monte-Carlo statistics of the solutions $(X_t, Y_t)$ at time at. The default at = T.            |
| digits    | integer, used for number formatting.   |
| ...       | potentially further arguments for (non-default) methods.   |

**Details**

The function `snssde2d` returns a matrix `x` of length  $N+1$ ; i.e. solution of the 2-dim sde  $(X_t, Y_t)$  of Ito or Stratonovich types; If `Dt` is not specified, then the best discretization  $\Delta t = \frac{T-t_0}{N}$ .

The 2-dim Ito stochastic differential equation is:

$$dX(t) = a(t, X(t), Y(t))dt + b(t, X(t), Y(t))dW_1(t)$$

$$dY(t) = a(t, X(t), Y(t))dt + b(t, X(t), Y(t))dW_2(t)$$

2-dim Stratonovich sde :

$$dX(t) = a(t, X(t), Y(t))dt + b(t, X(t), Y(t)) \circ dW_1(t)$$

$$dY(t) = a(t, X(t), Y(t))dt + b(t, X(t), Y(t)) \circ dW_2(t)$$

$W_1(t), W_2(t)$  two standard Brownian motion independent.

The methods of approximation are classified according to their different properties. Mainly two criteria of optimality are used in the literature: the strong and the weak (orders of) convergence. The method of simulation can be one among: Euler-Maruyama Order 0.5, Milstein Order 1, Milstein Second-Order, Predictor-Corrector method, Itô-Taylor Order 1.5, Heun Order 2 and Runge-Kutta Order 1, 2 and 3.

An overview of this package, see [browseVignettes\('Sim.DiffProc'\)](#) for more informations.

**Value**

snssde2d returns an object inheriting from `class "snssde2d"`.

`X`, `Y` an invisible mts (2-dim) object (X(t),Y(t)).

`driftx`, `drifty` drift coefficient of X(t) and Y(t).

`diffx`, `diffy` diffusion coefficient of X(t) and Y(t).

`type` type of sde.

`method` the numerical method used.

**Author(s)**

A.C. Guidoum, K. Boukhetala.

**References**

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- Saito, Y, and Mitsui, T. (1993). Simulation of Stochastic Differential Equations. *The Annals of the Institute of Statistical Mathematics*, **3**, 419–432.

**See Also**

[snssde3d](#) for 3-dim sde.  
[simulate](#) in package "yuima".

## Examples

```
## Example 1: Ito sde
## dX(t) = 4*(-1-X(t))*Y(t) dt + 0.2 dW1(t)
## dY(t) = 4*(1-Y(t))*X(t) dt + 0.2 dW2(t)
set.seed(1234)

fx <- expression(4*(-1-x)*y , 4*(1-y)*x )
gx <- expression(0.25*y,0.2*x)

mod2d1 <- snssde2d(drift=fx,diffusion=gx,x0=c(x0=1,y0=-1),M=1000)
mod2d1
summary(mod2d1)
##
dev.new()
plot(mod2d1,type="n")
mx <- apply(mod2d1$X,1,mean)
my <- apply(mod2d1$Y,1,mean)
lines(time(mod2d1),mx,col=1)
lines(time(mod2d1),my,col=2)
legend("topright",c(expression(E(X[t])),expression(E(Y[t]))),lty=1,inset = .01,col=c(1,2),cex=0.95)
##
dev.new()
plot2d(mod2d1) ## in plane (0,X,Y)
lines(my~mx,col=2)

## Example 2: Stratonovich sde
## dX(t) = Y(t) dt + 0 o dW1(t)
## dY(t) = (4*(1-X(t)^2)*Y(t) - X(t) ) dt + 0.2 o dW2(t)
set.seed(1234)

fx <- expression( y , (4*( 1-x^2 ))* y - x))
gx <- expression( 0 , 0.2)

mod2d2 <- snssde2d(drift=fx,diffusion=gx,type="str",T=100,N=10000)
mod2d2
plot(mod2d2,pos=2)
dev.new()
plot(mod2d2,union = FALSE)
dev.new()
plot2d(mod2d2,type="n") ## in plane (0,X,Y)
points2d(mod2d2,col=rgb(0,100,0,50,maxColorValue=255), pch=16)
```

**Description**

The (S3) generic function `snssde3d` of simulation of solutions to 3-dim stochastic differential equations of Itô or Stratonovich type, with different methods.

**Usage**

```
snssde3d(N, ...)
## Default S3 method:
snssde3d(N = 1000, M = 1, x0=c(0,0,0), t0 = 0, T = 1, Dt=NULL,
  drift, diffusion, alpha = 0.5, mu = 0.5, type = c("ito", "str"),
  method = c("euler", "milstein","predcorr", "smilstein", "taylor",
    "heun", "rk1", "rk2", "rk3"), ...)

## S3 method for class 'snssde3d'
summary(object, at, digits=NULL,...)
## S3 method for class 'snssde3d'
time(x, ...)
## S3 method for class 'snssde3d'
mean(x, at, ...)
## S3 method for class 'snssde3d'
Median(x, at, ...)
## S3 method for class 'snssde3d'
Mode(x, at, ...)
## S3 method for class 'snssde3d'
quantile(x, at, ...)
## S3 method for class 'snssde3d'
kurtosis(x, at, ...)
## S3 method for class 'snssde3d'
skewness(x, at, ...)
## S3 method for class 'snssde3d'
min(x, at, ...)
## S3 method for class 'snssde3d'
max(x, at, ...)
## S3 method for class 'snssde3d'
moment(x, at, ...)
## S3 method for class 'snssde3d'
cv(x, at, ...)
## S3 method for class 'snssde3d'
bconfint(x, at, ...)

## S3 method for class 'snssde3d'
plot(x, ...)
## S3 method for class 'snssde3d'
lines(x, ...)
## S3 method for class 'snssde3d'
points(x, ...)
## S3 method for class 'snssde3d'
plot3D(x, display = c("persp","rgl"), ...)
```

**Arguments**

|           |  |
|-----------|--|
| N         | number of simulation steps.  |
| M         | number of trajectories.  |
| x0        | initial value of the process $X_t, Y_t$ and $Z_t$ at time $t_0$ .  |
| t0        | initial time.  |
| T         | ending time.   |
| Dt        | time step of the simulation (discretization). If it is <code>NULL</code> a default $\Delta t = \frac{T-t_0}{N}$ .            |
| drift     | drift coefficient: an <b>expression</b> of four variables $t, x, y$ and $z$ for process $X_t, Y_t$ and $Z_t$ .               |
| diffusion | diffusion coefficient: an <b>expression</b> of four variables $t, x, y$ and $z$ for process $X_t, Y_t$ and $Z_t$ .           |
| alpha, mu | weight of the predictor-corrector scheme; the default alpha = 0.5 and mu = 0.5.  |
| type      | if type="ito" simulation sde of Itô type, else type="str" simulation sde of Stratonovich type; the default type="ito".       |
| method    | numerical methods of simulation, the default method = "euler".   |
| x, object | an object inheriting from class "snssde3d".  |
| at        | time between $t_0$ and $T$ . Monte-Carlo statistics of the solutions $(X_t, Y_t, Z_t)$ at time $at$ . The default $at = T$ . |
| digits    | integer, used for number formatting.   |
| display   | "persp" perspective or "rgl" plots.  |
| ...       | potentially further arguments for (non-default) methods.   |

**Details**

The function `snssde3d` returns a matrix  $x$  of length  $N+1$ ; i.e. solution of the 3-dim sde  $(X_t, Y_t, Z_t)$  of Ito or Stratonovich types; If  $Dt$  is not specified, then the best discretization  $\Delta t = \frac{T-t_0}{N}$ .

The 3-dim Ito stochastic differential equation is:

$$dX(t) = a(t, X(t), Y(t), Z(t))dt + b(t, X(t), Y(t), Z(t))dW_1(t)$$

$$dY(t) = a(t, X(t), Y(t), Z(t))dt + b(t, X(t), Y(t), Z(t))dW_2(t)$$

$$dZ(t) = a(t, X(t), Y(t), Z(t))dt + b(t, X(t), Y(t), Z(t))dW_3(t)$$

3-dim Stratonovich sde :

$$dX(t) = a(t, X(t), Y(t), Z(t))dt + b(t, X(t), Y(t), Z(t)) \circ dW_1(t)$$

$$dY(t) = a(t, X(t), Y(t), Z(t))dt + b(t, X(t), Y(t), Z(t)) \circ dW_2(t)$$

$$dZ(t) = a(t, X(t), Y(t), Z(t))dt + b(t, X(t), Y(t), Z(t)) \circ dW_3(t)$$

$W_1(t), W_2(t), W_3(t)$  three standard Brownian motion independent.

The methods of approximation are classified according to their different properties. Mainly two criteria of optimality are used in the literature: the strong and the weak (orders of) convergence. The method of simulation can be one among: Euler-Maruyama Order 0.5, Milstein Order 1, Milstein Second-Order, Predictor-Corrector method, Itô-Taylor Order 1.5, Heun Order 2 and Runge-Kutta Order 1, 2 and 3.

An overview of this package, see `browseVignettes('Sim.DiffProc')` for more informations.

**Value**

snssde3d returns an object inheriting from `class "snssde3d"`.

`X`, `Y`, `Z` an invisible mts (3-dim) object  $(X(t), Y(t), Z(t))$ .

`driftx`, `drifty`, `driftz`  
drift coefficient of  $X(t)$ ,  $Y(t)$  and  $Z(t)$ .

`diffx`, `diffy`, `diffz`  
diffusion coefficient of  $X(t)$ ,  $Y(t)$  and  $Z(t)$ .

`type` type of sde.

`method` the numerical method used.

**Author(s)**

A.C. Guidoum, K. Boukhetala.

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**See Also**

[snssde1d](#) and [snssde2d](#) for 1- and 2-dim sde.  
[sde.sim](#) in package "sde". [simulate](#) in package "yuima".

**Examples**

```
## Example 1: Ito sde
## dX(t) = (2*(Y(t)>0)-2*(Z(t)<=0)) dt + 0.2 * dW1(t)
## dY(t) = -2*Y(t) dt + 0.2 * dW2(t)
## dZ(t) = -2*Z(t) dt + 0.2 * dW3(t)
## W1(t), W2(t) and W3(t) three independent Brownian motion
set.seed(1234)

fx <- expression(2*(y>0)-2*(z<=0) , -2*y, -2*z)
gx <- rep(expression(0.2),3)

mod3d1 <- snssde3d(x0=c(0,2,-2),drift=fx,diffusion=gx,M=500,Dt=0.003)
mod3d1
summary(mod3d1)
##
dev.new()
plot(mod3d1,type="n")
mx <- apply(mod3d1$X,1,mean)
my <- apply(mod3d1$Y,1,mean)
mz <- apply(mod3d1$Z,1,mean)
lines(time(mod3d1),mx,col=1)
lines(time(mod3d1),my,col=2)
lines(time(mod3d1),mz,col=3)
legend("topright",c(expression(E(X[t])),expression(E(Y[t])),
  expression(E(Z[t]))),lty=1,inset = .01,col=c(1,2,3),cex=0.95)
##
dev.new()
plot3D(mod3d1,display="persp") ## in space (0,X,Y,Z)

## Example 2: Stratonovich sde
## dX(t) = Y(t)* dt
## dY(t) = (4*( 1-X(t)^2 )* Y(t) - X(t))* dt + 0.2 o dW2(t)
## dZ(t) = (4*( 1-X(t)^2 )* Z(t) - X(t))* dt + 0.2 o dW3(t)
set.seed(1234)

fx <- expression( y , (4*( 1-x^2 )* y - x), (4*( 1-x^2 )* z - x))
gx <- expression( 0 , 0.2, 0.2)

mod3d2 <- snssde3d(drift=fx,diffusion=gx,N=10000,T=100,type="str")
mod3d2
##
dev.new()
plot(mod3d2,pos=2)
##
dev.new()
plot(mod3d2,union = FALSE)
```

```
##
dev.new()
plot3D(mod3d2,display="persp") ## in space (0,X,Y,Z)
```

---

st.int

*Stochastic Integrals*


---

### Description

The (S3) generic function `st.int` of simulation of stochastic integrals of Itô or Stratonovich type.

### Usage

```
st.int(expr, ...)
## Default S3 method:
st.int(expr, lower = 0, upper = 1, M = 1, subdivisions = 1000L,
       type = c("ito", "str"), ...)

## S3 method for class 'st.int'
summary(object, at ,digits=NULL, ...)
## S3 method for class 'st.int'
time(x, ...)
## S3 method for class 'st.int'
mean(x, at, ...)
## S3 method for class 'st.int'
Median(x, at, ...)
## S3 method for class 'st.int'
Mode(x, at, ...)
## S3 method for class 'st.int'
quantile(x, at, ...)
## S3 method for class 'st.int'
kurtosis(x, at, ...)
## S3 method for class 'st.int'
min(x, at, ...)
## S3 method for class 'st.int'
max(x, at, ...)
## S3 method for class 'st.int'
skewness(x, at, ...)
## S3 method for class 'st.int'
moment(x, at, ...)
## S3 method for class 'st.int'
cv(x, at, ...)
## S3 method for class 'st.int'
bconfint(x, at, ...)

## S3 method for class 'st.int'
plot(x, ...)
```

```

## S3 method for class 'st.int'
lines(x, ...)
## S3 method for class 'st.int'
points(x, ...)

```

### Arguments

|              |   |
|--------------|---|
| expr         | an <a href="#">expression</a> of two variables t (time) and w (w: standard Brownian motion).                    |
| lower, upper | the lower and upper end points of the interval to be integrate.   |
| M            | number of trajectories (Monte-Carlo).   |
| subdivisions | the maximum number of subintervals.   |
| type         | Itô or Stratonovich integration.  |
| x, object    | an object inheriting from class "st.int".   |
| at           | time between lower and upper. Monte-Carlo statistics of stochastic integral at time at. The default at = upper. |
| digits       | integer, used for number formatting.  |
| ...          | potentially further arguments for (non-default) methods.  |

### Details

The function `st.int` returns a [ts](#) x of length N+1; i.e. simulation of stochastic integrals of Itô or Stratonovich type.

The Itô interpretation is:

$$\int_{t_0}^t f(s) dW_s = \lim_{N \rightarrow \infty} \sum_{i=1}^N f(t_{i-1})(W_{t_i} - W_{t_{i-1}})$$

The Stratonovich interpretation is:

$$\int_{t_0}^t f(s) \circ dW_s = \lim_{N \rightarrow \infty} \sum_{i=1}^N f\left(\frac{t_i + t_{i-1}}{2}\right)(W_{t_i} - W_{t_{i-1}})$$

An overview of this package, see [browseVignettes\('Sim.DiffProc'\)](#) for more informations.

### Value

`st.int` returns an object inheriting from [class](#) "st.int".

|              |   |
|--------------|---|
| X            | the final simulation of the integral, an invisible <a href="#">ts</a> object. |
| fun          | function to be integrated.  |
| type         | type of stochastic integral.  |
| subdivisions | the number of subintervals produced in the subdivision process.               |

### Author(s)

A.C. Guidoum, K. Boukhetala.

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## See Also

[snssde1d](#), [snssde2d](#) and [snssde3d](#) for 1,2 and 3-dim sde.

## Examples

```
## Example 1: Ito integral
## f(t,w(t)) = int(exp(w(t) - 0.5*t) * dw(s)) with t in [0,1]
set.seed(1234)

f <- expression( exp(w-0.5*t) )
mod1 <- st.int(expr=f, type="ito", M=50, lower=0, upper=1)
mod1
summary(mod1)
## Display
plot(mod1)
lines(time(mod1), apply(mod1$X, 1, mean), col=2, lwd=2)
lines(time(mod1), apply(mod1$X, 1, bconfint, level=0.95)[1, ], col=4, lwd=2)
lines(time(mod1), apply(mod1$X, 1, bconfint, level=0.95)[2, ], col=4, lwd=2)
legend("topleft", c("mean path", paste("bound of", 95, " percent confidence")),
      inset = .01, col=c(2,4), lwd=2, cex=0.8)

## Example 2: Stratonovich integral
## f(t,w(t)) = int(w(s) o dw(s)) with t in [0,1]
set.seed(1234)

g <- expression( w )
mod2 <- st.int(expr=g, type="str", M=50, lower=0, upper=1)
mod2
summary(mod2)
## Display
plot(mod2)
lines(time(mod2), apply(mod2$X, 1, mean), col=2, lwd=2)
lines(time(mod2), apply(mod2$X, 1, bconfint, level=0.95)[1, ], col=4, lwd=2)
lines(time(mod2), apply(mod2$X, 1, bconfint, level=0.95)[2, ], col=4, lwd=2)
legend("topleft", c("mean path", paste("bound of", 95, " percent confidence")),
      inset = .01, col=c(2,4), lwd=2, cex=0.8)
```

## Description

These methods produces the related LaTeX table and mathematic expression for Sim.DiffProc environment.

## Usage

```
TEX.sde(object, ...)
```

```
## Default S3 method:
```

```
TEX.sde(object, ...)
```

## Arguments

`object` an objects from class `MCM.sde` and `MEM.sde`. Or an R vector of `expression` of SDEs, i.e., drift and diffusion coefficients.

`...` arguments to be passed to `kable` function if object from class `MCM.sde`.

## Details

New tools for constructing tables and mathematical expressions with Sim.DiffProc package.

An overview of this package, see `browseVignettes('Sim.DiffProc')` for more informations.

## Author(s)

A.C. Guidoum

## References

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## See Also

`kable` create tables in LaTeX, HTML, Markdown and reStructuredText.

`toLatex` converting R Objects to BibTeX or LaTeX.

**Examples**

```

## LaTeX mathematic for an R expression of SDEs
## Copy and paste the following output in your LaTeX file

# Example 1

f <- expression(-mu1 * x)
g <- expression(mu2 * sqrt(x))
TEX.sde(object = c(drift = f, diffusion = g))

# Example 2

f <- expression(mu1*cos(mu2+z),mu1*sin(mu2+z),0)
g <- expression(sigma,sigma,alpha)
TEX.sde(object = c(drift = f, diffusion = g))

## LaTeX mathematic for object of class 'MEM.sde'
## Copy and paste the following output in your LaTeX file

# Example 3

mem.mod3d <- MEM.sde(drift = f, diffusion = g)
TEX.sde(object = mem.mod3d)

## LaTeX table for object of class 'MCM.sde'
## Copy and paste the following output in your LaTeX file

# Example 4

## Not run:
mu1=0.25; mu2=3; sigma=0.05; alpha=0.03
mod3d <- snssde3d(drift=f,diffusion=g,x0=c(x=0,y=0,z=0),M=100,T=10)

stat.fun3d <- function(data, i){
  d <- data[i,]
  return(c(mean(d$x),mean(d$y),mean(d$z),
           var(d$x),var(d$y),var(d$z)))
}
mcm.mod3d = MCM.sde(mod3d,statistic=stat.fun3d,R=10,parallel="snow",ncpus=parallel::detectCores(),
                   names=c("m1","m2","m3","S1","S2","S3"))

TEX.sde(object = mcm.mod3d, booktabs = TRUE, align = "r", caption = "\LaTeX~
table for Monte Carlo results generated by \code{TEX.sde()} method.")

## End(Not run)

```

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