# Package 'betafunctions'

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Type Package

<b>Title</b> Functions for Working with Two- And Four-Parameter Beta Probability Distributions
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<b>Description</b> Package providing a number of functions for working with Two- and Four-parameter Beta distributions, including alternative parameterizations, calculation of moments, and fitting of Beta distributions to vectors of values. Also includes functions for estimating classification accuracy, diagnostic performance and consistency, using what is generally known as the "Livingston and Lewis approach" in the psychometric literature as the foundational method, which makes extensive use of the Beta distribution. Livingston and Lewis (1995) <doi:10.1111 j.1745-3984.1995.tb00462.x="">. Hanson (1991) <a href="https://files.eric.ed.gov/fulltext/ED344945.pdf">https://files.eric.ed.gov/fulltext/ED344945.pdf</a>&gt;. Glas, Lijmer, Prins, Bonsel and Bossuyt (2003) <doi:10.1016 s0895-4356(03)00177-x="">.</doi:10.1016></doi:10.1111>
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AMS

2 AMS

AMS		Alpha Shape-Parameter Given Location-Parameters, Skewness. Kurtosis and Beta Shape-Parameter of a	
Index			3
	•		
	1		
	1		
	•		
	*		
	cba		1
	caStats		1
	BMS		1
	1		
	U 1 . 1		
	Beta.gfx.polv.qdf		

### Description

Calculates the Beta value required to produce a Beta probability density distribution with defined moments and parameters. Be advised that not all combinations of moments and parameters can be satisfied (e.g., specifying mean, variance, skewness and kurtosis uniquely determines both location-parameters, meaning that the value of the lower-location parameter will take on which ever value it must, and cannot be specified).

Beta PDD.

### Usage

```
AMS(
mean = NULL,
variance = NULL,
skewness = NULL,
```

AMS 3

```
kurtosis = NULL,
    1 = 0,
    u = 1,
    beta = NULL,
    sd = NULL
)
```

#### **Arguments**

mean	The mean (first raw moment) of the target Standard Beta probability density distribution.
variance	The variance (second central moment) of the target Standard Beta probability density distribution.
skewness	The skewness (third standardized moment) of the target Beta probability density distribution.
kurtosis	The kurtosis (fourth standardized moment) of the target Beta probability density distribution.
1	The lower-bound of the Beta distribution. Default is 0 (i.e., the lower-bound of the Standard, two-parameter Beta distribution).
u	The upper-bound of the Beta distribution. Default is 1 (i.e., the upper-bound of the Standard, two-parameter Beta distribution).
beta	Optional specification of the Beta shape-parameter of the target Beta distribution. Finds then the Alpha parameter necessary to produce a distribution with the specified mean, given specified Beta, l, and u parameters.
sd	Optional alternative to specifying var. The standard deviation of the target Standard Beta probability density distribution.

### Value

A numeric value representing the required value for the Alpha shape-parameter in order to produce a Beta probability density distribution with the target mean and variance, given specified lower- and upper bounds of the Beta distribution.

```
# Generate some fictional data. Say, 100 individuals take a test with a # maximum score of 100 and a minimum score of 0, rescaled to proportion # of maximum. set.seed(1234) testdata <- rbinom(100, 100, rBeta.4P(100, .25, .75, 5, 3)) / 100 hist(testdata, xlim = c(0, 1)) # To find the alpha shape-parameter of a Standard (two-parameter) Beta # distribution with the same mean and variance as the observed-score # distribution using AMS(): AMS(mean(testdata), var(testdata))
```

4 AUC

AUC

Area Under the ROC Curve.

### Description

Given a vector of false-positive rates and a vector of true-positive rates, calculate the area under the Receiver Operator Characteristic (ROC) curve.

### Usage

```
AUC(FPR, TPR)
```

### **Arguments**

FPR Vector of False-Positive Rates.

TPR Vector of True-Positive Rates.

#### Value

A value representing the area under the ROC curve.

#### Note

Script originally retrieved and modified from https://blog.revolutionanalytics.com/2016/11/calculating-auc.html.

```
# Generate some fictional data. Say, 100 individuals take a test with a
# maximum score of 100 and a minimum score of 0.
set.seed(1234)
testdata <- rbinom(100, 100, rBeta.4P(100, .25, .75, 5, 3))
hist(testdata, xlim = c(0, 100))

# Suppose the cutoff value for attaining a pass is 50 items correct, and
# that the reliability of this test was estimated to 0.7. To calculate the
# necessary (x, y) coordinates to compute the area under the curve statistic
# one can use the LL.ROC() function with the argument
# raw.out = TRUE.
coords <- LL.ROC(x = testdata, reliability = .7, truecut = 50, min = 0,
max = 100, raw.out = TRUE)

# To calculate and retrieve the Area Under the Curve (AUC) with the AUC()
# function, feed it the raw coordinates calculated above.
AUC(coords[, "FPR"], coords[, "TPR"])</pre>
```

Beta.2p.fit 5

Beta.2p.fit	Method of Moment Estimates of Shape-Parameters of the Two-
	Parameter (Standard) Beta Distribution.

### **Description**

An implementation of the method of moments estimation of two-parameter beta distribution parameters. Given a vector of values, calculates the shape parameters required to produce a two-parameter beta distribution with the same mean and variance (i.e., the first two moments) as the observed-score distribution.

#### Usage

```
Beta.2p.fit(scores, mean = NULL, variance = NULL, 1 = 0, u = 1)
```

### **Arguments**

scores	A vector of values to which the two-parameter beta distribution is to be fitted. The values ought to fall within the $[0, 1]$ interval.
mean	The mean of the target Beta distribution. Alternative to feeding the function raw scores.
variance	The variance of the target Beta distribution. Alternative to feeding the function raw scores.
1	Optional specification of a lower-bound parameter of the Beta distribution. Default is 0 (i.e., the lower-bound of the Standard two-parameter Beta distribution).
u	Optional specification of an upper-bound parameter of the Beta distribution. Default is 1 (i.e., the lower-bound of the Standard two-parameter Beta distribution).

### Value

A list of parameter-values required to produce a Standard two-parameter beta distribution with the same first two moments as the observed distribution.

```
# Generate some fictional data. Say, 100 individuals take a test with a
# maximum score of 100 and a minimum score of 0.
set.seed(1234)
testdata <- rbinom(100, 100, rBeta.4P(100, .25, .75, 5, 3)) / 100
hist(testdata, xlim = c(0, 1), freq = FALSE)

# To fit and retrieve the parameters for a two-parameter beta distribution
# to the observed-score distribution using Beta.2p.fit():
(params.2p <- Beta.2p.fit(testdata))
curve(dbeta(x, params.2p$alpha, params.2p$beta), add = TRUE)</pre>
```

Beta.4p.fit

Beta.4p.fit	Method of Moment Estimates of Shape- and Location Parameters of the Four-Parameter Beta Distribution.

### **Description**

An implementation of the method of moments estimation of four-parameter beta distribution parameters presented by Hanson (1991). Given a vector of values, calculates the shape- and location parameters required to produce a four-parameter beta distribution with the same mean, variance, skewness and kurtosis (i.e., the first four moments) as the observed-score distribution.

### Usage

```
Beta.4p.fit(
   scores,
   mean = NULL,
   variance = NULL,
   skewness = NULL,
   kurtosis = NULL
)
```

#### **Arguments**

scores	A vector of values to which the four-parameter beta distribution is to be fitted.
mean	If scores are not supplied: specification of the mean for the target four-parameter Beta distribution.
variance	If scores are not supplied: specification of the variance for the target four- parameter Beta distribution.
skewness	If scores are not supplied: specification of the skewness for the target four- parameter Beta distribution.
kurtosis	If scores are not supplied: specification of the kurtosis for the target four-parameter Beta distribution.

### Value

A list of parameter-values required to produce a four-parameter beta distribution with the same first four moments as the observed distribution.

### References

Hanson, Bradley A. (1991). Method of Moments Estimates for the Four-Parameter Beta Compound Binomial Model and the Calculation of Classification Consistency Indexes. American College Testing Research Report Series.

Lord, Frederic M. (1965). A Strong True-Score Theory, With Applications. Psychometrika, 30(3).

Beta.gfx.poly.cdf 7

#### **Examples**

```
# Generate some fictional data. Say, 100 individuals take a test with a
# maximum score of 100 and a minimum score of 0.
set.seed(1234)
testdata <- rbinom(100, 100, rBeta.4P(100, .25, .75, 5, 3))
hist(testdata, xlim = c(0, 100), freq = FALSE)

# To fit and retrieve the parameters for a four-parameter beta distribution
# to the observed-score distribution using Beta.4p.fit():
(params.4p <- Beta.4p.fit(testdata))
curve(dBeta.4P(x, params.4p$1, params.4p$u, params.4p$alpha, params.4p$beta), add = TRUE)</pre>
```

Beta.gfx.poly.cdf

Coordinate Generation for Marking an Area Under the Curve for the Beta Cumulative Probability Density Distribution.

### **Description**

Plotting tool, producing a two-column matrix with values of y corresponding to locations on x. Useful for shading areas under the curve when tracing the line for the Standard Beta cumulative probability function.

#### Usage

```
Beta.gfx.poly.cdf(from, to, by, alpha, beta, l = 0, u = 1)
```

#### **Arguments**

from	The point of the x-axis from where to start producing y-density values.
to	The point of the x-axis to where y-density values are to be produced.
by	The resolution (or spacing) at which to produce y-density values.
alpha	The Alpha shape-parameter value for the Standard Beta cumulative probability distribution.
beta	The Beta shape-parameter for the Standard Beta cumulative probability distribution.
1	The lower-bound location parameter of the Beta distribution.
u	The upper-bound location parameter of the Beta distribution.

#### Value

A two-column matrix with cumulative probability-values of y to plot against corresponding location values of x.

8 Beta.gfx.poly.pdf

#### **Examples**

```
# To box in an area under a four-parameter Beta cumulative distribution with # location parameters l=.25 and u=75, and shape parameters # alpha = 5 and beta = 3, from .4 to .6: plot(NULL, xlim = c(0, 1), ylim = c(0, 1)) coords <- Beta.gfx.poly.cdf(from = .4, to = .6, by = .001, alpha = 5, beta = 3, l=.25, u=.75) polygon(coords)
```

Beta.gfx.poly.pdf

Coordinate Generation for Marking an Area Under the Curve for the Beta Probability Density Distribution.

### **Description**

Plotting tool, producing a two-column matrix with values of y corresponding to locations on x. Useful for shading areas under the curve when tracing the line for the Standard Beta probability density function.

#### Usage

```
Beta.gfx.poly.pdf(from, to, by, alpha, beta, l = 0, u = 1)
```

#### **Arguments**

from	The point of the x-axis from where to start producing y-density values.
to	The point of the x-axis to where y-density values are to be produced.
by	The resolution (or spacing) at which to produce y-density values.
alpha	The Alpha shape-parameter value for the Standard Beta probability density distribution.
beta	The Beta shape-parameter for the Standard Beta probability density distribution.
1	The lower-bound location parameter of the Beta distribution.
u	The upper-bound location parameter of the Beta distribution.

#### Value

A two-column matrix with density-values of y to plot against corresponding location values of x.

```
# To box in an area under a four-parameter beta distribution with location # parameters l=.25 and u=.75, and shape parameters # alpha = 5 and beta = 3, from .4 to .6: plot(NULL, xlim = c(0, 1), ylim = c(0, 7)) coords <- Beta.gfx.poly.pdf(from = .4, to = .6, by = .001, alpha = 5, beta = 3, l=.25, u=.75) polygon(coords)
```

Beta.gfx.poly.qdf

	poordinate Generation for Marking an Area Under the Curve for the eta Quantile Density Distribution.
--	--

### **Description**

Plotting tool, producing a two-column matrix with values of y corresponding to locations on x. Useful for shading areas under the curve when tracing the line for the Standard Beta probability quantile function.

### Usage

```
Beta.gfx.poly.qdf(from, to, by, alpha, beta, l = 0, u = 1)
```

#### **Arguments**

from	The point of the x-axis from where to start producing y-quantile values.
to	The point of the x-axis to where y-quantile values are to be produced.
by	The resolution (or spacing) at which to produce y-density values.
alpha	The Alpha shape-parameter value for the Standard Beta probability distribution.
beta	The Beta shape-parameter for the Standard Beta probability distribution.
1	The lower-bound location parameter of the Beta distribution.
u	The upper-bound location parameter of the Beta distribution.

#### Value

A two-column matrix with quantile-values of y to plot against corresponding location values of x.

```
# To box in an area under a four-parameter beta quantile distribution with # location parameters l=.25 and u=75, and shape parameters # alpha = 5 and beta = 3, from .4 to .6: plot(NULL, xlim = c(0, 1), ylim = c(0, 1)) coords <- Beta.gfx.poly.qdf(from = .4, to = .6, by = .001, alpha = 5, beta = 3, l=.25, u=.75) polygon(coords)
```

Beta.tp.fit

Beta.tp.fit	Estimate Beta true-score distribution based on observed-score raw-
	moments and the effective test length.

### Description

Estimator for the Beta true-score distribution shape-parameters from the observed-score distribution and Livingston and Lewis' effective test length. Returns a list with entries representing the lower- and upper shape parameters (l and u), and the shape parameters (alpha and beta) of the four-parameters beta distribution.

### Usage

```
Beta.tp.fit(
    x,
    min,
    max,
    etl,
    reliability = NULL,
    true.model = "4P",
    failsafe = FALSE,
    l = 0,
    u = 1,
    alpha = NA,
    beta = NA,
    output = "parameters"
)
```

### **Arguments**

X	Vector of observed-scores.
min	The minimum possible score to attain on the test.
max	The maximum possible score to attain on the test.
etl	The value of Livingston and Lewis' effective test length. See ?ETL().
reliability	Optional specification of the test-score reliability coefficient. If specified, overrides the input of the etl argument.
true.model	The type of Beta distribution which is to be fit to the moments of the true-score distribution. Options are "4P" and "2P", where "4P" refers to the four-parameter (with the same mean, variance, skewness, and kurtosis) and "2P" the two-parameter solution (with the same mean and variance).
failsafe	Logical. Whether to revert to a failsafe two-parameter solution should the four-parameter solution contain invalid parameter estimates.
1	If failsafe = TRUE or true.model = "2P": The lower-bound of the Beta distribution. Default is 0 (i.e., the lower-bound of the Standard, two-parameter Beta distribution).

Beta.tp.fit

u	If failsafe = TRUE or true.model = "2P": The upper-bound of the Beta distribution. Default is 1 (i.e., the upper-bound of the Standard, two-parameter Beta distribution).
alpha	If failsafe = TRUE or true.model = "2P": The Alpha shape-parameter of the Beta distribution. Default is NA (i.e., estimate).
beta	If failsafe = TRUE or true.model = "2P": The Beta shape-parameter of the Beta distribution. Default is NA (i.e., estimate).
output	Option to specify true-score distribution moments as output if the value of the output argument does not equal "parameters".

#### Value

A list with the parameter values of a four-parameter Beta distribution. "I" is the lower location-parameter, "u" the upper location-parameter, "alpha" the first shape-parameter, and "beta" the second shape-parameter.

#### Note

This estimator is based on the S-Plus code provided by Rogosa and Finkelman (2004). It includes an option for implementing a failsafe should the four-parameter solution be invalid (e.g., 1 < 0 or u > 1, alpha < 1 or beta < 1).

#### References

Hanson, B. A. (1991). Method of Moments Estimates for the Four-Parameter Beta Compound Binomial Model and the Calculation of Classification Consistency Indexes. American College Testing Research Report Series. Retrieved from https://files.eric.ed.gov/fulltext/ED344945.pdf

Lord, F. M. (1965). A strong true-score theory, with applications. Psychometrika. 30(3). pp. 239–270. doi: 10.1007/BF02289490

Rogosa, D. & Finkelman, M. (2004). How Accurate Are the STAR Scores for Individual Students? – An Interpretive Guide. Retrieved from http://statweb.stanford.edu/~rag/accguide/guide04.pdf

```
# Generate some fictional data. Say 1000 individuals take a 100-item test # where all items are equally difficult, and the true-score distribution # is a four-parameter Beta distribution with location parameters l=.25, # u=.75, alpha = 5, and beta = 3: set.seed(12) testdata <- rbinom(1000, 100, rBeta.4P(1000, .25, .75, 5, 3)) # Since this test contains items which are all equally difficult, the true # effective test length (etl) is the actual test length. I.e., etl = 100. # To estimate the four-parameter Beta distribution parameters underlying # the draws from the binomial distribution: Beta.tp.fit(testdata, 0, 100, 100) # Imagine a case where the fitting procedure produces an impermissible # estimate (e.g., l < 0 or u > 1).
```

12 betamoments

```
set.seed(1234)
testdata <- rbinom(1000, 50, rBeta.4P(1000, .25, .75, 5, 3))
Beta.tp.fit(testdata, 0, 50, 50)

# This example produced an 1-value estimate less than 0. One way of
# dealing with such an occurance is to revert to a two-parameter
# model, specifying the 1 and u parameters and estimating the
# alpha and beta parameters necessary to produce a Beta distribution
# with the same mean and variance as the observed-score distribution.

# Suppose you have good theoretical reasons to fix the 1 parameter at a
# value of 0.25 (e.g., the test is composed of multiple-choice questions
# with four response-options, resulting in a 25% chance of guessing the
# correct answer). The 1-parameter could be specified to this theoretically
# justified value, and the u-parameter could be specified to be equal to the
# estimate above (u = 0.7256552) as such:
Beta.tp.fit(testdata, 0, 50, 50, true.model = "2P", 1 = 0.25, u = 0.7256552)</pre>
```

betamoments

Compute Moments of Two-to-Four Parameter Beta Probability Density Distributions.

### **Description**

Computes Raw, Central, or Standardized moment properties of defined Standard Beta probability density distributions.

### Usage

```
betamoments(
  alpha,
  beta,
  l = 0,
  u = 1,
  types = c("raw", "central", "standardized"),
  orders = 4
)
```

#### **Arguments**

alpha	The Alpha shape parameter of the PDD.
beta	The Beta shape parameter of the PDD.
1	The first (lower) location parameter of a four-parameter distribution.
u	The second (upper) location parameter of a four-parameter distribution.
types	A character vector determining which moment-types are to be calculated. Permissible values are "raw", "central", and "standardized".
orders	The number of moment-orders to be calculated for each of the moment-types.

BMS 13

#### Value

A list of moment types, each a list of moment orders.

#### References

Hanson, B. A (1991). Method of Moments Estimates for the Four-Parameter Beta Compound Binomial Model and the Calculation of Classification Consistency Indexes. American College Testing Research Report Series.

#### **Examples**

```
# Assume some variable follows a four-parameter beta distribution with # location parameters l=0.25 and u=.75, and shape # parameters a=5 and b=3. To compute the first four # raw, central, and standardized moments of this distrubution using # betamoments(): betamoments(a = 5, b = 3, l = .25, u = .75, types = c("raw", "central", "standardized"), orders = 4)
```

**BMS** 

Beta Shape-Parameter Given Location-Parameters, Mean, Variance, Skewness, Kurtosis and Alpha Shape-Parameter of a Four-Parameter Beta PDD.

### **Description**

Calculates the Beta value required to produce a Beta probability density distribution with defined moments and parameters. Be advised that not all combinations of moments and parameters can be satisfied (e.g., specifying mean, variance, skewness and kurtosis uniquely determines both location-parameters, meaning that the value of the lower-location parameter will take on which ever value it must, and cannot be specified).

### Usage

```
BMS(
mean = NULL,
variance = NULL,
skewness = NULL,
kurtosis = NULL,
l = 0,
u = 1,
alpha = NULL,
sd = NULL
```

14 BMS

### **Arguments**

mean	The mean (first raw moment) of the target Standard Beta probability density distribution.
variance	The variance (second central moment) of the target Standard Beta probability density distribution.
skewness	The skewness (third standardized moment) of the target Beta probability density distribution.
kurtosis	The kurtosis (fourth standardized moment) of the target Beta probability density distribution.
1	The lower-bound of the Beta distribution. Default is 0 (i.e., the lower-bound of the Standard, two-parameter Beta distribution).
u	The upper-bound of the Beta distribution. Default is 1 (i.e., the upper-bound of the Standard, two-parameter Beta distribution).
alpha	Optional specification of the Alpha shape-parameter of the target Beta distribution. Finds then the Beta parameter necessary to produce a distribution with the specified mean, given specified Alpha, l, and u parameters.
sd	Optional alternative to specifying var. The standard deviation of the target Standard Beta probability density distribution.

#### Value

A numeric value representing the required value for the Beta shape-parameter in order to produce a Standard Beta probability density distribution with the target mean and variance, given specified lower- and upper bounds of the Beta distribution.

```
# Generate some fictional data. Say, 100 individuals take a test with a # maximum score of 100 and a minimum score of 0, rescaled to proportion # of maximum. set.seed(1234) testdata <- rbinom(100, 100, rBeta.4P(100, .25, .75, 5, 3)) / 100 hist(testdata, xlim = c(0, 1)) # To find the beta shape-parameter of a Standard (two-parameter) Beta # distribution with the same mean and variance as the observed-score # distribution using BMS(): BMS(mean(testdata), var(testdata)) # To find the beta shape-parameter of a four-parameter Beta # distribution with specified lower- and upper-bounds of l = 0.25 and # u = 0.75 using BMS: BMS(mean(testdata), var(testdata), .25, .75)
```

caStats 15

Classification Assurance Statistics	-+-	225+2+2
caStats Classification Accuracy Statistics.	ats	castats

#### **Description**

Provides a set of statistics often used for conveying information regarding the certainty of classifications based on tests.

### Usage

```
caStats(tp, tn, fp, fn)
```

#### **Arguments**

tp	The frequency or rate of true-positive classifications.
tn	The frequency or rate of true-negative classifications.
fp	The frequency or rate of false-positive classifications.
fn	The frequency or rate of false-negative classifications.

#### Value

A list of diagnostic performance statistics based on true/false positive/negative statistics. Specifically, the sensitivity, specificity, positive likelihood ratio (LR.pos), negative likelihood ratio (LR.neg), positive predictive value (PPV), negative predictive value (NPV), Youden's J. (Youden.J), and Accuracy.

#### References

Glas et al. (2003). The Diagnostic Odds Ratio: A Single Indicator of Test Performance, Journal of Clinical Epidemiology, 1129-1135, 56(11). doi: 10.1016/S0895-4356(03)00177-X

```
# Generate some fictional data. Say, 100 individuals take a test with a
# maximum score of 100 and a minimum score of 0.
set.seed(1234)
testdata <- rbinom(100, 100, rBeta.4P(100, .25, .75, 5, 3))
hist(testdata, xlim = c(0, 100))

# Suppose the cutoff value for attaining a pass is 50 items correct, and
# that the reliability of this test was estimated to 0.7. First, compute the
# estimated confusion matrix using LL.CA():
cmat <- LL.CA(x = testdata, reliability = .7, cut = 50, min = 0,
max = 100)$confusionmatrix

# To estimate and retrieve diagnostic performance statistics using caStats(),
# feed it the appropriate entries of the confusion matrix.
caStats(tp = cmat["True", "Fail"], tn = cmat["True", "Pass"],
fp = cmat["False", "Fail"], fn = cmat["False", "Pass"])</pre>
```

16 cba

cba

Calculate Cronbach's Alpha from supplied variables.

### Description

Calculates Cronbach's Alpha, a very commonly used index for assessing the reliability / internal consistency of a sum-score. Often interpreted as the mean correlation across all possible split-half alternate forms of the test.

### Usage

cba(x)

#### **Arguments**

Х

A data-frame or matrix of numerical values where rows are across-items withinrespondent observation vectors, and columns are within-item across-respondents observation vectors.

#### Value

Cronbach's Alpha for the sum-score of supplied variables.

### Note

Missing values are treated by passing na.rm = TRUE to the var function call.

Be aware that this function does not issue a warning if there are negative correlations between variables in the supplied data-set.

#### References

Cronbach, L.J. (1951). Coefficient alpha and the internal structure of tests. Psychometrika 16, 297–334. doi: 10.1007/BF02310555

```
# Generate some fictional data. Say 100 students take a 50-item long test
# where all items are equally difficult.
set.seed(1234)
p.success <- rBeta.4P(100, .25, .75, 5, 3)
for (i in 1:50) {
   if (i == 1) {
      rawdata <- matrix(nrow = 100, ncol = 50)
   }
   rawdata[, i] <- rbinom(100, 1, p.success)
}
# To calculate Cronbach's Alpha for this test:
cba(rawdata)</pre>
```

ccStats 17

ccStats

Classification Consistency Statistics.

#### **Description**

Provides a set of statistics often used for conveying information regarding the consistency of classifications based on tests.

#### Usage

```
ccStats(ii, ij, ji, jj)
```

### **Arguments**

The frequency or rate of consistent classifications into category "i".
The frequency or rate of inconsistent classifications into categories "i" and "j".
The frequency or rate of inconsistent classifications into categories "j" and "i".
The frequency or rate of consistent classifications into category "j".

#### Value

A list of classification consistency statistics. Specifically, the coefficient of consistent classification (p), the coefficient of consistent classification by chance  $(p_c)$ , and Cohen's Kappa coefficient.

#### References

Hanson, Bradley A. (1991). Method of Moments Estimates for the Four-Parameter Beta Compound Binomial Model and the Calculation of Classification Consistency Indexes. American College Testing.

```
# Generate some fictional data. Say, 100 individuals take a test with a
# maximum score of 100 and a minimum score of 0.
set.seed(1234)
testdata <- rbinom(100, 100, rBeta.4P(100, .25, .75, 5, 3))
hist(testdata, xlim = c(0, 100))

# Suppose the cutoff value for attaining a pass is 50 items correct, and
# that the reliability of this test was estimated to 0.7. First, compute the
# estimated consistency matrix using LL.CA():
cmat <- LL.CA(x = testdata, reliability = .7, cut = 50, min = 0,
max = 100)$consistencymatrix

# To estimate and retrieve consistency statistics using ccStats(),
# feed it the appropriate entries of the consistency matrix.
ccStats(ii = cmat["i", "i"], ij = cmat["i", "j"],
ji = cmat["j", "i"], jj = cmat["j", "j"])</pre>
```

18 dBeta.pBeta

dBeta.4P

Probability Density under the Four-Parameter Beta PDD.

### **Description**

Gives the density at desired values of x under the Four-Parameter Beta PDD.

### Usage

```
dBeta.4P(x, 1, u, alpha, beta)
```

### **Arguments**

X	Value of x.
1	The first (lower) location parameter.
u	The second (upper) location parameter.
alpha	The first shape parameter.
beta	The second shape parameter.

#### Value

The value for the probability density at specified values of x.

#### **Examples**

```
# Assume some variable follows a four-parameter beta distribution with # location parameters l=0.25 and u=.75, and shape # parameters alpha = 5 and beta = 3. To compute the # probability density at a specific point of the distribution (e.g., .5) # using dBeta.4P(): dBeta.4P(x=.5, y=.75, alpha = 5, beta = 3)
```

dBeta.pBeta

An implementation of the Beta-density Compound Cumulative-Beta Distribution.

### Description

The Beta Compound Beta distribution: The product of the four-parameter Beta probability density function and the beta cumulative probability function. Used in the Livingston and Lewis approach to classification accuracy and consistency, the output can be interpreted as the population density of passing scores produced at "x" (a value of true-score).

#### Usage

```
dBeta.pBeta(x, 1, u, alpha, beta, n, c, lower.tail = FALSE)
```

dBeta.pBinom

#### **Arguments**

Χ	x-axis input for which p (proportion or probability) is to be computed.
1	The lower-bound of the four-parameter Beta distribution.
u	The upper-bound of the four-parameter Beta distribution.
alpha	The alpha shape-parameter of the Beta density distribution.
beta	The beta shape-parameter of the Beta density distribution.
n	The number of trials for the Beta cumulative probability distribution.
С	The "true-cut" (proportion) of on the Beta cumulative probability distribution.
lower.tail	Logical. Whether to compute the lower or upper tail of the Beta cumulative probability distribution. Default is FALSE (i.e., upper tail).

#### References

Hanson, Bradley A. (1991). Method of Moments Estimates for the Four-Parameter Beta Compound Binomial Model and the Calculation of Classification Consistency Indexes. American College Testing Research Report Series.

Livingston, Samuel A. and Lewis, Charles. (1995). Estimating the Consistency and Accuracy of Classifications Based on Test Scores. Journal of Educational Measurement, 32(2).

Lord, Frederic M. (1965). A Strong True-Score Theory, With Applications. Psychometrika, 30(3).

```
# Given a four-parameter Beta distribution with parameters 1 = 0.25, u = 0.75,
# alpha = 5, and beta = 3, and a Beta error distribution with number of
# trials (n) = 10 and a cutoff-point (c) at 50% correct (i.e., proportion correct
# of 0.5), the population density of passing scores produced at true-score
# (x) = 0.5 can be calculated as:
dBeta.pBeta(x = 0.5, l = 0.25, u = 0.75, a = 5, b = 3, n = 10, c = 0.5)
# Conversely, the density of failing scores produced at x can be calculated
# by passing the additional argument "lower.tail = TRUE" to the function.
# That is:
dBeta.pBeta(x = 0.5, l = 0.25, u = 0.75, a = 5, b = 3, n = 10, c = 0.5, lower.tail = TRUE)
# By integration, the population proportion of (e.g.) passing scores in some
# region of the true-score distribution (e.g. between 0.25 and 0.5) can be
# calculated as:
integrate(function(x) { dBeta.pBeta(x, 0.25, .75, 5, 3, 10, 0.5) }, lower = 0.25, upper = 0.5)
```

```
dBeta.pBinom An implementation of the Beta-density Compound Cumulative-
Binomial Distribution.
```

20 dBeta.pBinom

#### **Description**

The Beta Compound Binomial distribution: The product of the four-parameter Beta probability density function and the binomial cumulative probability mass function. Used in the Livingston and Lewis approach to classification accuracy and consistency, the output can be interpreted as the population density of passing scores produced at "x" (a value of true-score).

### Usage

```
dBeta.pBinom(x, l, u, alpha, beta, n, c, lower.tail = FALSE)
```

#### **Arguments**

x	x-axis input for which p (proportion or probability) is to be computed.
1	The lower-bound of the four-parameter Beta distribution.
u	The upper-bound of the four-parameter Beta distribution.
alpha	The alpha shape-parameter of the Beta distribution.
beta	The beta shape-parameter of the Beta distribution.
n	The number of trials for the Binomial distribution.
С	The "true-cut" (proportion) of on the Binomial distribution.
lower.tail	Logical. Whether to compute the lower or upper tail of the Binomial distribution. Default is FALSE (i.e., upper tail).

#### References

Hanson, Bradley A. (1991). Method of Moments Estimates for the Four-Parameter Beta Compound Binomial Model and the Calculation of Classification Consistency Indexes. American College Testing Research Report Series.

Livingston, Samuel A. and Lewis, Charles. (1995). Estimating the Consistency and Accuracy of Classifications Based on Test Scores. Journal of Educational Measurement, 32(2).

Lord, Frederic M. (1965). A Strong True-Score Theory, With Applications. Psychometrika, 30(3).

```
# Given a four-parameter Beta distribution with parameters 1 = 0.25, u = 0.75,
# alpha = 5, and beta = 3, and a Binomial error distribution with number of
# trials (n) = 10 and a cutoff-point (c) at 50% correct (i.e., proportion correct
# of 0.5), the population density of passing scores produced at true-score
# (x) = 0 can be calculated as:
dBeta.pBinom(x = 0.5, 1 = 0.25, u = 0.75, a = 5, b = 3, n = 10, c = 0.5)
# Conversely, the density of failing scores produced at x can be calculated
# by passing the additional argument "lower.tail = TRUE" to the function.
# That is:
dBeta.pBinom(x = 0.5, 1 = 0.25, u = 0.75, a = 5, b = 3, n = 10, c = 0.5, lower.tail = TRUE)
#By integration, the population proportion of (e.g.) passing scores in some
#region of the true-score distribution (e.g. between 0.25 and 0.5) can be
```

dBetaMS 21

```
#calculated as: integrate(function(x) { dBeta.pBinom(x, 0.25, .75, 5, 3, 10, 0.5) }, lower = 0.25, upper = 0.5)
```

dBetaMS	Density Under a Specific Point of the Standard Beta PDD with Specific Mean and Variance or Standard Deviation.

### Description

Calculates the density under specific points of the Standard Beta probability density distribution with defined mean and variance or standard deviation.

### Usage

```
dBetaMS(x, mean, variance = NULL, sd = NULL)
```

#### **Arguments**

X	A specific point on the x-axis of the Standard Beta PDD.
mean	The mean of the target Standard Beta probability density distribution.
variance	The variance of the target Standard Beta probability density distribution.
sd	The standard deviation of the target Standard Beta probability density distribution.

### Value

A numeric value representing the required value for the Beta Shape-parameter in order to produce a Standard Beta probability density distribution with the target mean and variance.

```
# To compute the density at a specific point (e.g., .5) along the Standard # (two-parameter) PDD with mean of .6 and variance of .04: dBetaMS(x = .5, mean = .6, variance = .04)
```

22 ETL

ETL

Livingston and Lewis' "Effective Test Length".

#### **Description**

According to Livingston and Lewis (1995), "The effective test length corresponding to a test score is the number of discrete, dichotomously scored, locally independent, equally difficult items required to produce a total score of the same reliability."

#### Usage

```
ETL(mean, variance, l = 0, u = 1, reliability)
```

### **Arguments**

mean	The mean of the observed-score distribution.
variance	The variance of the observed-score distribution.
1	The lower-bound of the observed-score distribution. Default is 0 (assuming observed scores represent proportions).
u	The upper-bound of the observed-score distribution. Default is 1 (assuming observed scores represent proportions).
reliability	The reliability of the observed scores (proportion of observed-score distribution variance shared with true-score distribution).

#### Value

An estimate of the effective length of a test, given the stability of the observations it produces.

#### References

Livingston, Samuel A. and Lewis, Charles. (1995). Estimating the Consistency and Accuracy of Classifications Based on Test Scores. Journal of Educational Measurement, 32(2).

```
# Generate some fictional data. Say, 100 individuals take a test with a # maximum score of 100 and a minimum score of 0. set.seed(1234) testdata <- rbinom(100, 100, rBeta.4P(100, .25, .75, 5, 3)) hist(testdata, xlim = c(0, 100))

# Suppose the reliability of this test was estimated to 0.7. To estimate and # retrieve the effective test length using ETL(): ETL(mean = mean(testdata), variance = var(testdata), l = 0, u = 100, reliability = .7)
```

LABMSU 23

LABMSU	Lower Location Parameter Given Shape Parameters, Mean, Variance,
	and Upper Location Parameter of a Four-Parameter Beta PDD.

### Description

Calculates the lower-bound value required to produce a Beta probability density distribution with defined moments and parameters. Be advised that not all combinations of moments and parameters can be satisfied (e.g., specifying mean, variance, skewness and kurtosis uniquely determines both location-parameters, meaning that the value of the lower-location parameter will take on which ever value it must, and cannot be specified).

### Usage

```
LABMSU(
    alpha = NULL,
    beta = NULL,
    u = NULL,
    mean = NULL,
    variance = NULL,
    skewness = NULL,
    kurtosis = NULL,
    sd = NULL
)
```

### **Arguments**

alpha	The Alpha shape-parameter of the target Beta probability density distribution.
beta	The Beta shape-parameter of the target Beta probability density distribution.
u	The upper-bound of the Beta distribution. Default is NULL (i.e., does not take a specified u-parameter into account).
mean	The mean (first raw moment) of the target Standard Beta probability density distribution.
variance	The variance (second central moment) of the target Standard Beta probability density distribution.
skewness	The skewness (third standardized moment) of the target Beta probability density distribution.
kurtosis	The kurtosis (fourth standardized moment) of the target Beta probability density distribution.
sd	Optional alternative to specifying var. The standard deviation of the target Standard Beta probability density distribution.

#### Value

A numeric value representing the required value for the Beta shape-parameter in order to produce a Standard Beta probability density distribution with the target mean and variance, given specified lower- and upper bounds of the Beta distribution.

24 LL.CA

#### **Examples**

```
# Generate some fictional data.
set.seed(1234)
testdata <- rBeta.4P(100000, .25, .75, 5, 3)
hist(testdata, xlim = c(0, 1), freq = FALSE)

# Suppose you know three of the four necessary parameters to fit a four-
# parameter Beta distribution (i. e., u = 0.75, alpha = 5, beta = 3) to this
# data. To find the value for the necessary 1 parameter, estimate the mean
# and variance of the distribution:
M <- mean(testdata)

$2 <- var(testdata)

# To find the 1 parameter necessary to produce a four-parameter Beta
# distribution with the target mean, variance, and u, alpha, and beta
# parameters using the LMSBAU() function:
(1 <- LABMSU(alpha = 5, beta = 3, mean = M, variance = S2, u = 0.75))
curve(dBeta.4P(x, 1, .75, 5, 3), add = TRUE, lwd = 2)</pre>
```

LL.CA

An Implementation of the Livingston and Lewis (1995) Approach to Estimate Classification Consistency and Accuracy based on Observed Test Scores and Test Reliability.

#### **Description**

An implementation of what has been come to be known as the "Livingston and Lewis approach" to classification consistency and accuracy, which by employing a compound beta-binomial distribution assumes that true-scores conform to the four-parameter beta distribution, and errors of measurement to the binomial distribution. Under these assumptions, the expected classification consistency and accuracy of tests can be estimated from observed outcomes and test reliability.

#### Usage

```
LL.CA(
    x = NULL,
    reliability,
    cut,
    min = 0,
    max = 1,
    true.model = "4P",
    error.model = "binomial",
    truecut = NULL,
    output = c("accuracy", "consistency"),
    failsafe = FALSE,
    l = 0,
    u = 1,
    override = NULL
)
```

#### **Arguments**

A vector of observed scores for which a beta-distribution is to be fitted, or a Х list of pre-defined true-score distribution parameter values. If a list is provided, the list entries must be named after the parameters: 1 and u for the location parameters, and alpha and beta for the shape parameters. reliability The observed-score squared correlation (i.e., proportion of shared variance) with the true-score. The cutoff value for classifying observations into pass or fail categories. cut The minimum value possible to attain on the test. Default is 0 (assuming x min represent proportions). The maximum value possible to attain on the test. Default is 1 (assuming x max represent proportions). true.model The probability distribution to be fitted to the moments of the true-score distribution. Options are "4P" (default) and "2P", referring to four- and two-parameter Beta distributions. The "4P" method produces a four-parameter Beta distribution with the same first four moments (mean, variance, skewness, and kurtosis) as the estimated true-score distribution, while the "2P" method produces a twoparameter Beta distribution with the first two moments (mean and variance) as the estimated true-score distribution. The probability distribution to be used for producing the sampling distributions error.model at different points of the true-score scale. Options are binomial and beta. The binomial distribution is discrete, and is the distribution used originally by Livingston and Lewis. Use of the binomial distribution involves a rounding of the effective test length to the nearest integer value. The Beta distribution is continuous, and does not involve rounding of the effective test length. Optional specification of a "true" cutoff. Useful for producing ROC curves (see truecut documentation for the LL.ROC() function). output Character vector indicating which types of statistics (i.e, accuracy and/or consistency) are to be computed and included in the output. Permissible values are "accuracy" and "consistency". failsafe Logical value indicating whether to engage the automatic failsafe defaulting to the two-parameter Beta true-score distribution if the four-parameter fitting procedure produces impermissible parameter estimates. Default is FALSE (i.e., the function will not engage failsafe, and will likely produce an error if impermissible parameter estimates were produced. If true.model = "2P" or failsafe = TRUE, the lower-bound location parameter 1 to be used in the two-parameter fitting procedure. Default is 0 (i.e., the) If true.model = "2P" or failsafe = TRUE, the upper-bound location parameter u to be used in the two-parameter fitting procedure. Inert artifact from betafunctions version 1.3.1 (replaced by the failsafe arguoverride ment). Will be removed completely in a later update.

#### Value

A list containing the estimated parameters necessary for the approach (i.e., the effective test-length and the beta distribution parameters), the confusion matrix containing estimated proportions of

26 LL.CA

true/false pass/fail categorizations for a test, diagnostic performance statistics, and / or a classification consistency matrix and indices. Accuracy output includes a confusion matrix and diagnostic performance indices, and consistency output includes a consistency matrix and consistency indices p (expected proportion of agreement between two independent test administrations), p\_c (proportion of agreement on two independent administrations expected by chance alone), and Kappa (Cohen's Kappa).

#### Note

It should be noted that this implementation differs from the original articulation of Livingston and Lewis (1995) in some respects. First, the procedure includes a number of diagnostic performance (accuracy) indices which the original procedure enables but that were not included. Second, the possibility of employing a two-parameter Beta error distribution in place of the binomial error distribution is not part of the original procedure. Third, the way consistency is calculated differs substantially from the original articulation of the procedure, which made use of a split-half approach. Rather, this implementation uses the approach to calculating classification consistency outlined by Hanson (1991).

#### References

Livingston, Samuel A. and Lewis, Charles. (1995). Estimating the Consistency and Accuracy of Classifications Based on Test Scores. Journal of Educational Measurement, 32(2).

Hanson, Bradley A. (1991). Method of Moments Estimates for the Four-Parameter Beta Compound Binomial Model and the Calculation of Classification Consistency Indexes. American College Testing.

```
# Generate some fictional data. Say, 100 individuals take a test with a
# maximum score of 100 and a minimum score of 0.
set.seed(1234)
testdata <- rbinom(100, 100, rBeta.4P(100, 0.25, 0.75, 5, 3))
hist(testdata, xlim = c(0, 100))
# Suppose the cutoff value for attaining a pass is 50 items correct, and
# that the reliability of this test was estimated to 0.7. To estimate and
# retrieve the estimated parameters, confusion matrix, consistency and
# accuracy statistics using LL.CA():
LL.CA(x = testdata, reliability = .7, cut = 50, min = 0, max = 100)
# Suppose the true-score parameter estimation procedure arrived at
# impermissible parameter estimates (i.e., 1 < 0, u > 1, alpha < 0, or
# beta < 0). For example:</pre>
set.seed(9)
testdata <- rbinom(100, 25, rBeta.4P(100, 0.25, 1, 5, 3))
Beta.tp.fit(testdata, 0, 25, 25, failsafe = TRUE)
# Suppose further that you have good grounds for assuming that the lower-
# bound parameter is equal to 0.25 (e.g., the test consists of multiple-
# choice questions with four response options, leading to a 25% probability
# of guessing the correct answer per question), and good reason to believe
```

LL.ROC 27

```
# that the upper-bound parameter is equal to 1 (i.e., there is no reason to
# believe that there are no members of the population who will attain a
# perfect score across all possible test-forms.) To set these lower and
# upper bounds for the fitting procedure in the LL.CA() function, set
# the argument true.model = "2p", and specify the location parameters
# 1 = 0.25 and u = 1:
LL.CA(testdata, 0.6287713, 12, 0, 25, true.model = "2p", 1 = 0.25, u = 1)
# Alternatively to supplying scores to which a true-score distribution is
# to be fit, a list with true-score distribution parameter values can be
# supplied manually along with the effective test length (see documentation
# for the ETL() function), foregoing the need for actual data. The list
# entries must be named. "1" is the lower-bound and "u" the upper-bound
# location parameters of the true-score distribution, "alpha" and "beta" for
# the shape parameters, and "etl" for the effective test-length..
trueparams <- list("1" = 0.25, "u" = 0.75, "alpha" = 5, "beta" = 3, "etl" = 50)
LL.CA(x = trueparams, cut = 50, min = 0, max = 100)
```

LL.ROC

ROC curves for the Livingston and Lewis approach.

### **Description**

Generate a ROC curve plotting the false-positive rate against the true-positive rate at different cutoff values across the observed proportion-score scale.

### Usage

```
LL.ROC(
  x = NULL
  reliability,
  min = 0.
  max = 1,
  truecut,
  true.model = "4P",
  error.model = "Binomial",
  AUC = FALSE,
  maxJ = FALSE,
  raw.out = FALSE,
  grainsize = 100
)
```

#### **Arguments**

min

A vector of observed results.

reliability The reliability coefficient of the test.

The minimum possible value to attain on the observed-score scale. Default is 0

(assuming x represent proportions).

LL.ROC

The maximum possible value to attain on the observed-score scale. Default is 1 (assuming x represent proportions).

truecut The true point along the x-scale that marks the categorization-threshold.

true.model The probability distribution to be fitted to the moments of the true-score distribu-

tion. Options are "4P" (default) and "2P", referring to four- and two-parameter Beta distributions. The "4P" method produces a four-parameter Beta distribution with the same first four moments (mean, variance, skewness, and kurtosis) as the estimated true-score distribution, while the "2P" method produces a two-parameter Beta distribution with the first two moments (mean and variance) as

the estimated true-score distribution.

error.model The probability distribution to be used for producing the sampling distributions

at different points of the true-score scale. Options are binomial and beta. The binomial distribution is discrete, and is the distribution used originally by Livingston and Lewis. Use of the binomial distribution involves a rounding of the effective test length to the nearest integer value. The Beta distribution is contin-

uous, and does not involve rounding of the effective test length.

AUC Calculate and include the area under the curve? Default is FALSE.

maxJ Mark the point along the curve where Youden's J statistic is maximized? Default

is FALSE.

raw.out Give raw coordinates as output rather than plot? Default is FALSE.

grainsize Specify the number of cutoff-points for which the ROC curve is to be calculated.

The greater this number the greater the accuracy. Default is 100 points.

### Value

A plot tracing the ROC curve for the test, or matrix of coordinates if raw.out is TRUE.

```
# Generate some fictional data. Say, 100 individuals take a test with a
# maximum score of 100 and a minimum score of 0.
set.seed(1234)
testdata <- rbinom(100, 100, rBeta.4P(100, .25, .75, 5, 3))
hist(testdata, xlim = c(0, 100))

# Suppose the cutoff value for attaining a pass is 50 items correct, and
# that the reliability of this test was estimated to 0.7. To produce a plot
# with an ROC curve using LL.ROC(), along with the AUC statistics and the
# points at which Youden's J. is maximized:
LL.ROC(x = testdata, reliability = .7, truecut = 50, min = 0, max = 100,
AUC = TRUE, maxJ = TRUE)</pre>
```

**MLA** 29

 $\mathsf{MLA}$ 

Most Likely True Alpha Value Given Observed Outcome.

#### **Description**

Given a fitted Standard (two-parameter) Beta Distribution, return the alpha shape-parameter value where the observed mean becomes the mode.

### Usage

```
MLA(alpha, beta, x = NULL, n = NULL)
```

### Arguments

alpha	Observed alpha-parameter value for fitted Standard Beta PDD.
beta	Observed beta-parameter value for fitted Standard Beta PDD.
Х	Observed proportion-correct outcome.
n	Test-length.

### Value

The Alpha shape-parameter value for the Standard Beta probability density distribution where the observed mean is the expected mode.

### Examples

```
# Assuming a prior Standard (two-parameter) Beta distribution is fit, which
# yield an alpha parameter of 10 and a beta parameter of 8, calculate the
# true-alpha parameter most likely to have produced the observations:
MLA(a = 10, b = 8)
```

MLB

Most Likely True Beta Value Given Observed Outcome.

### **Description**

Assuming a prior standard (two-parameter) Beta Distribution, return the beta shape-parameter value where the observed mean becomes the mode.

### Usage

```
MLB(alpha, beta, x = NULL, n = NULL)
```

30 MLM

### **Arguments**

alpha	Observed alpha-parameter value for fitted Standard Beta PDD.
beta	Observed beta-parameter value for fitted Standard Beta PDD.
х	Observed proportion-correct outcome.
n	Test-length.

#### Value

The Beta shape-parameter value for the Standard Beta probability density distribution where the observed mean is the expected mode.

### **Examples**

```
# Assuming a prior Standard (two-parameter) Beta distribution is fit, which # yield an alpha parameter of 10 and a beta parameter of 8, calculate the # true-beta parameter most likely to have produced the observations: MLB(a = 10, b = 8)
```

MLM	Most Likely Mean of the Standard Beta PDD, Given that the Observa-
	tion is Considered the Most Likely Observation of the Standard Beta PDD (i.e., Mode).

#### **Description**

Assuming a prior Standard (two-parameter) Beta Distribution, returns the expected mean of the distribution under the assumption that the observed value is the most likely value of the distribution.

#### Usage

```
MLM(alpha, beta, x = NULL, n = NULL)
```

### **Arguments**

alpha	Observed alpha value for fitted Standard Beta PDD.
beta	Observed beta value for fitted Standard Beta PDD.
x	Observed proportion-correct outcome.
n	Test-length.

### Value

The expected mean of the Standard Beta probability density distribution, for which the observed mean is the most likely value.

observed moments 31

#### **Examples**

```
# Assuming a prior Standard (two-parameter) Beta distribution is fit, which # yield an alpha parameter of 10 and a beta parameter of 8, calculate the # true-mean most likely to have produced the observations: MLM(a = 10, b = 8)
```

observedmoments

Compute Moments of Observed Value Distribution.

#### **Description**

Computes Raw, Central, or Standardized moment properties of a vector of observed scores.

#### Usage

```
observedmoments(
   x,
   type = c("raw", "central", "standardized"),
   orders = 4,
   correct = TRUE
)
```

#### **Arguments**

A vector of values, the distribution of which moments are to be calculated.

A character vector determining which moment-types are to be calculated. Permissible values are "raw", "central", and "standardized".

Orders

The number of moment-orders to be calculated for each of the moment-types.

Logical. Whether to include bias correction in estimation of orders. Default is TRUE.

#### Value

A list of moment types, each a list of moment orders.

```
# Generate some fictional data. Say, 100 individuals take a test with a # maximum score of 100 and a minimum score of 0. set.seed(1234) testdata <- rbinom(100, 100, rBeta.4P(100, .25, .75, 5, 3)) hist(testdata, xlim = c(0, 100))

# To compute the first four raw, central, and standardized moments for this # distribution of observed scores using observedmoments(): observedmoments(x = testdata, type = c("raw", "central", "standardized"), orders = 4, correct = TRUE)
```

pBeta.4P

pBeta.4P	Cumulative Probability Function under the Four-Parameter Beta
•	Probability Density Distribution.

### Description

Function for calculating the proportion of observations up to a specifiable quantile under the Four-Parameter Beta Distribution.

### Usage

```
pBeta.4P(q, 1, u, alpha, beta, lower.tail = TRUE)
```

### Arguments

q	The quantile or a vector of quantiles for which the proportion is to be calculated.
1	The first (lower) location parameter.
u	The second (upper) location parameter.
alpha	The first shape parameter.
beta	The second shape parameter.
lower.tail	Whether the proportion to be calculated is to be under the lower or upper tail. Default is TRUE (lower tail).

### Value

A vector of proportions of observations falling under specified quantiles under the four-parameter beta distribution.

```
# Assume some variable follows a four-parameter beta distribution with # location parameters 1 = 0.25 and u = .75, and shape # parameters alpha = 5 and beta = 3. To compute the # cumulative probability at a specific point of the distribution (e.g., .5) # using pBeta.4P(): pBeta.4P(q = .5, 1 = .25, u = .75, alpha = 5, beta = 3)
```

pBetaMS 33

pBetaMS	Probability of Some Specific Observation under the Standard Beta PDD with Specific Mean and Variance.

### Description

Calculates the probability of some specific observation falling under a specified interval ([0, x] or [x, 1]) under the Standard Beta probability density distribution with defined mean and variance or standard deviation.

### Usage

```
pBetaMS(q, mean, variance = NULL, sd = NULL, lower.tail = TRUE)
```

### Arguments

q	A specific point on the x-axis of the Standard Beta probability density distribution with a defined mean and variance.
mean	The mean of the target Standard Beta probability density distribution.
variance	The variance of the target Standard Beta probability density distribution.
sd	The standard deviation of the target Standard Beta probability density distribution.
lower.tail	Whether the density that should be considered is between the lower-end (i.e., $[0 -> x]$ ) or the higher-end of the distribution (i.e., $[x -> 1]$ ).

### Value

A value representing the probability of a random draw from the Standard Beta probability density distribution with a defined mean and variance being from one of two defined intervals (i.e., [0 -> x] or [x -> 1]).

```
# To compute the proportion of the density under the lower-end tail of a # point along the Standard (two-parameter) PDD (e.g., .5) with mean of .6 # and variance of .04: pBetaMS(q = .5, mean = .6, variance = .04)
```

34 qBeta.4P

tion.	qBeta.4P	Quantile Given Probability Under the Four-Parameter Beta Distribution.
-------	----------	--

### **Description**

Function for calculating the quantile (i.e., value of x) for a given proportion (i.e., the value of y) under the Four-Parameter Beta Distribution.

### Usage

```
qBeta.4P(p, 1, u, alpha, beta, lower.tail = TRUE)
```

### **Arguments**

p	A vector (or single value) of proportions or probabilities for which the corresponding value of x (i.e., the quantiles) are to be calculated.
1	The first (lower) location parameter.
u	The second (upper) location parameter.
alpha	The first shape parameter.
beta	The second shape parameter.
lower.tail	Logical. Whether the quantile(s) to be calculated is to be under the lower or upper tail. Default is TRUE (lower tail).

### Value

A vector of quantiles for specified probabilities or proportions of observations under the four-parameter beta distribution.

```
# Assume some variable follows a four-parameter beta distribution with # location parameters 1 = 0.25 and u = .75, and shape # parameters alpha = 5 and beta = 3. To compute the # quantile at a specific point of the distribution (e.g., .5) # using qBeta.4P(): qBeta.4P(p = .5, 1 = .25, u = .75, alpha = 5, beta = 3)
```

qBetaMS 35

qBetaMS	Quantile Containing Specific Proportion of the Distribution, Given a Specific Probability of the Standard Beta PDD with Specific Mean and
	Variance or Standard Deviation.

### Description

Calculates the quantile corresponding to a specific probability of some observation falling within the [0, x] (1t = TRUE) or [x, 1] (1t = FALSE) interval under the Standard Beta probability density distribution with defined mean and variance or standard deviation.

### Usage

```
qBetaMS(p, mean, variance = NULL, sd = NULL, lower.tail = TRUE)
```

### **Arguments**

р	A value of probability marking the point of the Y-axis to correspond to the X-axis.
mean	The mean of the target Standard Beta probability density distribution.
variance	The variance of the target Standard Beta probability density distribution.
sd	The standard deviation of the target Standard Beta probability density distribution.
lower.tail	Logical. Specifies which end of the tail for which to calculate quantile. Default is TRUE (meaning, find q for lower tail.)

#### Value

A numeric value representing the quantile for which the specified proportion of observations fall within.

```
\# To compute the quantile at a specific point (e.g., .5) along the Standard \# (two-parameter) PDD with mean of .6 and variance of .04: qBetaMS(p = .5, mean = .6, variance = .04)
```

36 rBetaMS

rBeta.4P	Random Number Generation under the Four-Parameter Beta Probability Density Distribution.

### **Description**

Function for generating random numbers from a specified Four-Parameter Beta Distribution.

#### Usage

```
rBeta.4P(n, l, u, alpha, beta)
```

### **Arguments**

n	Number of draws.
1	The first (lower) location parameter.
u	The second (upper) location parameter.
alpha	The first shape parameter.
beta	The second shape parameter.

#### Value

A vector with length n of random values drawn from the Four-Parameter Beta Distribution.

#### **Examples**

```
# Assume some variable follows a four-parameter beta distribution with # location parameters l=0.25 and u=.75, and shape # parameters alpha = 5 and beta = 3. To draw a random # value from this distribution using rBeta.4P(): rBeta.4P(n=1, l=.25, u=.75, alpha = 5, beta = 3)
```

rBetaMS

Random Draw from the Standard Beta PDD With Specific Mean and Variance.

### **Description**

Draws random samples of observations from the Standard Beta probability density distribution with defined mean and variance.

### Usage

```
rBetaMS(n, mean, variance = NULL, sd = NULL)
```

UABMSL 37

### **Arguments**

n	Number of observations to be drawn from under the Standard Beta PDD.
mean	The mean of the target Standard Beta probability density distribution.
variance	The variance of the target Standard Beta probability density distribution.
sd	The standard deviation of the target Standard probability density distribution.

#### Value

A vector of length n, each value representing a random draw from the Standard Beta probability density distribution with defined mean and variance.

**UABMSL** 

Upper Location Parameter Given Shape Parameters, Mean, Variance, and Lower Location Parameter of a Four-Parameter Beta PDD.

### **Description**

Calculates the upper-bound value required to produce a Beta probability density distribution with defined moments and parameters. Be advised that not all combinations of moments and parameters can be satisfied (e.g., specifying mean, variance, skewness and kurtosis uniquely determines both location-parameters, meaning that the value of the upper-location parameter will take on which ever value it must, and cannot be specified).

### Usage

```
UABMSL(
   alpha = NULL,
   beta = NULL,
   mean = NULL,
   variance = NULL,
   skewness = NULL,
   kurtosis = NULL,
   l = NULL,
   sd = NULL
)
```

#### **Arguments**

alpha	The Alpha shape-parameter of the target Beta probability density distribution.
beta	The beta shape-parameter of the target Beta probability density distribution.
mean	The mean (first raw moment) of the target Standard Beta probability density distribution.
variance	The variance (second central moment) of the target Standard Beta probability

density distribution.

38 UABMSL

skewness	The skewness (third standardized moment) of the target Beta probability density distribution.
kurtosis	The kurtosis (fourth standardized moment) of the target Beta probability density distribution.
1	The lower-bound of the Beta distribution. Default is NULL (i.e., does not take a specified l-parameter into account).
sd	Optional alternative to specifying var. The standard deviation of the target Standard Beta probability density distribution.

#### Value

A numeric value representing the required value for the Beta shape-parameter in order to produce a Standard Beta probability density distribution with the target mean and variance, given specified lower- and upper bounds of the Beta distribution.

```
# Generate some fictional data.
set.seed(1234)
testdata <- rBeta.4P(100000, .25, .75, 5, 3)
hist(testdata, xlim = c(0, 1), freq = FALSE)

# Suppose you know three of the four necessary parameters to fit a four-
# parameter Beta distribution (i. e., l = 0.25, alpha = 5, beta = 3) to this
# data. To find the value for the necessary u parameter, estimate the mean
# and variance of the distribution:
M <- mean(testdata)
S2 <- var(testdata)

# To find the l parameter necessary to produce a four-parameter Beta
# distribution with the target mean, variance, and u, alpha, and beta
# parameters using the LMSBAU() function:
(u <- UABMSL(alpha = 5, beta = 3, mean = M, variance = S2, l = 0.25))
curve(dBeta.4P(x, .25, u, 5, 3), add = TRUE, lwd = 2)</pre>
```

## **Index**

```
AMS, 2
AUC, 4
Beta.2p.fit,5
Beta.4p.fit,6
{\tt Beta.gfx.poly.cdf, 7}
Beta.gfx.poly.pdf, 8
{\tt Beta.tp.fit}, \textcolor{red}{10}
betamoments, 12
BMS, 13
caStats, 15
cba, 16
ccStats, 17
dBeta.4P, 18
dBeta.pBeta, 18
{\tt dBeta.pBinom,}\ {\tt 19}
dBetaMS, 21
ETL, 22
LABMSU, 23
LL.CA, 24
LL.ROC, 27
MLA, 29
MLB, 29
MLM, 30
observedmoments, 31
pBeta.4P, 32
pBetaMS, 33
qBeta.4P, 34
qBetaMS, 35
rBeta.4P, 36
rBetaMS, 36
UABMSL, 37
```