Package ‘causaloptim’

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Title An Interface to Specify Causal Graphs and Compute Bounds on Causal Effects
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Description When causal quantities are not identifiable from the observed data, it still may be possible to bound these quantities using the observed data. We outline a class of problems for which the derivation of tight bounds is always a linear programming problem and can therefore, at least theoretically, be solved using a symbolic linear optimizer. We extend and generalize the approach of Balke and Pearl (1994) <doi:10.1016/B978-1-55860-332-5.50011-0> and we provide a user friendly graphical interface for setting up such problems via directed acyclic graphs (DAG), which only allow for problems within this class to be depicted. The user can then define linear constraints to further refine their assumptions to meet their specific problem, and then specify a causal query using a text interface. The program converts this user defined DAG, query, and constraints, and returns tight bounds. The bounds can be converted to R functions to evaluate them for specific datasets, and to latex code for publication. The methods and proofs of tightness and validity of the bounds are described in a preprint by Sachs, Gabriel, and Sjölander (2021) <https://sachsmc.github.io/causaloptim/articles/CausalBoundsMethods.pdf>.

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An Interface to Specify Causal Graphs and Compute Bounds on Causal Effects

Description

Specify causal graphs using a visual interactive interface and then analyze them and compute symbolic bounds for the causal effects in terms of the observable parameters.

Details

Run the shiny app by results <- specify_graph(). See detailed instructions in the vignette browseVignettes("causaloptim").

Author(s)

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References


See Also

browseVignettes('causaloptim')

Analyze the causal graph to determine constraints and objective

Description

The graph must contain certain edge and vertex attributes which are documented in the Details below. The shiny app run by specify_graph will return a graph in this format.

Usage

analyze_graph(graph, constraints, effectt)
Arguments

graph An `aaaigraphpackage` object that represents a directed acyclic graph with certain attributes. See Details.

constraints A vector of character strings that represent the constraints on counterfactual quantities

effectt A character string that represents the causal effect of interest

Details

The graph object must contain the following named vertex attributes:

- **name** The name of each vertex must be a valid R object name starting with a letter and no special characters. Good candidate names are for example, Z1, Z2, W2, X3, etc.
- **leftside** An indicator of whether the vertex is on the left side of the graph, 1 if yes, 0 if no.
- **latent** An indicator of whether the variable is latent (unobserved). There should always be a variable U1 on the left side that is latent and a parent of all variables on the left side, and another latent variable Ur on the right side that is a parent of all variables on the right side.
- **nvals** The number of possible values that the variable can take on, the default and minimum is 2 for 2 categories (0,1).

In addition, there must be the following edge attributes:

- **rlconnect** An indicator of whether the edge goes from the right side to the left side. Should be 0 for all edges.
- **edge.monotone** An indicator of whether the effect of the edge is monotone, meaning that if V1 -> V2 and the edge is monotone, then a > b implies V2(V1 = a) >= V2(V1 = b). Only available for binary variables (nvals = 2).

The `effectt` parameter describes your causal effect of interest. The `effectt` parameter must be of the form

\[ p\{V11(X=a); V12(X=a)=b; \ldots\} op1 p\{V21(X=b)=a; V22(X=c)=b; \ldots\} op2 \ldots \]

where \( V_{ij} \) are names of variables in the graph, \( a, b \) are numeric values from 0:(nvals - 1), and \( op \) are either - or +. You can specify a single probability statement (i.e., no operator). Note that the probability statements begin with little p, and use curly braces, and items inside the probability statements are separated by `. The variables may be potential outcomes which are denoted by parentheses. Variables may also be nested inside potential outcomes. Pure observations such as \( p\{Y = 1\} \) are not allowed if the left side contains any variables. If the left side contains any variables, then they must be ancestors of the intervention set variables (or the intervention variables themselves). All of the following are valid effect statements:

\[ p\{Y(X = 1) = 1\} - p\{Y(X = 0) = 1\} \]
\[ p\{X(Z = 1) = 1; X(Z = 0) = 0\} \]
\[ p\{Y(M(X = 0), X = 1) = 1\} - p\{Y(M(X = 0), X = 0) = 1\} \]

The constraints are specified in terms of potential outcomes to constrain by writing the potential outcomes, values of their parents, and operators that determine the constraint (equalities or inequalities). For example, \( X(Z = 1) >= X(Z = 0) \)
Value

A an object of class "linearcausalproblem", which is a list with the following components. This list can be passed to `optimize_effect` which interfaces with Balke’s code. Print and plot methods are also available.

**variables** Character vector of variable names of potential outcomes, these start with ’q’ to match Balke’s notation

**parameters** Character vector of parameter names of observed probabilities, these start with ’p’ to match Balke’s notation

**constraints** Character vector of parsed constraints

**objective** Character string defining the objective to be optimized in terms of the variables

**p.vals** Matrix of all possible values of the observed data vector, corresponding to the list of parameters.

**q.vals** Matrix of all possible values of the response function form of the potential outcomes, corresponding to the list of variables.

**parsed.query** A nested list containing information on the parsed causal query.

**objective.nonreduced** The objective in terms of the original variables, before algebraic variable reduction. The nonreduced variables can be obtained by concatenating the columns of q.vals.

**response.functions** List of response functions.

**graph** The graph as passed to the function.

**R** A matrix with coefficients relating the p.vals to the q.vals, \( p = R \cdot q \)

**c0** A vector of coefficients relating the q.vals to the objective function, \( \theta = c0 \cdot q \)

**iqR** A matrix with coefficients to represent the inequality constraints

Examples

```r
### confounded exposure and outcome
b <- igraph::graph_from_literal(X -> Y, Ur -> X, Ur -> Y)
V(b)$leftside <- c(0,0,0)
V(b)$latent <- c(0,0,1)
V(b)$nvals <- c(2,2,2)
E(b)$rlconnect <- E(b)$edge.monotone <- c(0, 0, 0)
analyze_graph(b, constraints = NULL, effectt = "p(Y(X = 1) = 1) - p(Y(X = 0) = 1)")
```

---

**btm_var**

Recursive function to get the last name in a list

**Description**

Recursive function to get the last name in a list

**Usage**

```
btm_var(x, name = NULL)
```
constant_term

Arguments

x          a list
name       name of the top element of the list

Value

The name of the deepest nested list element

const.to.sets                      Translate lists of constraints to lists of vectors

Description

Translate lists of constraints to lists of vectors

Usage

const.to.sets(constr, objterms)

Arguments

constr   List of constraint terms as character strings
objterms Vector of terms in the objective function

constant_term                      Compute the scalar product of two numeric vectors of the same length

Description

A helper function for evaluate_objective.

Usage

constant_term(numbers1, numbers2)

Arguments

numbers1, numbers2

Value

A string consisting of the value of the scalar product of numbers1 and numbers2.
create_effect_vector

Description
Translate target effect to vector of response variables

Usage
create_effect_vector(effect, graph, obsvars, respvars, q.list, variables)

Arguments
- effect: Effect list, as returned by parse_effect
- graph: The graph
- obsvars: Vector of observed variable vertices from the graph
- respvars: Response function, as returned by create_response_function
- q.list: List with q matrices, as returned by create_q_matrix
- variables: Vector of qs names

Value
A list with the target effect in terms of qs

create_q_matrix

Description
Translate response functions into matrix of counterfactuals

Usage
create_q_matrix(respvars, right.vars, cond.vars, constraints)

Arguments
- respvars: A list of functions as returned by create_response_function
- right.vars: Vertices of graph on the right side
- cond.vars: Vertices of graph on the left side
- constraints: A vector of character strings that represent the constraints

Value
A list of 3 data frames of counterfactuals and their associated labels
create_response_function

Translate regular DAG to response functions

Description

Translate regular DAG to response functions

Usage

create_response_function(graph, right.vars, cond.vars)

Arguments

<table>
<thead>
<tr>
<th>Argument</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>graph</td>
<td>An <code>igraph-package</code> object that represents a directed acyclic graph must contain edge attributes named &quot;leftside&quot; and &quot;lrconnect&quot; that takes values 0 and 1. Only one edge may have a value 1 for lrconnect. The shiny app returns a graph in this format.</td>
</tr>
<tr>
<td>right.vars</td>
<td>Vertices of graph on the right side</td>
</tr>
<tr>
<td>cond.vars</td>
<td>Vertices of graph on the left side</td>
</tr>
</tbody>
</table>

Value

A list of functions representing the response functions

create_R_matrix

Create constraint matrix

Description

Matrix and text representation of constraints on observed probabilities

Usage

create_R_matrix(graph, obsvars, respvars, p.vals, parameters, q.list, variables)
**evaluate_objective**

**Arguments**

- graph: The graph
- obsvars: Vector of observed variable vertices from the graph
- respvars: Response function, as returned by `create_response_function`
- p.vals: Observed probability matrix
- parameters: Vector of ps names
- q.list: List with q matrices, as returned by `create_q_matrix`
- variables: Vector of qs names

**Value**

A list with the R matrix and the string representation

---

**evaluate_objective**  
*Compute the scalar product of a vector of numbers and a vector of both numbers and strings*

**Description**

A helper function for `opt_effect`.

**Usage**

```r
evaluate_objective(c1_num, p, y)
```

**Arguments**

- c1_num: A numeric column matrix.
- p: A character vector.
- y: A numeric vector whose length is the sum of the lengths of `c1_num` and `p`.

**Value**

A string consisting of an affine expression in `p` corresponding to the scalar product of `c(c1_num, p)` with `y`. 
expand_cond

**Description**

Expand potential outcome conditions

**Usage**

```
expand_cond(cond, obsnames)
```

**Arguments**

- `cond`: Text string of the condition
- `obsnames`: Vector of names of observed variables

find_cycles

**Description**

Find cycles in a graph

**Usage**

```
find_cycles(g)
```

**Arguments**

- `g`: an igraph object

**Value**

A list of vectors of integers, indicating the vertex sequences for the cycles found in the graph
interpret_bounds  

Convert bounds string to a function

Description

Convert bounds string to a function

Usage

interpret_bounds(bounds, parameters)

Arguments

bounds The bounds element as returned by optimize_effect
parameters Character vector defining parameters, as returned by analyze_graph

Value

A function that takes arguments for the parameters, i.e., the observed probabilities and returns a vector of length 2: the lower bound and the upper bound.

Examples

b <- graph_from_literal(X -+ Y, Ur -+ X, Ur -+ Y)
V(b)$leftside <- c(0,0,0)
V(b)$latent <- c(0,0,1)
V(b)$nvals <- c(2,2,2)
E(b)$rlconnect <- E(b)$edge.monotone <- c(0, 0, 0)
obj <- analyze_graph(b, constraints = NULL, effectt = "p{Y(X = 1) = 1} - p{Y(X = 0) = 1}"
bounds <- optimize_effect(obj)
bounds_func <- interpret_bounds(bounds$bounds, obj$parameters)
bounds_func(.1, .1, .4, .3)
# vectorized
do.call(bounds_func, lapply(1:4, function(i) runif(5)))

latex_bounds  

Latex bounds equations

Description

Latex bounds equations

Usage

latex_bounds(bounds, parameters, prob.sym = "P", brackets = c("(" , ")"))
linear_expression

Arguments

bounds Vector of bounds as returned by optimize_effect
parameters The parameters object as returned by analyze_graph
prob.sym Symbol to use for probability statements in latex, usually "P" or "pr"
brackets Length 2 vector with opening and closing bracket, usually c("(" , ")")

Value

A character string with latex code for the bounds

Examples

b <- graph_from_literal(X -+ Y, Ur -+ X, Ur -+ Y)
V(b)$leftside <- c(0,0,0)
V(b)$latent <- c(0,0,1)
V(b)$nvals <- c(2,2,2)
E(b)$rlconnect <- E(b)$edge.monotone <- c(0, 0, 0)
obj <- analyze_graph(b, constraints = NULL, effectt = "p(Y(X = 1) = 1) - p(Y(X = 0) = 1)"
bounds <- optimize_effect(obj)
latex_bounds(bounds$bounds, obj$parameters)
latex_bounds(bounds$bounds, obj$parameters, "Pr")

linear_expression

Compute the scalar product of a vector of numbers and a vector of strings

Description

A helper function for evaluate_objective.

Usage

linear_expression(numbers, strings)

Arguments

numbers A numeric vector.
strings A character vector of the same length as numbers.

Value

A string consisting of the corresponding linear combination, including the sign of its first term.
**linear_term**

*Compute the product of a single numeric scalar and a single string*

**Description**

A helper function for `linear_expression`.

**Usage**

`linear_term(number, string)`

**Arguments**

- **number** A numeric vector of length 1.
- **string** A character vector of length 1.

**Value**

A string consisting of the concatenation of `number` and `string`, including its sign.

---

**list_to_path**

*Recursive function to translate an effect list to a path sequence*

**Description**

Recursive function to translate an effect list to a path sequence

**Usage**

`list_to_path(x, name = NULL)`

**Arguments**

- **x** A list of vars as returned by `parse_effect`
- **name** The name of the outcome variable

**Value**

A list of characters describing the path sequence
**numberOfValues**  
*Get the number of values of a given variable in the graph*

**Description**
Get the number of values of a given variable in the graph

**Usage**
```r
numberOfValues(graph, varname)
```

**Arguments**
- `graph`: An igraph object.
- `varname`: A string. The name of a vertex in 'graph'.

**Value**
An integer greater than 1. The number of values of 'varname'.

---

**optimize_effect**  
*Run the Balke optimizer*

**Description**
Given a object with the linear programming problem set up, compute the bounds using the c++ code developed by Alex Balke. Bounds are returned as text but can be converted to R functions using `interpret_bounds`, or latex code using `latex_bounds`.

**Usage**
```r
optimize_effect(obj)
```

**Arguments**
- `obj`: Object as returned by `analyze_graph`

**Value**
An object of class "balkebound" that contains the bounds and logs as character strings
Examples

```r
b <- graph_from_literal(X -+ Y, Ur -+ X, Ur -+ Y)
V(b)$leftside <- c(0,0,0)
V(b)$latent <- c(0,0,1)
V(b)$nvals <- c(2,2,2)
E(b)$rlconnect <- E(b)$edge.monotone <- c(0, 0, 0)
obj <- analyze_graph(b, constraints = NULL, effectt = "p(Y(X = 1) = 1) - p(Y(X = 0) = 1)"
optimize_effect(obj)
```

---

**Description**

Given an object with the linear programming problem set up, compute the bounds using rcdd. Bounds are returned as text but can be converted to R functions using `interpret_bounds`, or latex code using `latex_bounds`.

**Usage**

```r
optimize_effect_2(obj)
```

**Arguments**

- `obj` Object as returned by `analyze_graph`

**Value**

An object of class "balkebound" that contains the bounds and logs as character strings

**Examples**

```r
b <- graph_from_literal(X -+ Y, Ur -+ X, Ur -+ Y)
V(b)$leftside <- c(0,0,0)
V(b)$latent <- c(0,0,1)
V(b)$nvals <- c(2,2,2)
E(b)$rlconnect <- E(b)$edge.monotone <- c(0, 0, 0)
obj <- analyze_graph(b, constraints = NULL, effectt = "p(Y(X = 1) = 1) - p(Y(X = 0) = 1)"
optimize_effect_2(obj)
```
opt_effect  \hspace{1cm} \textit{Compute a bound on the average causal effect}

\textbf{Description}

This helper function does the heavy lifting for \texttt{optimize_effect_2}. For a given casual query, it computes either a lower or an upper bound on the corresponding causal effect.

\textbf{Usage}

\texttt{opt_effect(opt, obj)}

\textbf{Arguments}

\begin{itemize}
    \item \texttt{opt} \hspace{1cm} A string. Either "min" or "max" for a lower or an upper bound, respectively.
    \item \texttt{obj} \hspace{1cm} An object as returned by the function \texttt{analyze_graph}. Contains the casual query to be estimated.
\end{itemize}

\textbf{Value}

An object of class \texttt{optbound}; a list with the following named components:

\begin{itemize}
    \item \texttt{expr} is the \textit{main} output; an expression of the bound as a print-friendly string,
    \item \texttt{type} is either "lower" or "upper" according to the type of the bound,
    \item \texttt{dual_vertices} is a numeric matrix whose rows are the vertices of the convex polytope of the dual LP,
    \item \texttt{dual_vrep} is a V-representation of the dual convex polytope, including some extra data.
\end{itemize}

parse_constraints \hspace{1cm} Parse text that defines a the constraints

\textbf{Description}

Parse text that defines a the constraints

\textbf{Usage}

\texttt{parse_constraints(constraints, obsnames)}

\textbf{Arguments}

\begin{itemize}
    \item \texttt{constraints} \hspace{1cm} A list of character strings
    \item \texttt{obsnames} \hspace{1cm} Vector of names of the observed variables in the graph
\end{itemize}
**Value**

A data frame with columns indicating the variables being constrained, what the values of their parents are for the constraints, and the operator defining the constraint (equality or inequalities).

**parse_effect**

*Parse text that defines a causal effect*

**Description**

Parse text that defines a causal effect

**Usage**

`parse_effect(text)`

**Arguments**

- `text` Character string

**Value**

A nested list that contains the following components:

- **vars** For each element of the causal query, this indicates potential outcomes as names of the list elements, the variables that they depend on, and the values that any variables are being fixed to.
- **oper** The vector of operators (addition or subtraction) that combine the terms of the causal query.
- **values** The values that the potential outcomes are set to in the query.
- **pcheck** List of logicals for each element of the query that are TRUE if the element is a potential outcome and FALSE if it is an observational quantity.

**pastestar**

*Paste with asterisk sep*

**Description**

Paste with asterisk sep

**Usage**

`pastestar(...)`

**Arguments**

- `...` Things to paste together
plot.linearcausalproblem

Plot the graph from the causal problem

Description

Plot the graph from the causal problem

Usage

## S3 method for class 'linearcausalproblem'
plot(x, ...)

Arguments

x object of class "linearcausaloptim"
...

Not used

Value

Nothing

plot_graphres

Plot the analyzed graph object

Description

Special plotting method for igraphs of this type

Usage

plot_graphres(graphres)

Arguments

graphres an igraph object

Value

None
### print.linearcausalproblem

*Print the causal problem*

#### Description

Print the causal problem

#### Usage

```r
## S3 method for class 'linearcausalproblem'
print(x, ...)
```

#### Arguments

- `x`: object of class "linearcausaloptim"
- `...`: Not used

#### Value

`x`, invisibly

### print_nvals

*Print the number of values of each variable/vertex of the analyzed graph object*

#### Description

Print the number of values of each variable/vertex of the analyzed graph object

#### Usage

```r
print_nvals(graphres)
```

#### Arguments

- `graphres`: an igraph object

#### Value

None
reduce.sets  Algebraically reduce sets

Description
Identifies and reduces redundant variables

Usage
reduce.sets(sets)

Arguments
sets  List of constraints as sets of variables

shortentxt  Shorten strings to 80 characters wide

Description
Shorten strings to 80 characters wide

Usage
shortentxt(x)

Arguments
x  String

Value
A string with line breaks to keep the width less than 80 characters
**simulate_bounds**

**Simulate bounds**

**Description**

Run a simple simulation based on the bounds. For each simulation, sample the set of counterfactual probabilities from a uniform distribution, translate into a multinomial distribution, and then compute the objective and the bounds in terms of the observable variables.

**Usage**

```r
simulate_bounds(obj, bounds, nsim = 1000)
```

**Arguments**

- **obj**: Object as returned by `analyze_graph`
- **bounds**: Object as returned by `optimize_effect`
- **nsim**: Number of simulation replicates

**Value**

A data frame with columns: objective, bound.lower, bound.upper

**Examples**

```r
b <- graph_from_literal(X -+ Y, Ur -+ X, Ur -+ Y)
V(b)$leftside <- c(0, 0, 0)
V(b)$latent <- c(0, 0, 1)
V(b)$nvals <- c(2, 2, 2)
E(b)$rlconnect <- E(b)$edge.monotone <- c(0, 0, 0)
obj <- analyze_graph(b, constraints = NULL, effectt = "p(Y(X = 1) = 1) - p(Y(X = 0) = 1)")
bounds <- optimize_effect(obj)
simulate_bounds(obj, bounds, nsim = 5)
```

**specify_graph**

**Shiny interface to specify network structure and compute bounds**

**Description**

This launches the Shiny interface in the system’s default web browser. The results of the computation will be displayed in the browser, but they can also be returned to the R session by assigning the result of the function call to an object. See below for information on what is returned.

**Usage**

```r
specify_graph()
```
Value

If the button "Exit and return graph object" is clicked, then only the graph is returned as an `aaaigraph-package` object.

If the bounds are computed and the button "Exit and return objects to R" is clicked, then a list is returned with the following elements:

- **graphres** The graph as drawn and interpreted, an `aaaigraph-package` object.
- **obj** The objective and all necessary supporting information. This object is documented in `analyze_graph`. This can be passed directly to `optimize_effect_2`.
- **bounds.obs** Object of class 'balkebound' as returned by `optimize_effect_2`.
- **constraints** Character vector of the specified constraints. NULL if no constraints.
- **effect** Text describing the causal effect of interest.
- **boundsFunction** Function that takes parameters (observed probabilities) as arguments, and returns a vector of length 2 for the lower and upper bounds.

---

`symb.subtract`  
*Symbolic subtraction*

**Description**

Like `setdiff` but doesn’t remove duplicates `x1 - x2`

**Usage**

`symb.subtract(x1, x2)`

**Arguments**

- `x1`  
  First term (subtract from)
- `x2`  
  Second term (subtract)

---

`update_effect`  
*Update the effect in a linearcausalproblem object*

**Description**

If you want to use the same graph and response function, but change the effect of interest, this can save some computation time.

**Usage**

`update_effect(obj, effectt)`
update_effect

Arguments

obj An object as returned by analyze_graph
effect A character string that represents the causal effect of interest

Value

A object of class linearcausalproblem, see analyze_graph for details
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