Package 'cmvnorm'

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Type Package

Title The Complex Multivariate Gaussian Distribution

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Author Robin K. S. Hankin

Depends emulator (>= 1.2-15)

Imports elliptic

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Description Various utilities for the complex multivariate Gaussian distribution.

VignetteBuilder elliptic, emulator

License GPL-2

URL https://github.com/RobinHankin/cmvnorm.git

BugReports https://github.com/RobinHankin/cmvnorm/issues

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`cmvnorm-package` *The Complex Multivariate Gaussian Distribution*

Description

Various utilities for the complex multivariate Gaussian distribution.

Details

The DESCRIPTION file:

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Type:	Package
Title:	The Complex Multivariate Gaussian Distribution
Version:	1.0-5
Date:	2018-08-15
Author:	Robin K. S. Hankin
Depends:	emulator (>= 1.2-15)
Imports:	elliptic
Maintainer:	Robin K. S. Hankin <hankin.robin@gmail.com>
Description:	Various utilities for the complex multivariate Gaussian distribution.
VignetteBuilder:	elliptic, emulator
License:	GPL-2
URL:	https://github.com/RobinHankin/cmvnorm.git
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```

Generalizing the real multivariate Gaussian distribution to the complex case is not straightforward but one common approach is to replace the real symmetric variance matrix with a Hermitian positive-definite matrix. The **cmvnorm** package provides some functionality for the resulting density function.

Author(s)

Robin K. S. Hankin

Maintainer: Robin K. S. Hankin <hankin.robin@gmail.com>

References

- N. R. Goodman 1963. "Statistical analysis based on a certain multivariate complex Gaussian distribution". *The Annals of Mathematical Statistics*. 34(1): 152–177
- R. K. S. Hankin 2015. "The complex multivariate Gaussian distribution". *R News*, volume 7, number 1.

Examples

```
S1 <- 4+diag(5)
S2 <- S1
S2[1,5] <- 4+1i
S2[5,1] <- 4-1i   # Hermitian


rcmvnorm(10,sigma=S1)
rcmvnorm(10,mean=rep(1i,5),sigma=S2)

dcmvnorm(rep(1,5),sigma=S2)
```

---

corr_complex   *Complex Gaussian processes*

Description

Various utilities for investigating complex Gaussian processes

Usage

```
corr_complex(z1, z2 = NULL, distance.function = complex_CF, means =
NULL, scales = NULL, pos.def.matrix = NULL)
complex_CF(z1,z2, means, pos.def.matrix)
scales.likelihood.complex(pos.def.matrix, scales, means,  zold, z,
               give_log = TRUE, func = regressor.basis)
interpolant.quick.complex(x, d, zold, Ainv, scales = NULL, pos.def.matrix = NULL,
               means=NULL, func = regressor.basis, give.Z = FALSE,
               distance.function = corr_complex, ...)
```

Arguments

`z, z1, z2` Points in C^n

`distance.function`
Function giving the (complex) covariance between two points in C^n

`means, pos.def.matrix, scales`
: In function `complex_CF()`, the mean and covariance matrix of the distribution whose characteristic function is used as to give the covariance matrix; `scales` is used to specify the diagonal of `pos.def.matrix` if the off-diagonal elements are zero

`zold, d, give_log, func, x, Ainv, give.Z,...`
: Direct analogues of the arguments in `interpolant()` and `scales.likelihood()` in the **emulator** package

Details

- Function `complex_CF()` returns a (slightly reparameterized) characteristic function of a complex Gaussian distribution. The covariance is given by

  $$c(\mathbf{t}) = \exp(i\mathrm{Re}(\mathbf{t}^*\mu) - \mathbf{t}^*B\mathbf{t})$$

  where $\mathbf{t} = \mathbf{x} - \mathbf{x}'$ is interpreted as the distance between two observations, μ is the mean of the distribution (which is in general a complex vector), and B a positive-definite matrix.

- Function `corr_complex()` is the complex analogue of `corr.matrix()`. It returns a matrix with entry (i, j) equal to the covariance of the process at observation i and observation j, or cov$(\eta(\mathbf{x}_i), \eta(\mathbf{x}_j))$. The elements are calculated by `complex_CF()`.

  This function includes only a single method, that of nested calls to `apply()`. I could not figure out how to generalize method 1 of `corr.mattrix()` to the complex case.

- Function `scales.likelihood.complex()` is a complex version of `scales.likelihood()` which takes a positive definite matrix and a mean. The formula used is

  $$(\sigma^2)^{-(n-q)}|A|^{-1}|H^*A^{-1}H|^{-1}$$

  . Here and elsewhere, A^* means the complex conjugate of the transpose.

- Function `interpolant.quick.complex()` is a complex version of `interpolant.quick()`.

  $$\mathbf{h}(\mathbf{x})^*\hat{\beta} + \mathbf{t}(\mathbf{x})^*A^{-1}(\mathbf{y} - H\hat{\beta})$$

  This is the complex version of Oakley's equation 2.30 or Hankin's equation 5.

More details are given in the package vignette.

Author(s)

Robin K. S. Hankin

References

- Hankin, R. K. S. 2005. "Introducing BACCO, an R bundle for Bayesian Analysis of Computer Code Output", *Journal of Statistical Software*, 14(15)
- J. Oakley 1999. *Bayesian uncertainty analysis for complex computer codes*, PhD thesis, University of Sheffield.

Examples

```
complex_CF(c(1,1i),c(1,-1i),means=c(1i,1i),pos.def.matrix=diag(2))

V <- latin.hypercube(7,2,complex=TRUE)

cm <- c(1,1+1i)                      # "complex mean"
cs <- matrix(c(2,1i,-1i,1),2,2)   # "complex scales"
tb <- c(1,1i,1-1i)                      # "true beta"

A <- corr_complex(V,means=cm,pos.def.matrix=cs)
Ainv <- solve(A)
z <- drop(rcmvnorm(n=1,mean=regressor.multi(V) %*% tb, sigma=A))


betahat.fun(V,Ainv,z)    # should be close to 'tb'

#scales.likelihood.complex(cs,cm,V,z)   # log-likelihood evaluated true parameters



interpolant.quick.complex(x=0.1i+V[1:3,],d=z,zold=V,Ainv=Ainv,pos.def.matrix=cs,means=cm)
```

`isHermitian` *Is a Matrix Hermitian?*

Description

Returns TRUE if a matrix is Hermitian or Hermitian positive-definite

Usage

```
isHermitian(x, tol = 100 * .Machine$double.eps)
ishpd(x,tol= 100 * .Machine$double.eps)
zapim(x,tol= 100 * .Machine$double.eps)
```

Arguments

x	A square matrix
tol	Tolerance for numerical scruff

Details

Functions `isHermitian()` and `ishpd()` return a Boolean. Function `zapim()` zaps small imaginary parts of components vector, returning real if all elements are so zapped.

Author(s)

Robin K. S. Hankin

Examples

```
v <- 2^(1:30)
zapim(v+1i*exp(-v))


ishpd(matrix(c(1,0.1i,-0.1i,1),2,2))   # should be TRUE
isHermitian(matrix(c(1,3i,-3i,1),2,2)) # should be TRUE
```

Mvcnorm *Multivariate complex Gaussian density and random deviates*

Description

Density function and a random number generator for the multivariate complex Gaussian distribution.

Usage

```
rcnorm(n)
dcmvnorm(z, mean, sigma, log = FALSE)
rcmvnorm(n, mean = rep(0, nrow(sigma)), sigma = diag(length(mean)),
    method = c("svd", "eigen", "chol"),
    tol= 100 * .Machine$double.eps)
```

Arguments

z	Complex vector or matrix of quantiles. If a matrix, each row is taken to be a quantile
n	Number of observations
mean	Mean vector
sigma	Covariance matrix, Hermitian positive-definite
tol	numerical tolerance term for verifying positive definiteness
log	In `dcmvnorm()`, Boolean with default `TRUE` meaning to return the Gaussian density function, and `FALSE` meaning to return the logarithm
method	Matrix decomposition used to determine the positive-definite matrix square root of `sigma`, possible methods are eigenvalue decomposition (“`eigen`”, default), and singular value decomposition (“`svd`”)

Details

Function dcmvnorm() is the density function of the complex multivariate normal (Gaussian) distribution:

$$p\left(\mathbf{z}\right)=\frac{\exp\left(-\mathbf{z}^{*}\Gamma\mathbf{z}\right)}{\left|\pi\Gamma\right|}$$

Function rcnorm() is a low-level function designed to generate observations drawn from a standard complex Gaussian. Function rcmvnorm() is a user-friendly wrapper for this.

Author(s)

Robin K. S. Hankin

References

N. R. Goodman 1963. "Statistical analysis based on a certain multivariate complex Gaussian distribution". *The Annals of Mathematical Statistics*. 34(1): 152–177

Examples

```
S <- emulator::cprod(rcmvnorm(3,mean=c(1,1i),sigma=diag(2)))

rcmvnorm(10,sigma=S)
rcmvnorm(10,mean=c(0,1+10i),sigma=S)


# Now try and estimate the mean (viz 1,1i) and variance (S) from a
#  random sample:


n <- 101
z <- rcmvnorm(n,mean=c(0,1+10i),sigma=S)
xbar <- colMeans(z)
Sbar <- cprod(sweep(z,2,xbar))/n
```

---

setreal *Manipulate real or imaginary components of an object*

Description

Manipulate real or imaginary components of an object

Usage

```
Im(x) <- value
Re(x) <- value
```

Arguments

`x`	Complex-valued object
`value`	Real-valued object

Author(s)

Robin K. S. Hankin

Examples

```
A <- matrix(c(1,0.1i,-0.1i,1),2,2)
Im(A) <- Im(A)*3
Re(A) <- matrix(c(5,2,2,5),2,2)
```

`var` *Variance and standard deviation of complex vectors*

Description

Complex generalizations of `stats::sd()` and `stats::var()`

Usage

```
var(x, y=NULL, na.rm=FALSE,use)
sd(x, na.rm=FALSE)
```

Arguments

`x,y`	Complex vector or matrix
`na.rm`	Boolean with default `FALSE` meaning to leave `NA` values present and `TRUE` meaning to remove them
`use`	Ignored

Details

Intended to be broadly compatible with `stats::sd()` and `stats::var()`.

If given real values, `var()` and `sd()` return the variance and standard deviation as per ordinary real analysis. If given complex values, returns the complex generalization in which Hermitian transposes are used.

If `z` is a complex matrix, `var(z)` returns the variance of the rows.

These functions use $n - 1$ on the denominator purely for consistency with `stats::var()` (for the record, I disagree with the rationale for $n - 1$).

Author(s)

Robin K. S. Hankin

Examples

```
sd(rcnorm(10)) # imaginary component suppressed by zapim()

var(rcmvnorm(1e5,mean=c(0,0)))
```

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