Modelling Competition in `comsimitv` package

Zoltán Botta-Dukát

Contents

During simulations seed production depends on the competition for resources within (sub)-communities. First strength of competition (α) is calculated for each pair of co-occurring individuals from the trait values related to resource use by competition kernels specified in `competition.kernel` parameter of `comm.simul` function. Then the matrix of pairwise competition coefficient are used in function specified by `fSeedProduction` parameter. This vignette shows the available symmetric (where $\alpha_{ij} = \alpha_{ji}$) and asymmetric (where $\alpha_{ij} \neq \alpha_{ji}$ if $i \neq j$) kernels, and `SeedProduction` function that recently the only available function in the package for this purpose.

1 Symmetric competition kernels

Recently the only available symmetric competition kernel is the Gaussian one:

$$\alpha_{ij} = \exp\left(-\frac{(B_i - B_j)^2}{\sigma_b}\right) \tag{1}$$

where B_i and B_j are the resource use related trait values of the two species, while σ_b determines how steeply decrease the strength of competition with increasing difference in resource use (Figure 1).

For $B_i = B_j$, $\alpha_{ij} = 1$ irrespectively to value of σ_b. If $\sigma_b = 0$ the strength of competition is zero any case of $B_i \neq B_j$. If $\sigma_b = \infty$, $\alpha_{ij} = 1$ for all species pairs.

Gaussian competition kernel can be used by setting `competition.kernel="Gaussian.competition,kernel"` (which is the default value of this parameter). Value of σ_b has to be set by parameter `sigma.b`.

According to MacArthur & Levins (1967), this competition kernel can be deduced as overlap of Gaussian resource use curves. Their general formula for overlap is

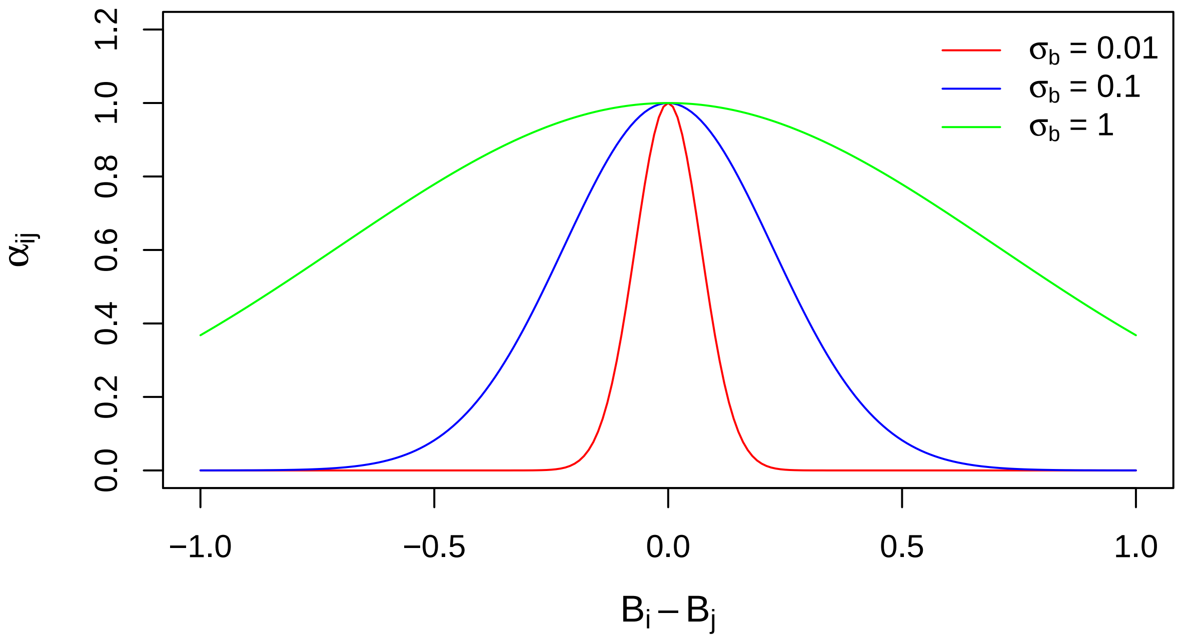

Figure 1: Shape of Gaussian competirion kernel with different σ_b values

$$\alpha_{ij} = \frac{\int_{-\infty}^{\infty} U_i(x)U_j(x)\,dx}{\int_{-\infty}^{\infty} U_i^2(x)\,dx} \tag{2}$$

where U is the resource use function, and x is the quality of the resource (e.g. seed size or rooting depth). Let both U_i and U_j be density function of normal (Gaussian) distribution with same standard deviation (σ), and let rescale x to be expected values equals to zero and $d = B_i - B_j$, respectively:

$$\alpha_{ij} = \frac{\int_{-\infty}^{\infty} \exp\left(-\frac{x^2}{\sigma^2}\right) \exp\left(-\frac{(x-d)^2}{\sigma^2}\right)\,dx}{\int_{-\infty}^{\infty} \left[\exp\left(-\frac{x^2}{\sigma^2}\right)\right]^2\,dx} = \frac{\int_{-\infty}^{\infty} \exp\left(-\frac{x^2+(x-d)^2}{\sigma^2}\right)\,dx}{\int_{-\infty}^{\infty} \exp\left(-\frac{2x^2}{\sigma^2}\right)\,dx} \tag{3}$$

Since $x^2 + (x-d)^2 = 2x^2 + d^2 - 2xd = 2\left(x - \frac{d}{2}\right)^2 + \frac{d^2}{2}$

$$\alpha_{ij} = \frac{\exp\left(\frac{d^2}{2\sigma^2}\right) \int_{-\infty}^{\infty} \exp\left(-\frac{2(x-d/2)^2}{\sigma^2}\right)\,dx}{\int_{-\infty}^{\infty} \exp\left(-\frac{2x^2}{\sigma^2}\right)\,dx} = \exp\left(\frac{d^2}{2\sigma^2}\right) \tag{4}$$

Note that σ_b in equation (1) equals to $2\sigma^2$ in equation (4)

2 Asymmetric competition kernels

Recently two types of asymmetric competition kernels are available via `asymmetric.competition.kernel` function:

- Kisdi's convex-concave function
- smooth function suggested by Nattrass et al. (2012)

2.1 Kisdi's convex-concave function

It is a function defined by equation (2) in Kisdi (1999), however the parametrization are slightly modyfied:

$$\alpha_{ij} = C\left(1 - \frac{1}{1 + v\exp\left(-\frac{B_i - B_j}{\sigma_b}\right)}\right) \tag{5}$$

Contrary to the Gaussian competition kernel, It has three parameters (C, v, σ_b) instead of the only one parameter of Gaussian competition kernel. Note that in the R function these parameters are called `ac.C`, `ac.v` and `sigma.b`, respectively. C and v have to be positive, while $\sigma_b \neq 0$. Possible values of the function ranges from zero to C. If $\sigma_b > 0$ it is a decreasing sigmoid (convex-concave) function (Figure 2) of trait difference $(B_i - B_j)$ with inflection point at $B_i - B_j = \sigma_b \ln v$, where the strength of competition is $C/2$.

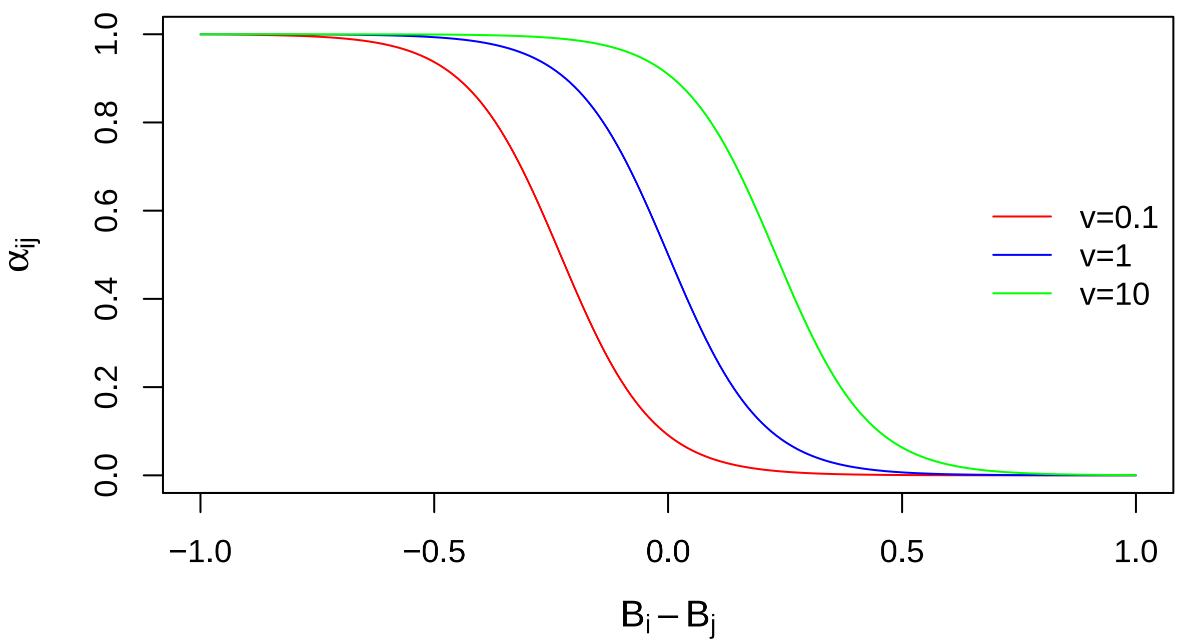

Figure 2: Shape of Kisdi's convex-convave function with different values of v (C=1,σ_b=0.1)

Strength of competition between functionally equivalent individulas (i.e. if $B_i = B_j$) is $\alpha_{ij} = C\left(\frac{v}{1+v}\right)$. If the other two parameter fixed, absolute value of parameter σ_b determines the steepness of the curve around its inflection point (Figure 3).

2.2 Smooth function poropsed by Nattrass et al. (2012)

This is also a sigmoid function defined by a formula similar to Kisdi's function:

$$\alpha_{ij} = 1 + C - \frac{2C}{1 + \exp(-\frac{2(B_i - B_j)}{\sigma_b})} \tag{6}$$

It ranges from $1 - C$ to $1 + C$. Position of its inflection point is $B_i - B_j = 0$, where its value is 1, irrespective to the parameter values. When its range (i.e. value of parameter C) is fixed, σ_b determines the steepness of the curve at the inflection point: lower σ_b results in steeper curve.

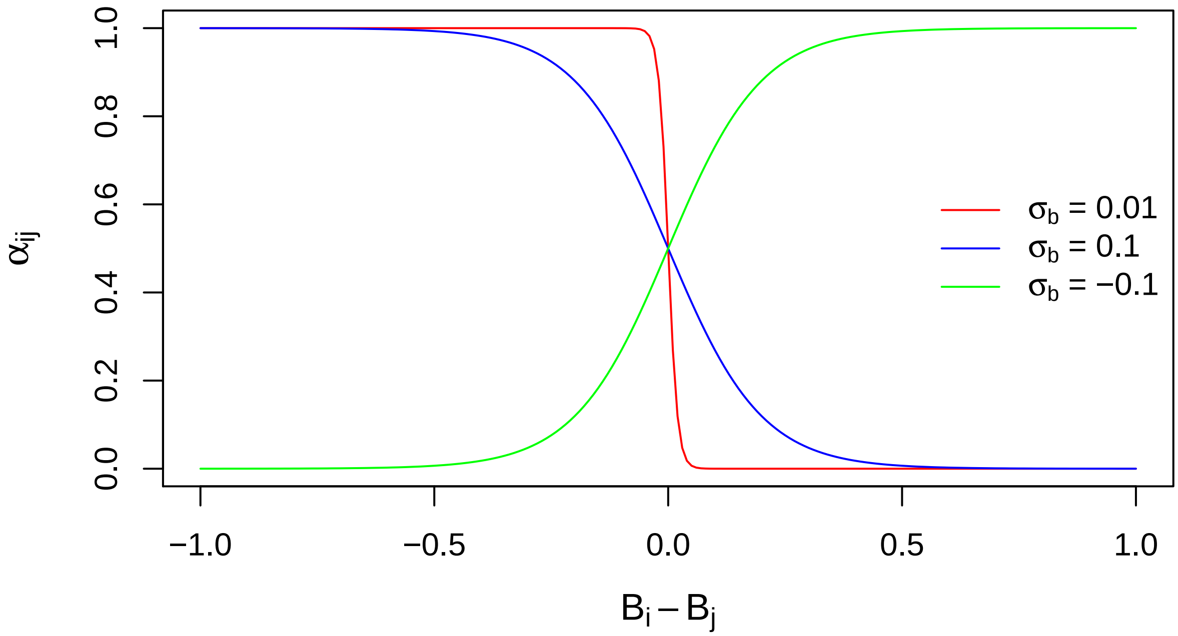

Figure 3: Shape of Kisdi's convex-convave function with different values of σ_b (C=1,v=1)

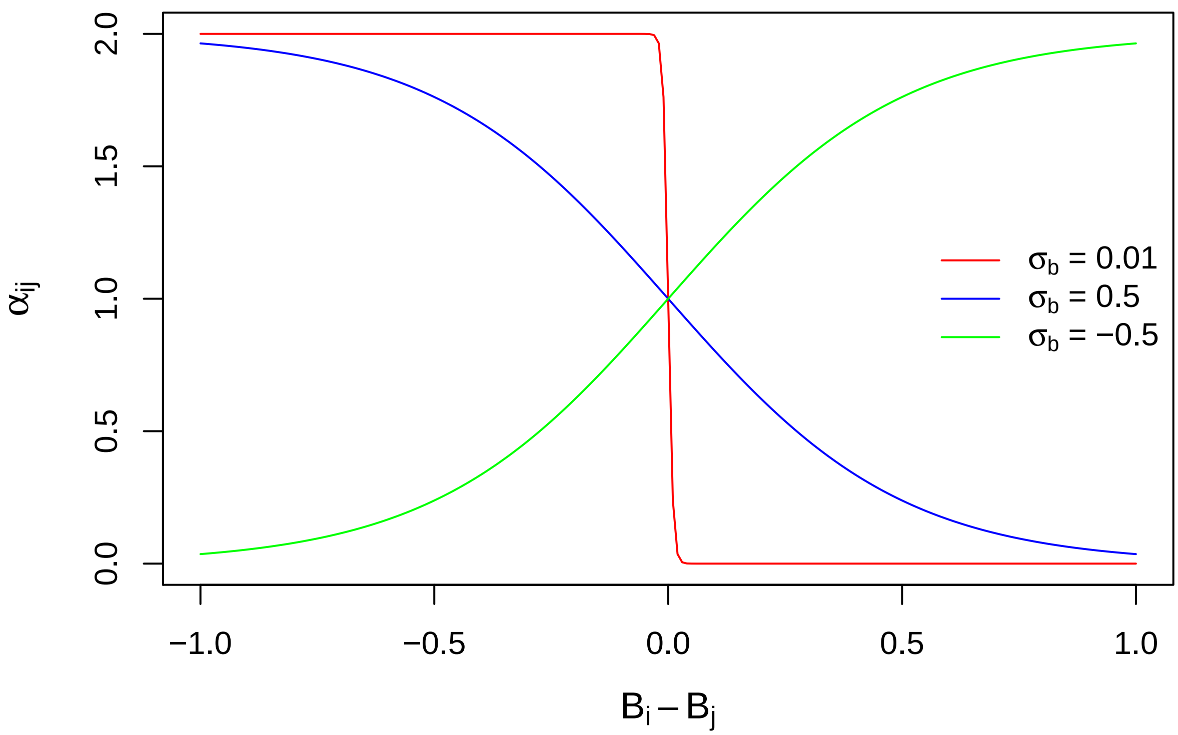

Figure 4: Shape of smooth function by Nattrass et al. with different values of σ_b (C=1)

However equation (6) is not defined when $sigma_b = 0$ and $B_j = B_i$, following suggestion of Nattrass et al. (2012) `asymmetric.competition.kernel` function set the strength of competition to 1 in this case.

3 `SeedProduction` function

In this function, each individual produces zero or one seed. The probability of reproduction of individual `i` in local community `k` depends on the competition for resources:

$$p_{ik} = b_0 * \max\left(\frac{K - \sum_{i \in k} \alpha_{ij}}{K}, 0\right) \tag{7}$$

Where: b_0 is the maximum probability of reproduction in competition free conditions; K is level of competition above which probability of reproduction becomes zero; α_{ij} = competitive effect of individual `j` on individual `i`, calculated from resource acquisition traits by the competition kernel functions.

References

Kisdi, É. (1999). Evolutionary Branching under Asymmetric Competition. *Journal of Theoretical Biology*, 197, 149–162.

MacArthur, R. & Levins, R. (1967). The Limiting Similarity, Convergence, and Divergence of Coexisting Species. *The American Naturalist*, 101, 377–385.

Nattrass, S., Baigent, S. & Murrell, D.J. (2012). Quantifying the Likelihood of Co-existence for Communities with Asymmetric Competition. *Bulletin of Mathematical Biology*, 74, 2315–2338.