

# Package ‘crqa’

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**Type** Package

**Title** Unidimensional and Multidimensional Methods for Recurrence  
Quantification Analysis

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**Description** Auto, Cross and Multi-dimensional recurrence quantification analysis.  
Different methods for computing recurrence, cross vs. multidimensional  
or profile it*i.e.*, only looking at the diagonal recurrent points,  
as well as functions for optimization and plotting are proposed.  
in-depth measures of the whole cross-recurrence plot,  
Please refer to Coco and others (2021) <[doi:10.32614/RJ-2021-062](https://doi.org/10.32614/RJ-2021-062)>,  
Coco and Dale (2014) <[doi:10.3389/fpsyg.2014.00510](https://doi.org/10.3389/fpsyg.2014.00510)>  
and Wallot (2018) <[doi:10.1080/00273171.2018.1512846](https://doi.org/10.1080/00273171.2018.1512846)>  
for further details about the method.

**Depends** R (>= 3.5.0)

**Imports** Matrix, pracma, rdist, tseriesChaos, gplots, dplyr, ggplot2

**License** GPL (>= 3)

**Collate** 'crqa.R' 'crqa\_helpers.R' 'drpfromts.R' 'lorenzattractor.R'  
'mdDelay.R' 'mdFnn.R' 'optimizeParam.R' 'piecewiseRQA.R'  
'plot\_rp.R' 'sims.R' 'spdiags.R' 'wincrqa.R' 'windowdrp.R'

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crqa-package	<i>Unidimensional and Multidimensional Methods for Recurrence Quantification Analysis</i>
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## Description

Auto, Cross and Multi-dimensional recurrence quantification analysis. Different methods for computing recurrence, cross vs. multidimensional or profile i.e., only looking at the diagonal recurrent points, as well as functions for optimization and plotting are proposed. in-depth measures of the whole cross-recurrence plot, Please refer to by Coco and Dale (2014) <doi:10.3389/fpsyg.2014.00510> and Wallot (2018) <doi: 10.1080/00273171.2018.1512846> for further details about the method.

## Details

`crqa`: Core recurrence function, which examines recurrent structures of a single `rqa`, two `crqa`, or multidimensional time-series `mcrqa`, which are time-delayed and embedded in higher dimensional space. The approach compares the phase space trajectories of the time-series in the same phase-space when delays are introduced. A distance matrix between the time-series, delayed and embedded is calculated. Several measures representative of the underlying dynamics of the system are extracted.

`drpfromts`: Method to explore the diagonal profile of the recurrence plot (Auto, Cross, or Multi-dimensional). It returns the recurrence for different delays, the maximal recurrence observed and the delay at which it occurred.

`lorenzattractor`: An implementation of the Lorenz dynamical system, which describes the motion of a possible particle, which will neither converge to a steady state, nor diverge to infinity; but rather stay in a bounded but 'chaotically' defined region, i.e., an attractor.

`mdDelay`: Estimates time delay for embedding of a multi-dimensional dataset.

`mdFnn`: Computes the percentage of false nearest neighbors for multidimensional time series as a function of embedding dimension.

`optimizeParam`: Iterative procedure to examine the values of delay, embedding dimension and radius to compute recurrence plots of one, two, or more time-series.

`piecewiseRQA`: This is a convenience function which breaks down the computation of large recurrence plots into a collection of smaller recurrence plots. It can ease speed and memory issues if an appropriate size for the block is found.

`plotRP`: A convenience function to plot the RP matrix returned by the `crqa`.

`simts`: A simple algorithm for producing a time-series that drives a second time-series using parameters, which change independent and conditional probability of an event to occur.

`wincrqa`: A recurrence plot is computed in overlapping windows of a certain size for a number of delays smaller than the size of the window; and measures of it extracted.

`windowdrp`: A recurrence plot is computed in overlapping windows of a specified size for a number of delays smaller than the size of the window. In every window, the recurrence value for the different delays is calculated. A mean is then taken across the delays to obtain a recurrence value in that particular window.

## Author(s)

Moreno I. Coco <moreno.cocoi@gmail.com> Dan Monster <danm@econ.au.dk> Giuseppe Leonardi <g.leonardi@vizja.pl> Rick Dale <rdale@ucla.edu> Sebastian Wallot <sebastian.wallot@ae.mpg.de>

## References

Webber Jr, C. L., and Zbilut, J. P. (2005). Recurrence quantification analysis of nonlinear dynamical systems. *Tutorials in contemporary nonlinear methods for the behavioral sciences*, 26-94.

Marwan, N., and Kurths, J. Nonlinear analysis of bivariate data with cross recurrence plots. *Physics Letters A* 302.5 (2002): 299-307.

Coco, M. I., Monster, D., Leonardi, G., Dale, R., & Wallot, S. (2021). Unidimensional and Multi-dimensional Methods for Recurrence Quantification Analysis with `crqa`. *R Journal*, 13(1).

## Examples

```
# use the available data
data(crqa)

listener = eyemovement$listener
narrator = eyemovement$narrator

delay = 1; embed = 1; rescale = 0; radius = .1;
normalize = 0; mindiagline = 2; minvertline = 2;
tw = 0; whiteline = FALSE; recpt = FALSE; side = "both"
method = 'crqa'; metric = 'euclidean';
datatype = "categorical"

ans = crqa(narrator, listener, delay, embed, rescale, radius, normalize,
           mindiagline, minvertline, tw, whiteline, recpt, side, method, metric,
           datatype)

print(ans[1:10]) ## last argument of list is the recurrence plot
```

---

crqa	<i>Auto, cross and multidimensional recurrence measures of one, two or multiple time-series, time-delayed and embedded in higher dimensional space</i>
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---

## Description

Core recurrence function, which examines recurrent structures of a single (rqa), two (crqa), or multidimensional time-series (mocrqa), which are time-delayed and embedded in higher dimensional space. The approach compares the phase space trajectories of the time-series in the same phase-space when delays are introduced. A distance matrix between the time-series, delayed and embedded is calculated. Several measures representative of the underlying dynamics of the system are extracted (explained below).

## Usage

```
crqa(ts1, ts2, delay, embed, rescale, radius, normalize,
     mindiagline, minvertline, tw, whiteline, recpt, side, method,
     metric, datatype)
```

## Arguments

ts1	First time-series dataset.
ts2	Second time-series dataset
delay	The delay unit by which the series are lagged.
embed	The number of embedding dimension for phase-reconstruction, i.e., the lag intervals.

rescale	Rescale the distance matrix; if rescale = 0 (do nothing); if rescale = 1 (mean distance of entire matrix); if rescale = 2 (maximum distance of entire matrix). if rescale = 3 (minimum distance of entire matrix). if rescale = 4 (euclidean distance of entire matrix).
radius	A threshold, cut-off, constant used to decide whether two points are recurrent or not.
normalize	Normalize the time-series; if normalize = 0 (do nothing); if normalize = 1 (Unit interval); if normalize = 2 (z-score).
mindiagline	A minimum diagonal length of recurrent points. Usually set to 2, as it takes a minimum of two points to define any line.
minvertline	A minimum vertical length of recurrent points.
tw	The Theiler window parameter
whiteline	A logical flag to calculate (TRUE) or not (FALSE) empty vertical lines.
recpt	A logical flag indicating whether measures of cross-recurrence are calculated directly from a recurrent plot (TRUE) or not (FALSE).
side	A string indicating whether recurrence measures should be calculated in the 'upper' triangle of the RP 'lower' triangle of the matrix, or 'both'. LOC is automatically excluded for 'upper' and 'lower'.
method	A string to indicate the type of recurrence analysis to perform. There are three options: rqa (autorecurrence); crqa(cross-recurrence); mdcrqa(multidimensional recurrence). Default value is crqa
metric	A string to indicate the type of distance metric used, default is euclidean but see help rdist() to list all other possible metrics.
datatype	a string (continuous or categorical) to indicate whether the nature of the data type

## Details

We recommend setting whiteline = FALSE, as the current version of the library does not make use of such information to extract recurrence measures.

## Value

If a recurrence plot (RP) can be calculated and hence recurrence observed the function will return a list with different measures extracted. Otherwise, the values for the output arguments will be either 0 or NA.

RR	The percentage of recurrent points falling within the specified radius (range between 0 and 100)
DET	Proportion of recurrent points forming diagonal line structures.
NRLINE	The total number of lines in the recurrent plot
maxL	The length of the longest diagonal line segment in the plot, excluding the main diagonal
L	The average length of line structures

ENTR	Shannon information entropy of diagonal line lengths longer than the minimum length
rENTR	Entropy measure normalized by the number of lines observed in the plot. Handy to compare across contexts and conditions
LAM	Proportion of recurrent points forming vertical line structures
TT	The average length of vertical line structures
catH	Entropy of categorical recurrence plots based on rectangular block structures
max_vertlength	The maximum vertical line length
RP	The Recurrence Plot sparse matrix data

### Note

Original bits of this code were translated from a Matlab version provided by Rick Dale, and created during the Non-Linear Methods for Psychological Science summer school held at the University of Cincinnati in 2012. The multi-dimensional method for the crqa function has been written together with Sebastian Wallot (sebastian.wallot at aesthetics.mpg.de )

### Author(s)

Moreno I. Coco (moreno.cocoi@gmail.com)

### References

- Coco, M. I., and Dale, R. (2014). Cross-recurrence quantification analysis of categorical and continuous time series: an R package. *Frontiers in psychology*, 5, 510.
- Wallot, S. (2018). Multidimensional Cross-Recurrence Quantification Analysis (MdCRQA) a method for quantifying correlation between multivariate time-series. *Multivariate behavioral research*, 1-19

### See Also

[spdiags](#), [simts](#)

### Examples

```
# use the available data
data(crqa)

listener = eyemovement$listener
narrator = eyemovement$narrator

delay = 1; embed = 1; rescale = 0; radius = .1;
normalize = 0; mindiagline = 2; minvertline = 2;
tw = 0; whiteline = FALSE; recpt = FALSE; side = "both"
method = 'crqa'; metric = 'euclidean';
datatype = "categorical"

ans = crqa(narrator, listener, delay, embed, rescale, radius, normalize,
           mindiagline, minvertline, tw, whiteline, recpt, side, method,
           metric, datatype)
```

```
print(ans[1:11]) ## last argument of list is the recurrence plot
```

---

drpfromts

*Diagonal recurrence profile*


---

### Description

Method to explore the diagonal profile of the recurrence plot (Auto, Cross, or Multi-dimensional). It returns the recurrence for different delays, the maximal recurrence observed and the delay at which it occurred.

### Usage

```
drpfromts(ts1, ts2, windowsize, radius,
          delay, embed, rescale, normalize, mindiagline, minvertline, tw,
          whiteline, recpt, side, method, metric, datatype)
```

### Arguments

ts1	First time-series
ts2	Second time-series
windowsize	A constant indicating the range of delays (positive and negative) to explore
radius	A threshold, cut-off, constant used to decide whether two points are recurrent or not.
delay	The delay unit by which the series are lagged.
embed	The number of embedding dimension for phase-reconstruction, i.e., the lag intervals.
rescale	Rescale the distance matrix; if rescale = 0 (do nothing); if rescale = 1 (mean distance of entire matrix); if rescale = 2 (maximum distance of entire matrix). if rescale = 3 (minimum distance of entire matrix). if rescale = 4 (euclidean distance of entire matrix).
normalize	Normalize the time-series; if normalize = 0 (do nothing); if normalize = 1 (Unit interval); if normalize = 2 (z-score).
mindiagline	A minimum diagonal length of recurrent points. Usually set to 2, as it takes a minimum of two points to define any line.
minvertline	A minimum vertical length of recurrent points.
tw	The Theiler window parameter
whiteline	A logical flag to calculate (TRUE) or not (FALSE) empty vertical lines.
recpt	A logical flag indicating whether measures of cross-recurrence are calculated directly from a recurrent plot (TRUE) or not (FALSE).

side	A string indicating whether recurrence measures should be calculated in the 'upper' triangle of the RP 'lower' triangle of the matrix, or 'both'. LOC is automatically excluded for 'upper' and 'lower'.
method	A string to indicate the type of recurrence analysis to perform. There are three options: rqa (autorecurrence); crqa(cross-recurrence); mdcrqa(multidimensional recurrence). Default value is crqa
metric	A string to indicate the type of distance metric used, default is euclidean but see help rdist() to list all other possible metrics.
datatype	a string (continuous or categorical) to indicate whether the nature of the data type

### Value

A list with the following arguments:

profile	A vector of recurrence (ranging from 0,1) with length equal to the number of delays explored
maxrec	Maximal recurrence observed between the two-series
maxlag	Delay at which maximal recurrence is observed

### Author(s)

Moreno I. Coco (moreno.cocoi@gmail.com)

### See Also

[windowdrp](#)

### Examples

```
# use the available data
data(crqa)

listener = eyemovement$listener
narrator = eyemovement$narrator

res = drpfromts(narrator, listener, windowsize = 100,
               radius = 0.001, delay = 1, embed = 1, rescale = 0,
               normalize = 0, mindiagline = 2, minvertline = 2,
               tw = 0, whiteline = FALSE, recpt = FALSE,
               side = 'both', method = 'crqa',
               metric = 'euclidean', datatype = 'continuous')

profile = res$profile

plot(seq(1,length(profile),1), profile, type = "l", lwd = 5,
      xaxt = "n", xlab = "Lag", ylab = "Recurrence")
```

---

eyemovement

*Eye-movement categorical time-series*

---

### Description

A two-columns dataset of eye-movement fixation scan-pattern, which are temporal sequences of fixated objects.

### Usage

eyemovement

### Format

A data frame with 1000 rows and 2 variables:

**listener** the listener time series

**narrator** the narrator time series

### References

Richardson, D. C., and Dale, R. (2005). Looking to understand: The coupling between speakers and listeners eye movements and its relationship to discourse comprehension. *Cognitive Science*, 29, 39-54.

---

Figure\_1

*Eye-movement categorical time-series*

---

### Description

Three time series (1250 observations) organised as columns and simulating data from: a periodical sine wave, one of the dimensions of the Lorenz attractor and a white noise signal.

### Usage

Figure\_1

### Format

A data frame with 1250 rows and 3 columns:

**sinus** A periodical sine wave

**lorenz** One dimension of a Lorenz attractor

**wnoise** White noise

---

Figure\_2                      *A unidimensional sinusoidal time series*

---

**Description**

A unidimensional sinusoidal time series

**Usage**

Figure\_2

**Format**

A data frame with 45 rows and 1 column:

**V1** A unidimensional sinusoidal time series

---

Figure\_3                      *Simulated time series of the three dimensions from the Lorenz system*

---

**Description**

Simulated time series of the three dimensions from the Lorenz system

**Usage**

Figure\_3

**Format**

A data frame with 2048 rows and 3 columns:

**V1** First dimension of the Lorenz system

**V2** Second dimension of the Lorenz system

**V3** Third dimension of the Lorenz system

Figure\_6

*Figure\_6***Description**

Speed (time in seconds) and memory (peak RAM in MB) performance of `crqa()` and `piecewiseRQA()` for simulated data of increasing size (from 3000 to 7000 data points), compared in blocks of different sizes (from 1000 to 6500 in increments of 500)

**Usage**

Figure\_6

**Format**

A data frame with 28 rows and 2 variables:

**speed** The time in seconds to perform a `crqa()` analysis

**memory** The peak MB occupied in the RAM to perform a `crqa()` analysis

**typeRQA** Whether the analysis contained all data points (full) or was computed piecewise (piece)

**datapoint** The number of datapoints

**blocksize** The dimension of the block

handmovement

*Continuous series of hand movements***Description**

Hand-movement velocity profiles of two participants (P1 and P2) for the dominant (d) and non-dominant (n) hand.

**Usage**

handmovement

**Format**

A dataframe of 5799 observations.

**P1\_TT\_d** Participant 1 dominant hand

**P1\_TT\_n** Participant 1 non-dominant hand

**P2\_TT\_d** Participant 2 dominant hand

**P2\_TT\_n** Participant 2 non-dominant

## References

Wallot, S., Mitkidis, P., McGraw, J. J. and Roepstorff, A. (2016). Beyond synchrony: joint action in a complex production task reveals beneficial effects of decreased interpersonal synchrony. *PloS one*, 11(12), e0168306.

---

lorenzattractor

*Simulate the Lorenz Attractor*

---

## Description

An implementation of the Lorenz dynamical system, which describes the motion of a possible particle, which will neither converge to a steady state, nor diverge to infinity; but rather stay in a bounded but 'chaotically' defined region, i.e., an attractor.

## Usage

```
lorenzattractor(numsteps, dt, sigma, r, b)
```

## Arguments

numsteps	The number of simulated points
dt	System parameter
sigma	System parameter
r	System parameter
b	System parameter

## Value

It returns a matrix with the 3 dimensions of the Lorenz

## Author(s)

Moreno I. Coco (moreno.cocoi@gmail.com)

## References

Lorenz, Edward Norton (1963). Deterministic nonperiodic flow. *Journal of the Atmospheric Sciences* 20(2) 130-141.

## Examples

```
## initialize the parameters
numsteps = 2 ^ 11; dt = .01; sigma = 10; r = 28; b = 8/3;

res = lorenzattractor(numsteps, dt, sigma, r, b)
```

---

mdDelay *Find optimal delay from a multi-dimensional dataset.*

---

### Description

Estimates time delay for embedding of a multi-dimensional dataset.

### Usage

```
mdDelay(data, nbins, maxlag, criterion, threshold)
```

### Arguments

data	The matrix containing all variables
nbins	The number of bins considered to estimate mutual information
maxlag	Number of lags considered
criterion	A string to indicate what delay optimizes mutual information: 'firstBelow' uses the lowest delay at which the AMI function drops below the value set by the threshold parameter. 'localMin' uses the position of the first local minimum of the AMI function. The categorical state on which phi is calculated
threshold	Value to select the delay when AMI drops below it.

### Value

It returns the recurrence phi-coefficient profile for state k for all delays considered

### Author(s)

Sebastian Wallot, Max Planck Institute for Empirical Aesthetics Dan Moenster, Aarhus University, Moreno I. Coco, University of East London

### References

Wallot, S., and Moenster, D. (2018). Calculation of average mutual information (AMI) and false-nearest neighbors (FNN) for the estimation of embedding parameters of multidimensional time-series in Matlab. *Front. Psychol. - Quantitative Psychology and Measurement*

### See Also

[mdFnn](#), [optimizeParam](#)

**Examples**

```

nbins = 10; maxlag = 10; criterion = "firstBelow"; threshold = exp(-1)

data(crqa) ## load the data

handset = handmovement[1:300, ] ## take less points

mdDelay(handset, nbins, maxlag, criterion, threshold)

```

---

mdFnn

*Find optimal embedding dimension of a multi-dimensional dataset.*


---

**Description**

Computes the percentage of false nearest neighbors for multidimensional time series as a function of embedding dimension.

**Usage**

```
mdFnn(data, tau, maxEmb, numSamples, Rtol, Atol)
```

**Arguments**

data	The matrix of data to estimate FNN.
tau	Time delay for embedding.
maxEmb	Maximum number of embedding dimensions considered
numSamples	Number of randomly drawn coordinates from phase-space used to estimate FNN
Rtol	First distance criterion for separating false neighbors
Atol	Second distance criterion for separating false neighbors

**Value**

It returns the percentage of false neighbors for each embedding.

**Author(s)**

Sebastian Wallot, Max Planck Insitute for Empirical Aesthetics Dan Moenster, Aarhus University, Moreno I. Coco, University of East London

**References**

Kennel, M. B., Brown, R., & Abarbanel, H. D. (1992). Determining embedding dimension for phase-space reconstruction using a geometrical construction. *Physical review A*, 45, 3403. Wallot, S., and Moenster, D. (2018). Calculation of average mutual information (AMI) and false-nearest neighbors (FNN) for the estimation of embedding parameters of multidimensional time-series in Matlab. *Front. Psychol. - Quantitative Psychology and Measurement*

**See Also**

[mdDelay](#), [optimizeParam](#)

**Examples**

```
tau = 1; maxEmb = 10; numSamples = 500; Rtol = 10; Atol = 2

data(crqa) ## load the data

handset = handmovement[1:300, ] ## take less points

mdFnn(handset, tau, maxEmb, numSamples, Rtol, Atol)
```

---

optimizeParam	<i>Estimate optimal delay, embedding dimension and radius for continuous time-series data</i>
---------------	---

---

**Description**

Iterative procedure to examine the values of delay, embedding dimension and radius to compute recurrence plots of one, two, or more time-series.

**Usage**

```
optimizeParam(ts1, ts2, par, min.rec, max.rec)
```

**Arguments**

ts1	First time-series
ts2	Second time-series
par	A list of parameters needed for the optimization, refer to the Details section.
min.rec	The minimum value of recurrence accepted. Default = 2
max.rec	The maximum value of recurrence accepted. Default = 5

**Details**

The optimization can be applied both to uni-dimensional time-series (method = crqa), or multi-dimensional (method = mdcrqa)

The procedure is identical in both cases:

- 1) Identify a delay that accommodates both time-series by finding the local minimum where mutual information between them drops, and starts to level off. When one ts has a considerably longer delay indicated than the another, the function selects the longer delay of the two to ensure that new information is gained for both. When the delays are close to each other, the function computes the mean of the two delays.

2) Determine embedding dimensions by using false nearest neighbors and checking when it bottoms out (i.e., there is no gain in adding more dimensions). If the embedding dimension for the two ts are different the algorithm selects the higher embedding dimension of the two to make sure that both time series are sufficiently unfolded.

3) Determine radius yielding a recurrence rate between 2-5 To do so, we first determine a starting radius that yields approximately 25 We generate a sampled sequence of equally spaced possible radii from such radius till 0, using as unit for the sequence step, the standard deviation of the distance matrix divided by a scaling parameter (radiusspan). The larger this parameter, the finer the unit.

For uni-dimensional time-series, the user has to decide how to choose the value of average mutual information (i.e., typeami = mindip, the lag at which minimal information is observed, or typeami = maxLag, the maximum lag at which minimal information is observed) and the relative percentage of information gained in FNN, relative to the first embedding dimension, when higher embeddings are considered (i.e., fnnpercent). Then, as crqa is integrated in the optimizeParam to estimate the radius, most of the arguments are the same (e.g., mindiagline or tw).

For multidimensional series, the user needs to specify the right RQA method (i.e., method = "mocrqa"). Then, for the estimation of the delay via AMI: (1) nbins the number of bins to compute the two-dimensional histogram of the original and delayed time series and (2) the criterion to select the delay (firstBelow to use the lowest delay at which the AMI function drops below the value set by the threshold argument, and localMin to use the position of the first local AMI minimum). The estimation of the embedding dimensions instead needs the following arguments: (1) maxEmb, which is the maximum number of embedding dimensions considered, (2) noSamples, which is the number of randomly drawn coordinates from phase-space used to estimate the percentage of false-nearest neighbors, (3) Rtol, which is the first distance criterion for separating false neighbors, and (4) Atol, which is the second distance criterion for separating false neighbors. The radius is estimated as before.

### Value

It returns a list with the following arguments:

radius	The optimal radius value found
emddim	Number of embedding dimensions
delay	The lag parameter.

### Note

As optimizeParam uses crqa to estimate the parameters: the additional arguments normalize, rescale, mindiagline, minvertline, whiteline, recpt should be supplied in the par list. Set up relatively large radiusspan (e.g. 100), for a decent coverage of radius values.

### Author(s)

Moreno I. Coco (moreno.cocoi@gmail.com), James A. Dixon (james.dixon@uconn.edu) Sebastian Wallot, Max Planck Insitute for Empirical Aesthetics Dan Moenster, Aarhus University

### References

Marwan, N., Carmen Romano, M., Thiel, M., and Kurths, J. (2007). Recurrence plots for the analysis of complex systems. Physics Reports, 438(5), 237-329.

**See Also**[crqa](#), [wincrqa](#)**Examples**

```

data(crqa) ## load the data

handset = handmovement[1:300, ] ## take less points

P1 = cbind(handset$P1_TT_d, handset$P1_TT_n)
P2 = cbind(handset$P2_TT_d, handset$P2_TT_n)

par = list(method = "mdcrqa", metric = "euclidean", maxlag = 20,
           radiusspan = 100, radiussample = 40, normalize = 0,
           rescale = 4, mindiagline = 10, minvertline = 10, tw = 0,
           whiteline = FALSE, recpt = FALSE, side = "both",
           datatype = "continuous", fnnpercent = NA,
           typeami = NA, nbins = 50, criterion = "firstBelow",
           threshold = 1, maxEmb = 20, numSamples = 500,
           Rtol = 10, Atol = 2)

results = optimizeParam(P1, P2, par, min.rec = 2, max.rec = 5)
print(unlist(results))

```

---

piecewiseRQA

---

*Compute recurrence plots for long time-series data series using a block (piece-wise) method.*


---

**Description**

This is a convenience function which breaks down the computation of large recurrence plots into a collection of smaller recurrence plots. It can ease speed and memory issues if an appropriate size for the block is found.

**Usage**

```

piecewiseRQA(ts1, ts2, blockSize, delay, embed, rescale, radius,
             normalize, mindiagline, minvertline, tw, whiteline, recpt, side,
             method, metric, datatype, typeRQA, windowsize)

```

**Arguments**

ts1	First time-series.
ts2	Second time-series.
blockSize	The dimension of the time-series subunit in which the-recurrence plot will be computed



**Author(s)**

Moreno I. Coco (moreno.cocoi@gmail.com) based on Matlab code by Sebastian Wallot

**See Also**

[crqa](#), [spdiags](#), [simts](#)

**Examples**

```
## Uncomment and run locally

## generate some data using pracma

# ts1 = seq(0.1, 200, .1)
# ts1 = sin(ts1) + linspace(0, 1,length(ts1));

# ts2 = ts1

## initialize the parameters
# blockSize = 100; delay = 15; embed = 2; rescale = 0; radius = 0.5;
# normalize = 0; mindiagline = 2; minvertline = 2;
# tw = 1; whiteline = FALSE; recpt = FALSE; side = 'both'
# method = "crqa"; metric = 'euclidean'; datatype = "continuous"
# typeRQA = "full"; windowsize = NA

# pieceRP = piecewiseRQA(ts1, ts2, blockSize, delay, embed, rescale,
#                         radius, normalize, mindiagline, minvertline,
#                         tw, whiteline, recpt, side,
#                         method, metric, datatype, typeRQA,
#                         windowsize)

# print(unlist(pieceRP[1:10]))
```

---

plot\_rp

*Plot a recurrence matrix*

---

**Description**

Given a recurrence plot object (RP) returned from the function `crqa()`, visualize it.

**Usage**

```
plot_rp(rp_matrix,
        title = "",
        xlabel = "Time",
        ylabel = "Time",
        pcolour = "black",
        geom = c("tile", "point", "void"),
        flip_y = FALSE)
```

**Arguments**

rp_matrix	A recurrence plot sparse matrix from crqa()
title	Main title of the plot (character). Default: "" (empty)
xlabel	The x-axis label (character). Default: "Time"
ylabel	The y-axis label (character). Default: "Time"
pcolour	The colour used for points in the plot (character). Default: "black"
geom	The ggplot2 geom used to draw the points (character). One of: "tile", "point" or "void". The default value, "tile", uses (geom_tile()) to plot the points. The value "point" uses geom_point() to plot the points, and the value "void" produces a plot without any geom, so users can add their own for a more customized plot (see examples below).
flip_y	Logical argument controlling whether to flip the directionality of the y axis (logical). Default: FALSE

**Details**

The argument geom is a character denoting what type of ggplot2 geom is used to plot the points in the recurrence plot. Allowed values are "tile", "point" and "void". The latter produces an empty plot, so the user is required to add their own geom to the output. See example below.

**Value**

A ggplot2 plot.

**Author(s)**

Mønster, D. <danm@econ.au.dk>

**See Also**

This plotting solution as substituted what was previously plotRP() available till version crqa.2.0.5

**Examples**

```
# Here is a very short example based on these two time series
ts_1 <- c(0, 0, 1, 1, 0, 0, 2, 2, 1, 1)
ts_2 <- c(1, 1, 2, 2, 0, 0, 1, 2, 2, 1)

# Create the cross recurrence matrix
short_rec <- crqa(ts_1, ts_2,
                 delay = 1, embed = 1, radius = 0.001,
                 rescale = 1, method = "crqa")

# Extract the cross recurrence matrix
small_rp <- short_rec$RP

# Make a cross recurrence plot with blue tiles
plot_rp(small_rp, pcolour = "blue", geom = "tile")
```

```

# Make a cross recurrence plot with blue points
plot_rp(small_rp, pcolour = "blue", geom = "point")

## Uncomment below if you want to have more example. Not runnable because
## of CRAN, globally unavailable function plus timing constraint to check vignette.

# Neither of the two plots above may be suitable
# Using geom = "void" we can add a customized geom, here blue tiles
# that are smaller than the default width and height
# plot_rp(small_rp, geom = "void") +
#   geom_tile(fill = "blue", height = 0.75, width = 0.75)

# Another custom geom uses tiles with a border colour
# plot_rp(small_rp, geom = "void") +
#   geom_tile(fill = "blue", colour = "pink", linewidth = 1)

# Use the eyemovement dataset to create a cross-recurrence plot
# large_crqa <- crqa(eyemovement$narrator, eyemovement$listener,
#                   delay = 1, embed = 1, radius = 0.001,
#                   method = "crqa", metric = "euclidean", datatype = "continuous")

# Extract the recurrence matrix from the result
# large_rp <- large_crqa$RP

# Create a recurrence plot using defaults
# plot_rp(large_rp)

# The same recurrence plot with flipped y axis and using geom_point()
# plot_rp(large_rp, flip_y = TRUE, geom = "point", title = "Flipped y axis")

# Add axes labels, a title and extra plot elements
# Note that we can add to the output, here added the line of synchronization
# using ggplot2's geom_abline() function.
# plot_rp(large_rp,
#         xlabel = "Narrator",
#         ylabel = "Listener",
#         title = "Coloured tiles with line of synchronization",
#         pcolour = "blue") +
#   geom_abline(intercept = 0, slope = 1)

```

---

## Description

A simple algorithm for producing a time-series that drives a second time-series (1 for event occurrence; 0 otherwise) using parameters, which change independent and conditional probability of an event to occur.

**Usage**

```
simts(BL1, BL2, BLR1, BLR2, BL2C1, tsL)
```

**Arguments**

BL1	Base event rate of the first time-series
BL2	Base event rate of the second time-series
BLR1	Rate of repetition in the first series
BLR2	Rate of repetition in the second series
BL2C1	Conditional probability of repetition.
tsL	Length of the simulated time-series

**Value**

A matrix with two-rows, where the first row is the 'driving' time-series and the second row is the second time-series. The columns are the number of simulated points as selected by the argument tsL.

**Author(s)**

Rick Dale and Moreno I. Coco (moreno.cocoi@gmail.com)

**Examples**

```
## set up parameters

BL1 = .08; BL2 = .05; BLR1 = .5; BLR2 = .5;
BL2C1 = .33; tsL = 100

ts = simts(BL1, BL2, BLR1, BLR2, BL2C1, tsL)
```

---

spdiags

*Extract diagonal matrices*

---

**Description**

Extracts all nonzero diagonals from the m-by-n matrix A. B is a min(m,n)-by-p matrix whose columns are the p nonzero diagonals of A.

**Usage**

```
spdiags(A)
```

**Arguments**

A	An m-by-n matrix with nonzero elements located on p diagonals.
---	--

**Details**

Compared to the original Matlab implementation: 1) it does not handle the case with more than one input, and 2) ( $m > n$ ) matrices give the **B** matrix columns in a different order, but the **d** vector of indices will also be changed accordingly, so the set of columns is OK, just ordered differently

**Value**

**B** A  $\min(m,n)$ -by- $p$  matrix, usually (but not necessarily) full, whose columns are the diagonals of **A**.

**d** A vector of length  $p$  whose integer components specify the diagonals in **A**.

**Note**

For computational efficiency `spdiags` is actually computed using a Fortran implementation (`jspd.f`)

**Author(s)**

John C. Nash ([nashjc@uottawa.ca](mailto:nashjc@uottawa.ca))

**Examples**

```
dta <- c(0, 5, 0, 10, 0, 0, 0, 0, 6, 0, 11, 0, 3, 0, 0,
7, 0, 12, 1, 4, 0, 0, 8, 0, 0, 2, 5, 0, 0, 9)

A1 <- matrix(dta, nrow=5, ncol=6, byrow=TRUE)

print(A1)
res1 <- spdiags(A1)
print(res1)
```

---

text

*Categorical sequence of words*


---

**Description**

A simple text of the nursery rhyme, 'the wheels on the bus'

**Usage**

```
text
```

**Format**

A vector of 120 words

**References**

Verna Hills (1939). The Wheels on the bus. *American Childhood* (25), 56

wincrqa

*Windowed Recurrence Measures***Description**

A recurrence plot is computed in overlapping windows of a certain size for a number of delays smaller than the size of the window; and measures of it extracted.

**Usage**

```
wincrqa(ts1, ts2, windowstep, windowsize, delay, embed,
radius, rescale, normalize, mindiagline, minvertline, tw, whiteline,
recpt, side, method, metric, datatype, trend)
```

**Arguments**

ts1	First time-series
ts2	Second time-series
windowstep	Interval by which the window is moved.
windowsize	The size of the window
delay	The delay unit by which the series are lagged.
embed	The number of embedding dimension for phase-reconstruction, i.e., the lag intervals.
radius	A threshold, cut-off, constant used to decide whether two points are recurrent or not.
rescale	Rescale the distance matrix; if rescale = 0 (do nothing); if rescale = 1 (mean distance of entire matrix); if rescale = 2 (maximum distance of entire matrix). if rescale = 3 (minimum distance of entire matrix). if rescale = 4 (euclidean distance of entire matrix).
normalize	Normalize the time-series; if normalize = 0 (do nothing); if normalize = 1 (Unit interval); if normalize = 2 (z-score).
mindiagline	A minimum diagonal length of recurrent points. Usually set to 2, as it takes a minimum of two points to define any line.
minvertline	A minimum vertical length of recurrent points.
tw	The Theiler window parameter
whiteline	A logical flag to calculate (TRUE) or not (FALSE) empty vertical lines.
recpt	A logical flag indicating whether measures of cross-recurrence are calculated directly from a recurrent plot (TRUE) or not (FALSE).
side	A string indicating whether recurrence measures should be calculated in the 'upper' triangle of the RP 'lower' triangle of the matrix, or 'both'. LOC is automatically excluded for 'upper' and 'lower'.

method	A string to indicate the type of recurrence analysis to perform. There are three options: rqa (autorecurrence); crqa(cross-recurrence); mdcrqa(multidimensional recurrence). Default value is crqa
metric	A string to indicate the type of distance metric used, default is euclidean but see help rdist() to list all other possible metrics.
datatype	a string (continuous or categorical) to indicate whether the nature of the data type
trend	a boolean (TRUE or FALSE) to indicate whether the TREND should be computed of the system

**Value**

It returns a matrix where the rows are the different windows explored, and the columns are the recurrence measures observed in that particular window. Refer to crqa for the values returned.

**Note**

If no-recurrence is found in a window, that window will not be saved, and a message about it will be warned. TREND is implemented following a solution proposed by Norbert Marwan, and translated here in R, for those who have asked him. He, however, warns that this measure might strongly depend on the chosen settings to calculate crq. Relying blindly on such measure can, therefore, produce misleading results. Also, we enabled the possibility to input directly a RP with recpt = T, and so extract the measures. This implies that it will not be the same as by conducting phase-space reconstruction on subsets of the time series. So, please, make sure why you are doing it and why.

**Author(s)**

Moreno I. Coco (moreno.cocoi@gmail.com) Alexandra Paxton <alexandra.paxton@uconn.edu>

**See Also**

[crqa](#)

**Examples**

```
data(crqa)

listener = eyemovement$listener[1:200]
narrator = eyemovement$narrator[1:200]

# NB, the parameters for windowsize and windowstep are large to allow
# faster running time, please set them carefully in your analysis.

delay = 1; embed = 1; rescale = 0; radius = 0.001;
normalize = 0; mindiagline = 2; minvertline = 2;
tw = 0; whiteline = FALSE; recpt = FALSE; side = "both"
method = 'crqa'; metric = 'euclidean';
datatype = "continuous";
windowsize = 100; windowstep = 50
trend = FALSE
```

```

ans = wincrqa(listener, narrator, windowstep, windowsize, delay, embed,
             radius, rescale, normalize, mindiagline, minvertline,
             tw, whiteline, recpt, side, method, metric,
             datatype, trend)

## other recurrence measures are available in ans
profile = as.numeric(ans$RR)

plot(profile, type = 'l')

```

---

windowdrp

*Windowed Recurrence Profile*


---

### Description

A recurrence plot is computed in overlapping windows of a specified size for a number of delays smaller than the size of the window. In every window, the recurrence value for the different delays is calculated. A mean is then taken across the delays to obtain a recurrence value in that particular window.

### Usage

```

windowdrp(ts1, ts2, windowstep, windowsize, lagwidth,
          radius, delay, embed, rescale, normalize, mindiagline, minvertline,
          tw, whiteline, recpt, side, method, metric, datatype)

```

### Arguments

ts1	First time-series
ts2	Second time-series
windowstep	Interval by which the window is moved.
windowsize	The size of the window
lagwidth	The number of delays to be considered within the window
radius	For numeric time-series, the cutoff distance to accept or reject two-points as recurrent
delay	The delay unit by which the series are lagged.
embed	The number of embedding dimension for phase-reconstruction, i.e., the lag intervals.
rescale	Rescale the distance matrix; if rescale = 0 (do nothing); if rescale = 1 (mean distance of entire matrix); if rescale = 2 (maximum distance of entire matrix). if rescale = 3 (minimum distance of entire matrix). if rescale = 4 (euclidean distance of entire matrix).

normalize	Normalize the time-series; if normalize = 0 (do nothing); if normalize = 1 (Unit interval); if normalize = 2 (z-score).
mindiaqline	A minimum diagonal length of recurrent points. Usually set to 2, as it takes a minimum of two points to define any line.
minvertline	A minimum vertical length of recurrent points.
tw	The Theiler window parameter
whiteline	A logical flag to calculate (TRUE) or not (FALSE) empty vertical lines.
recpt	A logical flag indicating whether measures of cross-recurrence are calculated directly from a recurrent plot (TRUE) or not (FALSE).
side	A string indicating whether recurrence measures should be calculated in the 'upper' triangle of the RP 'lower' triangle of the matrix, or 'both'. LOC is automatically excluded for 'upper' and 'lower'.
method	A string to indicate the type of recurrence analysis to perform. There are three options: rqa (autorecurrence); crqa(cross-recurrence); mdcrqa(multidimensional recurrence). Default value is crqa
metric	A string to indicate the type of distance metric used, default is euclidean but see help rdist() to list all other possible metrics.
datatype	a string (continuous or categorical) to indicate whether the nature of the data type

### Value

It returns a list of arguments where:

profile	Time-course windowed recurrence profile
maxrec	Maximal recurrence observed along the time-course
maxlag	The point where maximal recurrence is observed

### Author(s)

Moreno I. Coco (moreno.cocoi@gmail.com) and Rick Dale (rdale@ucmerced.edu)

### References

Boker, S. M., Rotondo, J. L., Xu, M., and King, K. (2002). Windowed cross-correlation and peak picking for the analysis of variability in the association between behavioral time series. *Psychological Methods*, 7(3), 338.

### See Also

[drpfromts](#)

**Examples**

```
# use the available data
data(crqa)

listener = eyemovement$listener
narrator = eyemovement$narrator

# NB, the parameters for window size and window step are large to allow
# faster running time, please set them carefully in your analysis.

delay = 1; embed = 1; rescale = 1; radius = 0.001;
normalize = 0; mindiagline = 2; minvertline = 2;
tw = 0; whiteline = FALSE; recpt = FALSE; side = "both"
method = 'crqa'; metric = 'euclidean';
datatype = "continuous"; window size = 100;
lagwidth = 10; window step = 200

ans = windowdrp(narrator, listener, window step, window size, lagwidth,
               radius, delay, embed, rescale, normalize,
               mindiagline, minvertline, tw,
               whiteline, recpt, side, method, metric,
               datatype)

profile = ans$profile; maxrec = ans$maxrec; maxlag = ans$maxlag

plot(profile, type = 'l')
```

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