diffpriv: An R Package for Easy Differential Privacy

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Abstract
The R package diffpriv provides tools for statistics and machine learning under differential privacy. Suitable for releasing analyses on privacy-sensitive data to untrusted third parties, differential privacy has become the framework of choice for privacy-preserving learning. diffpriv delivers: (a) implementations of generic mechanisms for privatizing non-private target functions, including the Laplace, Gaussian, exponential and Bernstein mechanisms; (b) a recent sensitivity sampler due to Rubinstein and Aldà (2017) that empirically estimates the sensitivity of non-private targets—obviating mathematical analysis for exact sensitivity bounds needed for most generic mechanisms; (c) an extensible framework for implementing differentially-private mechanisms. Together, the components of diffpriv permit easy high-utility privatization of complex analyses, learners and even black-box software programs.

Keywords: differential privacy, empirical process theory, R, open-source software

1. Introduction
Differential privacy (Dwork et al., 2006) has quickly become a key framework for semantic guarantees of data privacy when releasing analysis on privacy-sensitive data to untrusted third parties. A great deal of its popularity is owed to a suite of generic mechanisms for privatizing non-private target functions of data i.e., statistics, estimation procedures, and learners. Common to these generic mechanisms is the requirement that the non-private target’s sensitivity to dataset perturbation is known and bounded. In all except the most trivial analyses, bounding sensitivity is prohibitively involved. This paper describes the diffpriv R package that implements generic mechanisms for differential privacy, along with our recent sensitivity sampler that replaces exact sensitivity bounds with empirical estimates (Rubinstein and Aldà, 2017). As a result, diffpriv privatizes a wide range of procedures under random differential privacy (Hall et al., 2012), automatically without mathematical analysis and in many cases achieving high utility. diffpriv is available from https://github.com/brubinstein/diffpriv under an open-source license.

2. Generic Mechanisms for Differential Privacy
Fundamental to differential privacy is a privacy-sensitive dataset (or database) $D \in \mathcal{D}^n$ on domain $\mathcal{D}$. In diffpriv a dataset can be a list, matrix, data.frame, vector. We say that
a pair of databases $D, D' \in \mathcal{D}^n$ is neighboring if they differ on exactly one record. While individual records should not be revealed, we aim to release aggregate information on $D$ with a mechanism. A mechanism $M : \mathcal{D}^n \rightarrow \mathcal{R}$ is a random-valued function of databases taking values in a response set $\mathcal{R}$; implemented in diffpriv as virtual class DPMech.

**Definition 1 (Dwork et al., 2006)** For $\epsilon > 0$, mechanism $M : \mathcal{D}^n \rightarrow \mathcal{R}$ preserves $\epsilon$-differential privacy if for all neighboring pairs $D, D' \in \mathcal{D}^n$, measurable response sets $R \subseteq \mathcal{R}$, $\Pr(M(D) \in R) \leq \exp(\epsilon) \cdot \Pr(M(D') \in R)$. Alternatively for $\delta \in (0, 1)$, relaxed $(\epsilon, \delta)$-differential privacy holds if $\Pr(M(D) \in R) \leq \exp(\epsilon) \cdot \Pr(M(D') \in R) + \delta$.

Mechanism $M$ preserves differential privacy if its response distributions are close on neighboring pairs: an adversary cannot determine an unknown record from $M$ responses, even with knowledge of the other $n - 1$ records. Privacy parameters $\epsilon$ $(\epsilon, \delta)$ are encapsulated in class DPMech (DPParamsEps, DPParamsDel respectively). Virtual DPMech generic method releaseResponse(mechanism, privacyParams, X) takes privacy parameters along with sensitive dataset. Most generic mechanisms in differential privacy share a number of properties leveraged by the diffpriv package as follows.

**Privatizing a target function.** Many mechanisms $M : \mathcal{D}^n \rightarrow \mathcal{R}$ seek to privatize a non-private target function $f : \mathcal{D}^n \rightarrow \mathcal{B}$, with range $\mathcal{B}$ often (but not always!) coinciding with $\mathcal{R}$. Accordingly DPMech objects are initialized with target slot of type function. A mechanism’s releaseResponse() method calls target to form privacy-preserving responses.

**Normed target range space.** Target $f$’s output space $\mathcal{B}$ is typically imbued with a norm, denoted $\| \cdot \|_B$, needed for measuring the sensitivity of $f$’s outputs to input perturbation. diffpriv flexibly represents this norm within DPMech objects as described next.

**Sensitivity-induced privacy.** Many mechanisms achieve differential privacy by calibrating randomization to the sensitivity of the target function $f$. Targets that are relatively insensitive (sensitive) to perturbing input $D$ to neighboring $D'$ need relatively little (large) response randomization. On a pair of neighboring databases $D, D' \in \mathcal{D}^n$ the sensitivity of $f$ is measured as $\Delta(D, D') = \|f(D) - f(D')\|_B$. Global sensitivity is the largest such value $\Delta = \sup_{D, D'} \|f(D) - f(D')\|_B$ over all possible neighboring pairs. As we discuss in (Rubinstein and Aldà, 2017), a broad class of generic mechanisms, taking sensitivity $\Delta$ as a parameter, are sensitivity-induced private: for any neighboring pair $D, D' \in \mathcal{D}^n$ if $\Delta(D, D') \leq \Delta$ then the mechanism $M_\Delta$ run with parameter $\Delta$ achieves $\Pr(M_\Delta(D) \in R) \leq \exp(\epsilon) \cdot \Pr(M_\Delta(D') \in R)$ for all $R \subseteq \mathcal{R}$. When run with $\Delta = \Delta$, this condition holds for all neighboring pairs: $M_\Delta$ satisfies $\epsilon$-differential privacy. Similarly for $(\epsilon, \delta)$-differential privacy. This is essentially how differential privacy is proved for most generic mechanisms, included those in diffpriv. DPMech objects can be initialized with a sensitivity argument, stored in an S4 slot of the same name. If the user provides a manually-derived global sensitivity bound $\Delta$, then releaseResponse() responses preserve $\epsilon$- or $(\epsilon, \delta)$-DP (depending on the specific mechanism). We now demonstrate this use case.

**Example: Laplace mechanism.** diffpriv implements Laplace (Dwork et al., 2006), Gaussian (Dwork and Roth, 2014), exponential (McSherry and Talwar, 2007), and Bernstein (Aldà,
and Rubinstein, 2017) mechanisms as DPMech subclasses DPMechLaplace, DPMechGaussian, DPMechExponential, and DPMechBernstein. An exponential example is included below. DPMechLaplace releases numeric vectors, and requires that \( B \) be numeric \( \mathbb{R}^d \) for some \( d \) and adopts the \( L_1 \) norm (sum of absolutes) for \( \| \cdot \|_B \). The mechanism releases vectors in \( \mathbb{R}^d = B = \mathbb{R}^d \) by adding an i.i.d. sample of \( d \) Laplace-distributed random variables with means 0 and scales \( \frac{\Delta}{\epsilon} \) to \( f(D) \) to achieve \( \epsilon \)-differential privacy. DPMechGaussian also privatizes numeric responses but under \( L_2 \)-sensitivity and weaker \((\epsilon, \delta)\)-DP; DPMechBernstein leverages the Laplace mechanism to release multivariate real-valued functions.

We next demonstrate Laplace privatization of the sample mean on bounded data in \( D^n = [0, 1]^n \), for which \( B \) dimension \( d = 1 \). Global sensitivity is readily bounded as \( 1/n \): For any neighboring pair \( D, D' \in [0, 1]^n \), \( \Delta(D, D') = n^{-1} |\sum_{i=1}^n D_i - \sum_{i=1}^n D'_i| \). Since \( n - 1 \) records are identical, and all records are in \([0, 1]\), this is \( |D_n - D'_n|/n \leq 1/n \).

```r
library(diffpriv)
f <- function(X) mean(X) # target function
n <- 100 # dataset size
mechanism <- DPMechLaplace(target = f, sensitivity = 1/n, dims = 1)
D <- runif(n, min = 0, max = 1) # the sensitive database in [0,1]^n
pparams <- DPParamsEps(epsilon = 1) # desired privacy budget
r <- releaseResponse(mechanism, privacyParams = pparams, X = D)
cat("Private response r$response:", r$response,
    "\nNon-private response f(D): ", f(D))

#> Private response r$response: 0.5244495
#> Non-private response f(D): 0.5343454
```


When target global sensitivity is supplied as sensitivity within DPMech construction, responses are differentially private. Global sensitivity is known for idealizations of e.g., coefficients for regularized logistic regression (Chaudhuri and Monteleoni, 2009) and the SVM (Rubinstein et al., 2012; Chaudhuri et al., 2011). In complex applications such as privatizing a software function, however, target’s global sensitivity may not be readily available. For such cases, diffpriv implements the sensitivity sampler of Rubinstein and Aldà (2017) which forms a high-probability estimate of target sensitivity by repeated probing of sensitivity on random neighboring database pairs, leveraging tools from empirical process theory. Like sensitivity estimates, resulting privacy holds with high probability.

**Definition 2 (Hall et al., 2012)** A mechanism \( M \) preserves \((\epsilon, \gamma)\)-random differential privacy (with a corresponding form for \( \epsilon, \delta, \gamma \)) if \( \forall R \subseteq \mathcal{R}, \Pr(M(D) \in R) \leq \exp(\epsilon) \cdot \Pr(M(D') \in R) \) holds with probability at least \( 1 - \gamma \) over random database pairs \( D, D' \).

While weaker than \( \epsilon \)-DP, RDP is arguably more natural than \((\epsilon, \delta)\)-DP: The later safeguards all databases but not unlikely responses, while RDP protects against all responses but not pathological databases (as defined by the database sampling distribution). The sampling distribution can be anything meaningful e.g., uniform, a Bayesian prior, a density from data privately fit by the Bernstein mechanism (Aldà and Rubinstein, 2017), etc.
The DPMech method `sensitivitySampler(object, oracle, n, m, gamma)` requires:
a mechanism object; a function oracle which outputs i.i.d. random databases of given size
n which should match the size of input data supplied later in calls to `releaseResponse()`;
a sensitivity sample size m; and desired privacy confidence gamma. Either (but not both)
of m, gamma can be omitted: the omitted resource will be optimized automatically. For
example m taken small (few hundred) reflects limited sampling time; small gamma (e.g., 0.05)
prioritizes privacy. The sensitivity sampler calls appropriate DPMech `sensitivityNorm()`
which implements ∆(D, D') for the mechanism’s norm and stored target. New subclasses
of DPMech need only implement this method in order to take advantage of the sensitivity
sampler. After `sensitivitySampler()`, subsequent `releaseResponse()` results have a privacy parameter slot of type DPParamsGam indicating response RDP.

Example: Exponential mechanism. All DPMech subclasses are sensitivity-induced private
and can be sensitivity sampled; we demonstrate the exponential mechanism here. Exponential privately optimizes an application-specific objective (or score, utility) s(r) of
candidate response r ∈ R, with response distribution proportional to exp(ε · s(r)/(2Δ)).
Typically s depends on input D, and so DPMechExponential is initialized with target that
takes D and outputs a score function. That is, B = RR is a real-valued function space on
R and the class’s `sensitivityNorm()` implements the sup-norm (cf. Rubinstein and Aldà,
2017). In practice, users supply target as an R closure as demonstrated below. Given
global sensitivity of target, the mechanism preserves ε-DP; if `sensitivitySampler()` estimates sensitivity with some gamma, then RDP is preserved at confidence γ = gamma.

Applying these ideas to find the most frequent a–z character within a dataset of top-10
computer scientists from Semantic Scholar, subject to privacy of individuals. The exponential
mechanism privately maximizes total frequency. But without bounded name lengths,
this function has unbounded global sensitivity. We therefore use sensitivity sampler for
(1, 0.1)-RDP, with an oracle that samples representative U.S. names based on randomNames.

```r
library(randomNames) ## a package that generates representative random names
oracle <- function(n) randomNames(n)
D <- c("Michael Jordan", "Andrew Ng", "Andrew Zisserman", "Christopher Manning",
"Jitendra Malik", "Geoffrey Hinton", "Scott Shenker",
"Bernhard Scholkopf", "Jon Kleinberg", "Judea Pearl")
n <- length(D)
f <- function(X) { function(r) sum(r == unlist(base::strsplit(X, ""))) }
rSet <- as.list(letters) ## the response set, letters a--z, must be a list
mechanism <- DPMechExponential(target = f, responseSet = rSet)
mechanism <- sensitivitySampler(mechanism, oracle = oracle, n = n, gamma = 0.1)
pparams <- DPParamsEps(epsilon = 1)
r <- releaseResponse(mechanism, privacyParams = pparams, X = D)
cat("Private response r$response: ", r$response,
"Non-private f(D) maximizer: ", letters[which.max(sapply(rSet, f(D)))]
```

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References


