Package ‘mrfDepth’

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adjOutl ................................................. 3
bagdistance ......................................... 6
bagplot ................................................ 9
bloodfat ............................................. 11
cardata90 .............................................. 12
characterA .......................................... 13
characterI ........................................... 14
cmltest ............................................... 15
compBagplot ......................................... 16
depthContour .......................................... 19
dirOutl ............................................... 22
distSpace ............................................. 25
dprojdepth ........................................... 28
dprojmedian ........................................... 30
fheatmap ............................................. 31
fom .................................................... 32
fOutl .................................................. 34
geological ........................................... 37
glass .................................................. 38
hdepth ................................................ 38
hdepthmedian ........................................ 42
medcouple ............................................ 44
mfd ..................................................... 45
mfdmedian ............................................. 48
mrainbowplot ......................................... 50
mri ..................................................... 51
octane ............................................... 52
outlyingness ......................................... 53
plane .................................................. 57
plotContours .......................................... 58
projdepth .............................................. 59
projmedian ............................................ 61
rdepth ............................................... 63
rdepthmedian ......................................... 65
sdepth ............................................... 67
sprojdepth ........................................... 68
sprojmedian .......................................... 70
stars ................................................... 72
symtest ............................................... 73
tables ............................................... 74
wine ................................................... 75

Index 76
Description

Computes the skew-adjusted outlyingness of \( p \)-dimensional points \( z \) relative to a \( p \)-dimensional dataset \( x \). For each multivariate point \( z_i \), its adjusted outlyingness relative to \( x \) is defined as its maximal univariate adjusted outlyingness measured over all directions. To obtain the univariate adjusted outlyingness in the direction \( v \), the dataset \( x \) is projected on \( v \), and the robustly skew-adjusted standardized distance of \( v'z_i \) to the median of the projected data points \( xv \) is computed.

Usage

\[
\text{adjOutl}(x, z = \text{NULL}, \text{options} = \text{list()})
\]

Arguments

- \( x \): An \( n \) by \( p \) data matrix.
- \( z \): An optional \( m \) by \( p \) matrix containing rowwise the points \( z_i \) for which to compute the adjusted outlyingness. If \( z \) is not specified, it is set equal to \( x \).
- \text{options}: A list of available options:
  - \text{type}:
    Determines the desired type of invariance and should be one of "Affine", "Rotation" or "Shift". When the option "Affine" is used, the directions \( v \) are orthogonal to hyperplanes spanned by \( p \) observations from \( x \). When the option "Rotation" is used, the directions pass by two randomly selected observations from \( x \). With the option "Shift", directions are randomly generated. Defaults to "Affine".
  - \text{ndir}:
    Determines the number of directions \( v \) by setting \text{ndir} to a specific number or to "all". In the latter case, an exhaustive search over all possible directions (according to \text{type}) is performed. If \text{ndir} is larger than the number of possible directions, the algorithm will automatically use this setting. Defaults to 250\(p\) when \text{type}="Affine", to 5000 when \text{type}="Rotation" and to 12500 when \text{type}="Shift".
  - \text{seed}:
    A strictly positive integer specifying the seed to be used by the C++ code. Defaults to 10.

Details

The adjusted outlyingness (AO) of multivariate data was introduced in Brys et al. (2005) and studied in more detail in Hubert and Van der Veeken (2008). It extends the Stahel-Donoho outlyingness towards skewed distributions.
Depending on the dimension \( p \), different approximate algorithms are implemented. The affine invariant algorithm can only be used when \( n > p \). It draws \( ndir \) times at random \( p \) observations from \( x \) and considers the direction orthogonal to the hyperplane spanned by these \( p \) observations. At most \( p \) out of \( n \) directions can be considered. The orthogonal invariant version can be applied to high-dimensional data. It draws \( ndir \) times at random 2 observations from \( x \) and considers the direction through these two observations. Here, at most 2 out of \( n \) directions can be considered. Finally, the shift invariant version randomly draws \( ndir \) vectors from the unit sphere. The resulting AO values are invariant to affine transformations, rotations and shifts respectively provided that the seed is kept fixed at different runs of the algorithm. Note that the AO values are guaranteed to increase when more directions are considered provided the seed is kept fixed, as this ensures that the random directions are generated in a fixed order.

An observation from \( x \) and \( z \) is flagged as an outlier if its AO exceeds a cutoff value. This cutoff value is determined using the procedure in Rousseeuw et al. (2018). First, the logarithm of the AO values is taken to render their distribution more symmetric, after which a normal approximation yields a cutoff on these values. The cutoff is then transformed back by applying the exponential function.

It is first checked whether the data lie in a subspace of dimension smaller than \( p \). If so, a warning is given, as well as the dimension of the subspace and a direction which is orthogonal to it. Furthermore, the univariate adjusted outlyingness of the projected points \( xv \) is ill-defined when the scale in its denominator becomes zero. This can happen when many observations collapse. In these cases the algorithm will stop and give a warning. The returned values then include the direction \( v \) as well as an indicator specifying which of the observations of \( x \) belong to the hyperplane orthogonal to \( v \).

This function extends the adjOutlyingness function in the package robustbase. It has more options for choosing the directions, it allows to compute the adjusted outlyingness of points not belonging to the data matrix \( x \) and it is faster as it is fully implemented in C++. On the other hand, the constants (3 and -4) used in the definition of the adjusted outlyingness can not be modified in this implementation.

Value

A list with components:

- \texttt{outlyingnessX} Vector of length \( n \) giving the adjusted outlyingness of the observations in \( x \).
- \texttt{outlyingnessZ} Vector of length \( m \) giving the adjusted outlyingness of the points in \( z \) relative to \( x \).
- \texttt{cutoff} Points whose adjusted outlyingness exceeds this cutoff can be considered as outliers with respect to \( x \).
- \texttt{flagX} Observations of \( x \) whose adjusted outlyingness exceeds the cutoff receive a flag FALSE, regular observations receive a flag TRUE.
- \texttt{flagZ} Points of \( z \) whose adjusted outlyingness exceeds the cutoff receive a flag equal to FALSE, otherwise they receive a flag TRUE.
- \texttt{singularSubsets} When the input parameter type is equal to "Affine", the number of \( p \)-subsets that span a subspace of dimension smaller than \( p - 1 \). In such a case the orthogonal direction can not be uniquely determined. This is an indication that
the data are not in general position. When the input parameter type is equal to "Rotation" it is possible that two randomly selected points of the data coincide due to ties in the data. In such a case this value signals how many times this happens.

dimension
When the data \( x \) are lying in a lower dimensional subspace, the dimension of this subspace.

hyperplane
When the data \( x \) are lying in a lower dimensional subspace, a direction orthogonal to this subspace. When a direction \( v \) is found such that the robust skew-adjusted scale of \( xv \) is equal to zero, this equals \( v \).

inSubspace
When a direction \( v \) is found such that \( \text{AO}(xv) \) is ill-defined, the observations from \( x \) which belong to the hyperplane orthogonal to \( v \) receive a value TRUE. The other observations receive a value FALSE.

Author(s)

References


See Also
sprojdepth, sprojmedian, dirOutl, outlyingness
adjbox, adjOutlyingness from package robustbase.

Examples

```r
# Compute the adjusted outlyingness of a simple two-dimensional dataset. Outliers are plotted in red.
data("geological")
BivData <- geological[c("MnO","MgO")]
Result <- adjOutl(x = BivData)
IndOutliers <- which(!Result$flagX)
plot(BivData)
points(BivData[IndOutliers,], col = "red")

# The number of directions may be specified through the option list. The resulting adjusted outlyingness
```
# is monotone increasing in the number of directions.
Result1 <- adjOutl(x = BivData, 
                   options = list(ndir = 50))

Result2 <- adjOutl(x = BivData, 
                   options = list(ndir = 100))

which(Result2$outlyingnessX - Result1$outlyingnessX < 0)

# This is however not the case when the seed is changed
Result1 <- adjOutl(x = BivData, 
                   options = list(ndir = 50))

Result2 <- adjOutl(x = BivData, 
                   options = list(ndir = 100, seed = 950))

plot(Result2$outlyingnessX - Result1$outlyingnessX, 
      xlab = "Index", ylab = "Difference in AO")

# We can also consider directions through two data points. If the sample is small enough one may opt 
# to search over all choose(n,2) directions. 
# Note that the computational load increases dramatically 
# as n becomes larger.
data("bloodfat")
BivData <- bloodfat[1:100,] # Consider a small toy example.
Result <- adjOutl(x = BivData, 
                  options = list(type = "Rotation", 
                                 ndir = "all"))

IndOutliers <- which(!Result$flagX)
plot(BivData)
points(BivData[IndOutliers,], col = "red")

# Alternatively one may consider randomly generated directions.
data("bloodfat")
Result = adjOutl(x = bloodfat, 
                 options = list(type = "Shift", 
                                ndir = 1000))

IndOutliers <- which(!Result$flagX)
plot(bloodfat)
points(bloodfat[IndOutliers,], col = "red")

---

**bagdistance**

**Bagdistance of points relative to a dataset**

**Description**

Computes the bagdistance of \( p \)-dimensional points \( z \) relative to a \( p \)-dimensional dataset \( x \). To compute the bagdistance of a point \( z_i \), the bag of \( x \) which is defined as the depth region containing
the 50% observations with largest depth. Next, the ray from the halfspace median $\theta$ through $z_i$ is considered and $c_z$ is defined as the intersection of this ray and the boundary of the bag. The bagdistance of $z_i$ to $x$ is then given by the ratio between the Euclidean distance of $z_i$ to the halfspace median and the Euclidean distance of $c_z$ to the halfspace median.

**Usage**

```r
distanceHxL z [nullL options [listHII]
```

**Arguments**

- **x** An $n$ by $p$ data matrix.
- **z** An optional $m$ by $p$ matrix containing rowwise the points $z_i$ for which to compute the adjusted outlyingness. If $z$ is not specified, it is set equal to $x$. is not specified, it is set equal to $x$.
- **options** A list of available options:
  - **approx** In two dimension one may choose to use an approximate algorithm or the exact algorithm to find the bag. Defaults to `TRUE`.
  - **max.iter** The maximum number of steps in the bisection algorithm to find the intersection point $c_z$ (see Details). Defaults to 100.
  - **All options may be specified that are passed to the hdepth function, see hdepth for details. Note that the option parameter approx is by default set to `TRUE` to save computation time.**

**Details**

The bagdistance has been introduced in Hubert et al. (2015). It does not assume symmetry and is affine invariant. Note that when the halfspace is not computed in an affine invariant way, the bagdistance cannot be affine invariant either.

The function first computes the halfspace depth and the halfspace median of $x$. Additional options may be passed to the hdepth routine by specifying them in the option list argument.

It is first checked whether the data lie in a subspace of dimension smaller than $p$. If so, a warning is given, as well as the dimension of the subspace and a direction which is orthogonal to it.

Depending on the dimensions different algorithms are used. For $p = 1$ the bagdistance is computed exactly. For $p = 2$ the default setting (options$approx=TRUE) uses an approximated algorithm. Exact computation, based on the exact algorithm to compute the contours of the bag (see the depthContour function), is obtained by setting options$approx to `FALSE`. Note that this may lead to an increase in computation time.

For the approximated algorithm, the intersection point $c_z$ is approximated by searching on each ray the point whose depth is equal to the median of the depth values of $x$. As the halfspace depth is monotone decreasing along the ray, a bisection algorithm is used. Starting limits are obtained by projecting the data on the direction and considering the data point with univariate depth corresponding to the median of the halfspace depths of $x$. By definition the multivariate depth of this point has
to be lower or equal than its univariate depth. A second limit is obtained by considering the deepest location estimate. The maximum number of iterations bisecting the current search interval can be specified through the options argument `max.iter`.

An observation from \( z \) is flagged as an outlier if its bagdistance exceeds a cutoff value. This cutoff is equal to the squareroot of the 0.99 quantile of the chi-squared distribution with \( p \) degrees of freedom.

**Value**

A list with components:

- **bagdistance**: The bagdistance of the points of \( z \) with respect to the data matrix \( x \).
- **cutoff**: Points of \( z \) whose bagdistance exceeds this cutoff can be considered as outliers with respect to \( x \).
- **flag**: Points of \( z \) whose bagdistance exceeds the cutoff receive a flag equal to `FALSE`, otherwise they receive a flag `TRUE`.
- **converged**: Vector of length \( m \) indicating for each point of \( z \) whether the bisection algorithm converged within the maximum number of steps specified by `max.iter` in the options list.
- **dimension**: When the data \( x \) are lying in a lower dimensional subspace, the dimension of this subspace.
- **hyperplane**: When the data \( x \) are lying in a lower dimensional subspace, a direction orthogonal to this subspace.

**Author(s)**

P. Segaert.

**References**


**See Also**

- `depthContour`, `hdepth`, `bagplot`

**Examples**

```r
# Generate some bivariate data
nObs <- 500
N <- matrix(rnorm(nObs * 2), nrow = nObs, ncol = 2)
A <- matrix(c(1, 1, 5, 1), ncol = 2, nrow = 2)
X <- N
```
# In two dimensions we may either use the approximate
# or exact algorithm to compute the bag.
res.exact <- bagdistance(x = X, options = list(approx = FALSE))
res.approx <- bagdistance(x = X, options = list(approx = TRUE))
# The approximate algorithm leads to a good approximation.
plot(res.exact$bagdistance, res.approx$bagdistance)
abline(a = 0, b = 1)

# In Hubert et al. (2015) it was shown that for elliptical
# distributions the bagdistance^R relates to the Mahalanobis
# distances. This may easily be illustrated.
mahDist <- mahalanobis(x = X, colMeans(X), cov(X))
plot(res.exact$bagdistance^2, mahDist)

# Computation for the bagdistance relies on the calculation
# of halfspace depth using the hdepth function. Options for
# the hdepth routine can be passed down using the options
# arguments. Note that the affine invariance of the bagdistance
# depends on the affine invariant calculation of the halfspace
# depth. Choosing a different type for hdepth may lead to
# unsatisfying results.
options <- list(type = "Rotation",
                 ndir = 375,
                 approx = TRUE,
                 seed = 78341)
res.exact <- bagdistance(x = X, options = options)

---

**bagplot**

*Draws a bagplot, a bivariate boxplot*

**Description**

This function draws a bagplot of bivariate data, based on the result of a call to `compBagplot`. The bagplot is a generalisation of the univariate boxplot to bivariate data. It aims to visualize the location, spread, skewness and outliers of the data set.

**Usage**

```r
bagplot(compbag.result,
        colorbag = NULL, colorloop = NULL, colorchull = NULL,
        databag = TRUE, dataloop = TRUE, plot.fence = FALSE)
```

**Arguments**

- `compbag.result`  The return of a call to `compBagplot`.
- `colorbag`  The color of the bag (which contains the 50% observations with largest depth).
- `colorloop`  The color of the loop (which contains the regular observations).
When the bagplot is based on halfspace depth, the depth region with maximal depth is plotted. This argument controls its color.

databag Logical indicating whether data points inside the bag need to be plotted. Defaults to TRUE.

dataloop Logical indicating whether data points inside the fence need to be plotted. Defaults to TRUE.

plot.fence Logical indicating whether the fence should be plotted. Defaults to FALSE.

Details

The bagplot has been proposed by Rousseeuw et al. (1999) as a generalisation of the boxplot to bivariate data. It is constructed based on halfspace depth and as such is invariant under affine transformations. Similar graphical representations can be obtained by means of other depth functions, as illustrated in Hubert and Van der Veeken (2008) and in Hubert et al. (2015). See compBagplot for more details.

The deepest point is indicated with a "+" sign, the outlying observations with red stars.

The plot is made using ggplot2. The plot itself is returned by the function and is fully customisable using standard ggplot2 commands.

Author(s)

P. Segaert

References


See Also

compBagplot, hdepth, projdepth, sprojdepth.

Examples

data(bloodfat)

# The bagplot can be plotted for the halfspace depth,
# the projection depth or the skewness-adjusted projection depth.
# Note that the projection depth is not appropriate for skewed data.
bagplot(compBagplot(bloodfat))
bagplot(compBagplot(bloodfat, type = "projdepth"))
bagplot(compBagplot(bloodfat, type = "sprojdepth"))
bloodfat

# The mean features of the bagplot can easily be adjusted.
result <- compBagplot(bloodfat)
bagplot(result, databag = FALSE, dataloop = FALSE)
bagplot(result, colorbag = rgb(0.2,0.2,0.2), colorloop = "green")

data(cardata90)
result <- compBagplot(cardata90)
bagplot(result)

# Compared to the original paper on the bagplot,
# an additional outlier is identified. However this
# point lies very close to the fence and this may be
# attributed to differences in numerical rounding.
# This may be illustrated by plotting the fence.
plot <- bagplot(result, plot.fence = TRUE)
plot

# The returned object is a ggplot2 object and may be
# edited using standard ggplot2 commands.
library("ggplot2")
plot + ylab("Engine displacement") + xlab("Weight in pounds")

bloodfat  Blood data for patients with narrowing arteries

Description

Data were collected on the concentration of plasma cholesterol and plasma triglycerides (mg/dl) for 371 male patients evaluated for chest pain. For 51 of those patients, no evidence of heart disease was found. This subset corresponds to the remaining 320 patients for which there was evidence of narrowing arteries.

Usage

data(bloodfat)

Format

A dataframe containing following variables:

Cholesterol  Concentration of plasma cholesterol [mg/dl].
Triglycerides  Concentration of plasma triglycerides [mg/dl].

Source

References


Examples

data(bloodfat)
plot(bloodfat)

cardata90

*Car data from Consumer Reports in 1990*

Description


Usage

data(cardata90)

Format

A dataframe containing following variables:

- **Weight**  Weight of the car.
- **Disp**  Engine displacement in cubic inches.

Source


References


Examples

data(cardata90)
plot(cardata90)
Writing trajectories of the letter ‘a’.

Description

Subset of the ‘Character Trajectories Data Set’ from the UCI Machine Learning Repository. The data set consists of trajectories of the tip of a pen whilst writing the letter ‘a’. All samples are from the same writer. Original data has been processed.

Usage

data(characterA)

Format

Three dimensional array. The first dimension represents time. The second dimension corresponds to the observation number. The third dimensions contains the X and Y coordinates.

Source


References


Examples

data(characterA)
par(mfrow = c(1,2))
matplot(y = characterA[,,1],
     type ="l",col = "black", lty = 1, xlab = "Time", ylab="X position of the pen")
matplot(y = characterA[,,2],
     type ="l",col = "black", lty = 1, xlab = "Time", ylab="Y position of the pen")
par(mfrow = c(1,1))
characterI

Writing trajectories of the letter i.

Description

Subset of the 'Character Trajectories Data Set' from the UCI Machine Learning Repository. The data set consists of trajectories of the tip of a pen whilst writing the letter 'i'. All samples are from the same writer. Original data has been processed.

Usage

data(characterI)

Format

Three dimensional array. The first dimension represents time. The second dimension corresponds to the observation number. The third dimensions contain the X and Y coordinates.

Source


References


Examples

data(characterI)
par(mfrow = c(1,2))
matplot(y = characterI[,,1],
       type ="l",col = "black", lty = 1, xlab = "Time", ylab="X position of the pen")
matplot(y = characterI[,,2],
       type ="l",col = "black", lty = 1, xlab = "Time", ylab="Y position of the pen")
par(mfrow = c(1,1))
cmltest

Test for linearity of the conditional median in simple regression

Description
A test based on regression depth for linearity of the conditional median z given the simple regression dataset x. The calculation of the regression depth of z with respect to x is done by the function rdepth. The test is only valid when x contains no duplicates.

Usage
cmltest(x, z)

Arguments
- x: An n by 2 regression data matrix. The first column is the explanatory variable, the second column corresponds to the response variable.
- z: A parameter vector (intercept and slope).

Details
The following hypothesis test is performed:

\[ H_0: \text{The data come from a model with: } \text{med}(x_2|x_1) = z_1 + z_2 \times x_1 \]

The test statistic being used is the regression depth of z with respect to x.

Value
- pval: The p-value of the hypothesis test.

Author(s)
P. Segaert

References

See Also
- rdepth, rdepthmedian
Examples

data("stars")

# Compute the least squares fit. Due to outliers
# this fit will be bad and thus H0 should be rejected.
temp <- lsfit(x = stars[,1], y = stars[,2])$coefficients
intercept <- temp[1]
slope <- temp[2]
z <- matrix(c(intercept, slope), nrow = 1)
pvalue1 <- cmltest(x = stars, z = z)
pvalue1

# Lets now test the deepest regression line.
result <- rdepthmedian(x = stars)
pvalue2 <- cmltest(x = stars, z = matrix(result$deepest, nrow = 1))
pvalue2

plot(stars)
abline(a = intercept, b = slope)
abline(result$deepest, col = "red")
text(x = 3.8, y = 5.3, labels = paste("p-value", round(pvalue1, digits = 3)) )
text(x = 4.45, y = 4.8, labels = paste("p-value", round(pvalue2, digits = 3)), col = "red")

compBagplot

Location estimate based on the halfspace depth

Description

Computes all elements of the bagplot, a generalisation of the univariate boxplot to bivariate data. The bagplot can be computed based on halfspace depth, projection depth or skewness-adjusted projection depth. To make the actual plot, the function bagplot needs to be called on the result of compBagplot.

Usage

compBagplot(x, type = "hdepth", sizesubset = 500,
extra.directions = FALSE, options = NULL)

Arguments

x
An n by 2 data matrix.

type
Determines the depth function used to construct the bagplot: "hdepth" for halfspace depth, "projdepth" for projection depth and "sprojdepth" for skewness-adjusted projection depth.
Defaults to "hdepth".
When computing the bagplot based on halfspace depth, the size of the subset used to perform the main computations. See Details for more information. Defaults to 500.

Logical indicating whether additional directions should be considered in the computation of the fence for the bagplot based on projection depth or skewness-adjusted projection depth. If set to TRUE an additional 250 equispaced directions are added to the directions defined by the points in x themselves and the center. If FALSE only directions determined by the points in x are considered. Defaults to FALSE.

A list of options to pass to the projdepth or sprojdepth function. In addition the following option may be specified:

- max.iter
  The maximum number of iterations in the bisection algorithm used to compute the depth contour corresponding to the cutoff. See depthContour for more information. Defaults to 100.

The bagplot has been proposed by Rousseeuw et al. (1999) as a generalisation of the boxplot to bivariate data. It is constructed based on halfspace depth. In the original format the deepest point is indicated by a "+" and is contained in the bag which is defined as the depth region containing the 50% observations with largest depth. The fence is obtained by inflating the bag (relative to the deepest point) by a factor of three. The loop is the convex hull of the observations of x inside the fence. Observations outside the fence are flagged as outliers and plotted with a red star. This function only computes all the components constituting the bagplot. The bagplot itself can be drawn using the bagplot function.

The bagplot may also be defined using other depth functions. When using projection depth or skewness-adjusted projection depth the bagplot is build as follows. The center corresponds to the observation with largest depth. The bag is constructed as the convex hull of the fifty percent points with largest depth. Outliers are identified as points with a depth smaller than a cutoff value, see projdepth and sprojdepth for the precise definition. The loop is computed as the convex hull of the non-outlying points. The fence is approximated by the convex hull of those points that lie on rays from the center through the vertices of the bag and have a depth that equals the cutoff depth. For a better approximation the user can set the input parameter extradirections to TRUE such that an additional 250 equally spaced directions on the circle are considered.

The computation of the bagplot based on halfspace depth can be time consuming. Therefore it is possible to limit the bulk of the computations to a random subset of the data. Computations of the halfspace median and the bag are then based on this random subset. The number of points in this subset can be controlled by the optional argument sizesubset.

It is first checked whether the data is found to lie on a line. If so, the routine will give a warning, giving back the dimension of the subspace (being 1) together with the normal vector to that line.

A list with components:
center
Center of the data.
When type = "hdepth", this corresponds with the Tukey median. In other cases this point corresponds to the point with maximal depth.

chull
When type = "hdepth", these are the vertices of the region with maximal halfspace depth. In other cases this is a null vector.

bag
The coordinates of the vertices of the bag.

fence
The coordinates of the vertices of the fence.

datatype
An (nx3) matrix. The first two columns correspond with x. The third column indicates the position of each observation of x in the bagplot: 2 for observations in the bag, 1 for the observations in the fence and 3 for outliers. Note that points may not be in the same order as in x.

flag
A vector of length n which is 0 for outliers and 1 for regular observations of x.

depth
The depth of the observations of x.

dimension
If the data are lying in a lower dimensional subspace, the dimension of this subspace.

hyperplane
If the data are lying in a lower dimensional subspace, a direction orthogonal to this subspace.

type
Same as the input parameter type.

Author(s)
P. Segaert based on Fortran code by P.J. Rousseeuw, I. Ruts and A. Struyf.

References


See Also
bagplot, hdepth, projdepth, sprojdepth.

Examples
data(bloodfat)
Result <- compBagplot(bloodfat)
bagplot(Result)

# The sizesubset argument may be used to control the
# computation time when computing the bagplot based on
# halfspace depth. However results may be unreliable when
# choosing a small subset for the main computations.
system.time(Result1 <- compBagplot(bloodfat))
system.time(Result2 <- compBagplot(bloodfat, sizesubset = 100))
bagplot(Result1)

# When using the projection depth or skewness-adjusted
# projection depth to compute the bagplot a list of options
# may be passed down to the (adj)outlyingness routines.
options <- list(type = "Rotation",
                 ndir = 50,
                 stand = "unimcd",
                 h = floor(nrow(bloodfat)*3/4))

Result <- compBagplot(bloodfat,
                      type = "projdepth", options = options)

# The fence is computed using the depthContour function.
# To get a smoother fence, one may opt to consider extra
# directions.
options <- list(ndir = 500,
                seed = 36)

Result <- compBagplot(bloodfat,
                      type = "sprojdepth", options = options)

options <- list(ndir = 500,
                seed = 36)

Result <- compBagplot(bloodfat,
                      type = "sprojdepth", options = options,
                      extra.directions = TRUE)


---

### depthContour

**Depth contours of multivariate data**

**Description**

Computes the vertices of depth contours of multivariate data. The contours can be computed based on halfspace depth, projection depth or skewness-adjusted projection depth. To make the actual plot for bivariate data, the function `plotContours` needs to be called on the result of `depthContour`.

**Usage**

```
depthContour(x, alpha = NULL, type = "hdepth", directions = NULL, options = NULL)
```

**Arguments**

- **x**
  - An \(n \times p\) data matrix.
- **alpha**
  - A vector containing the depth values of which the depth contours have to be computed.
type

The depth used in the computation of the contours. Available options are "hdepth" for halfspace depth, "projdepth" for projection depth and "sprojdepth" for skewness-adjusted projection depth.

directions

An m by p matrix specifying the directions on which to compute the vertices (see Details).

options

A list of options to pass to hdepth, projdepth or sprojdepth. In addition the following option may be specified:

• max.iter
  The maximum number of iterations in the bisection algorithm used to compute the depth contour corresponding to level alpha.
  Defaults to 100.

Details

Depth contours of level $\alpha$ (or $\alpha$-depth contours) are the boundaries of depth regions. Depth regions of level alpha are defined as regions in space containing the multivariate points whose depth value is at least $\alpha$.

For bivariate data halfspace depth contours can be computed exactly following the algorithm in Ruts and Rousseeuw (1996). When the data are not in general position (i.e. when there is a line containing more than two observations) dithering is performed by adding random Gaussian noise to the data.

In all other cases an approximated method is performed using a bisection algorithm. Intersections with the depth contours are searched on lines originating from the depth median. The user can specify a set of directions corresponding to these lines. By defaults a random set of $250*p$ directions is considered. On each direction a point is searched having depth $\alpha$. Starting limits are obtained by projecting the data on the direction and considering the data point with univariate depth $\alpha$. By definition the multivariate depth of this point has to be lower or equal to $\alpha$. A second limit is obtained by considering the deepest location estimate. The maximum number of iterations bisecting the current search interval can be specified through the options argument max.iter. Note that this method is only affine or rotation equivariant if the chosen directions are affine or rotation invariant.

It is first checked whether the data is found to lie in a subspace of dimension lower than $p$. If so, the routine will give a warning, giving back the dimension of the subspace together with a direction describing a hyperplane containing this subspace.

Value

The output consists of a list. Each element of the list contains the following elements for each value $\alpha$ specified in the argument alpha.

depth

The depth of the depth contour of level $\alpha$. For halfspace depth this is equal to \(\text{floor}(\alpha n)/n\). For projection depth and skewness-adjusted projection depth this equals to $\alpha$.

vertices

The coordinates of the vertices of the depth contour.

empty

Logical indicating whether the corresponding depth region is empty. FALSE indicates the depth region is non-empty. TRUE indicates the depth region is empty.
dithered Logical indicating whether dithering has been applied in the exact bivariate algorithm based on halfspace depth. FALSE indicates no dithering has been applied. TRUE indicates dithering has been applied.

converged A vector of length m containing a flag indicating for each direction whether convergence was reached by the bisecting algorithm within the allowed max.iter number of steps.

type Same as input parameter type

dimension If the data are lying in a lower dimensional subspace, the dimension of this subspace.

hyperplane If the data are lying in a lower dimensional subspace, a direction orthogonal to this subspace.

Author(s)

P. Segaert based on Fortran code by P.J. Rousseeuw, I. Ruts and A. Struyf

References


See Also

plotcontours, bagdistance

Examples

# Compute the depth contours of a simple # two-dimensional dataset.
> data(cardata90)
> Result <- depthcontour(x = cardata90,
> alpha = c(0.125,0.25),
> type = "hddepth")
> plotcontours(x = cardata90, depthContour = Result)

# One may consider different depths such as # e.g. projection depth by changing the type.
> Result <- depthContour(x = cardata90,
> alpha = c(0.125,0.25),
> type = "projdepth")
> plotContours(x = cardata90, depthContour = Result)
# When there is skewness in the data projection depth # is less appropriate.

# For projection based types the directions to # search intersections may be specified by the user. # It is advised to consider enough to directions to have # a good image of the depth contours.
> directions <- matrix(c(0,1,
Computes the directional outlyingness of $p$-dimensional points $z$ relative to a $p$-dimensional dataset $x$. For each multivariate point $z_i$, its directional outlyingness relative to $x$ is defined as its maximal
univariate directional outlyingness measured over all directions. To obtain the univariate directional outlyingness in the direction \( v \), the dataset \( x \) is projected on \( v \), and the robustly skew-adjusted standardized distance of \( v'z_i \) to the median of the projected data points \( xv \) is computed. This is done through the estimation of 2 scales, one on each side of the median, using a 1-step M-estimator of scale.

Usage

\[ \text{dirOutl}(x, z = \text{NULL}, \text{options} = \text{list()}) \]

Arguments

- **x**
  An \( n \) by \( p \) data matrix.

- **z**
  An optional \( m \) by \( p \) matrix containing rowwise the points \( z_i \) for which to compute the adjusted outlyingness. If \( z \) is not specified, it is set equal to \( x \).

- **options**
  A list of available options:
  - **type**
    Determines the desired type of invariance and should be one of "Affine", "compWise". When the option "Affine" is used, the directions \( v \) are orthogonal to hyperplanes spanned by \( p \) observations from \( x \). With the option "compWise", the directional outlyingness is computed in the directions of the coordinate axes and combined through the Euclidean norm. Defaults to "Affine".
  - **ndir**
    When type is chosen to be "Affine", determines the number of directions \( v \) by setting ndir to a specific number. Defaults to \( 250p \).
  - **seed**
    A strictly positive integer specifying the seed to be used to select the directions.
    Defaults to 10.

Details

The directional outlyingness (DO) of multivariate data was introduced in Rousseeuw et al. (2018). It extends the Stahel-Donoho outlyingness towards skewed distributions.

Depending on the dimension \( p \), different approximate algorithms are implemented. The affine invariant algorithm can only be used when \( n > p \). It draws \( ndir \) times at random \( p \) observations from \( x \) and considers the direction orthogonal to the hyperplane spanned by these \( p \) observations. At most \( p \) out of \( n \) directions can be considered. The orthogonal invariant version can be applied to high-dimensional data. It draws \( ndir \) times at random 2 observations from \( x \) and considers the direction through these two observations. Here, at most 2 out of \( n \) directions can be considered. Finally, the shift invariant version randomly draws \( ndir \) vectors from the unit sphere.

The resulting DO values are invariant to affine transformations, rotations and shifts respectively provided that the seed is kept fixed at different runs of the algorithm. Note that the DO values are guaranteed to increase when more directions are considered provided the seed is kept fixed, as this ensures that the random directions are generated in a fixed order.
An observation from \( x \) and \( z \) is flagged as an outlier if its DO exceeds a cutoff value. This cutoff value is determined using the procedure in Rousseeuw et al. (2018). First, the logarithm of the DO values is taken to render their distribution more symmetric, after which a normal approximation yields a cutoff on these values. The cutoff is then transformed back by applying the exponential function.

It is first checked whether the data lie in a subspace of dimension smaller than \( p \). If so, a warning is given, as well as the dimension of the subspace and a direction which is orthogonal to it. Furthermore, the univariate adjusted outlyingness of the projected points \( xv \) is ill-defined when the scale in its denominator becomes zero. This can happen when many observations collapse. In these cases the algorithm will stop and give a warning. The returned values then include the direction \( v \) as well as an indicator specifying which of the observations of \( x \) belong to the hyperplane orthogonal to \( v \).

**Value**

A list with components:

- \texttt{outlyingnessX} Vector of length \( n \) giving the directional outlyingness of the observations in \( x \).
- \texttt{outlyingnessZ} Vector of length \( m \) giving the directional outlyingness of the points in \( z \) relative to \( x \).
- \texttt{cutoff} Points whose directional outlyingness exceeds this cutoff can be considered as outliers with respect to \( x \).
- \texttt{flagX} Observations of \( x \) whose directional outlyingness exceeds the cutoff receive a flag \texttt{FALSE}, regular observations receive a flag \texttt{TRUE}.
- \texttt{flagZ} Points of \( z \) whose directional outlyingness exceeds the cutoff receive a flag equal to \texttt{FALSE}, otherwise they receive a flag \texttt{TRUE}.
- \texttt{singularSubsets} When the input parameter type is equal to "Affine", the number of \( p \)-subsets that span a subspace of dimension smaller than \( p - 1 \). In such a case the orthogonal direction cannot be uniquely determined. This is an indication that the data are not in general position.
- \texttt{dimension} When the data \( x \) are lying in a lower dimensional subspace, the dimension of this subspace.
- \texttt{hyperplane} When the data \( x \) are lying in a lower dimensional subspace, a direction orthogonal to this subspace. When a direction \( v \) is found such that the robust skew-adjusted scale of \( xv \) is equal to zero, this equals \( v \).
- \texttt{inSubspace} When a direction \( v \) is found such that \( \text{DO}(xv) \) is ill-defined, the observations from \( x \) which belong to the hyperplane orthogonal to \( v \) receive a value \texttt{TRUE}. The other observations receive a value \texttt{FALSE}.

**Author(s)**

J. Raymaekers and P. Rousseeuw
distSpace

References


See Also
dprojdepth, dprojmedian, adjOutl, outlyingness

Examples

```r
# Compute the directional outlyingness of a simple two-dimensional dataset. Outliers are plotted # in red.
data("geological")
BivData <- geological[c("MnO","MgO")]
Result <- dirOutl(x = BivData)
IndOutliers <- which(!Result$flagX)
plot(BivData)
points(BivData[IndOutliers,], col = "red")

# The number of directions may be specified through # the option list. The resulting adjusted outlyingness # is monotone increasing in the number of directions.
Result1 <- dirOutl(x = BivData,options = list(ndir = 50))
Result2 <- dirOutl(x = BivData,options = list(ndir = 100))
which(Result2$outlyingnessX - Result1$outlyingnessX < 0)
# This is however not the case when the seed is changed
Result1 <- dirOutl(x = BivData,options = list(ndir = 50))
Result2 <- dirOutl(x = BivData,options = list(ndir = 100,seed = 950))

plot(Result2$outlyingnessX - Result1$outlyingnessX, xlab = "Index", ylab = "Difference in DO")

# Consider another example:
data("bloodfat")
BivData <- bloodfat[1:100,] # Consider a small toy example.
Result <- dirOutl(x = BivData,options = list(type = "Affine"))
IndOutliers <- which(!Result$flagX)
plot(BivData)
points(BivData[IndOutliers,], col = "red")
```

distSpace

Description

Calculation of distance space representation.
Usage

distSpace(trainingData, testData = NULL, type = "bagdistance", options = NULL)

Arguments

trainingData A list of nxp matrices containing the multivariate data or a list of t by n by p arrays containing the functional data.

testData An mxp matrix containing all multivariate training data or a t by m by p array for functional data.

type The distance used in the computations. For multivariate data one of the following options: "bagdistance", "outlyingness", or "adjOutl". For functional data one of the following options: "fBD", "fSDO" or "fAO" Defaults to "bagdistance".

options A list of options to pass to the function calculating the underlying distance. See "bagdistance", "outlyingness", "adjOutl" or fOutl for more information.

Details

The distance is a tool in supervised classification and was introduced in Hubert et al. (2016) as a generalisation of the depth-depth representation of a multivariate sample. Based on a distance transform, an observation (be it multivariate or functional) is mapped to its representation in distance space. The distance transformation consists of mapping the observation to a vector containing at coordinate i the distance to the training group i. After transformation, any multivariate classifier may be used to classify new observations in distance space. Typically the k-nearest neighbour algorithm is used.

Different options are available to calculate the distance to each of the training groups. For multivariate data, the user may choose between the bagdistance or any of the projection type distance including the Stahel-Donoho outlyingness, the adjusted outlyingness or the directional outlyingness. For functional data, the user may opt to employ the functional bagdistance (fBD), the functional Stahel-Donoho (fSDO) the functional adjusted-outlyingness (fAO) or the functional directional outlyingness (fDO). Options to available in each of the underlying distance routines may be passed down using the options argument.

Value

A matrix q x (p+1) composed of two blocks. The first block contains in each row an observations in the training set with in each column the distance to each of the training sets. The last column contains a label indicating the original group membership of the observation. The second block contains the observations in the test set, if any, with in each column the distance to the different training sets. The last column now contains an inddicator signalling the observations was part of the test set.

Author(s)

P. Segaert
References


Examples

```r
# We will use two multivariate toy data sets
data(cardata90)
data(bloodfat)

# Build the training data
trainingData <- list(set1 = cardata90, set2 = bloodfat)
# Transform the data into distsace
Result <- distSpace(trainingData = trainingData)
# Plot the results
plotColors <- c(rep("orange", nrow(cardata90)), rep("blue", nrow(bloodfat)))
plot(Result[, 1:2], col = plotColors, xlab = "distance to cardata90", ylab = "distance to bloodfat", main = "distsace representation of cardata90 and the bloodfat data.")

# By default the bagdistance is used to transform the data.
# This can be changed by using the type argument. Additional option to be
# passed to the underlying function calculating the distance may be passed in
# the option argument.
options <- list(type = "Affine", ndir = 1000, seed = 3)
Result <- distSpace(trainingData = trainingData, type = "adjOut1", options = options)
# Plot the results
plotColors <- c(rep("orange", nrow(cardata90)), rep("blue", nrow(bloodfat)))
plot(Result[, 1:2], col = plotColors, xlab = "distance to cardata90", ylab = "distance to bloodfat", main = "distsace representation of cardata90 and the bloodfat data.")

data(octane)
data(glass)
trainingData <- list(set1 = glass[1:100,, drop = FALSE], set2 = octane[1:100,, drop = FALSE])
# Transform the data into distsace
Result <- distSpace(trainingData = trainingData, type = "fAO")
```
**dprojdepth**  
*Directional projection depth of points relative to a dataset*

**Description**
Computes the directional projection depth of \( p \)-dimensional points \( z \) relative to a \( p \)-dimensional dataset \( x \).

**Usage**
\[
dprojdepth(x, z = \text{NULL}, \text{options} = \text{NULL})
\]

**Arguments**
- **\( x \)**: An \( n \) by \( p \) data matrix with observations in the rows and variables in the columns.
- **\( z \)**: An optional \( m \) by \( p \) matrix containing rowwise the points \( z_i \) for which to compute the projection depth. If \( z \) is not specified, it is set equal to \( x \).
- **options**: A list of options to pass to the underlying \textit{diroutl} routine. See \textit{diroutl} for the full list of options.

**Details**
Directional projection depth is based on the directional outlyingness and is computed as \( 1/(1 + DO) \). As directional outlyingness extends the Stahel-Donoho outlyingness towards skewed distributions, the directional projection depth is suited for both elliptical distributions and skewed multivariate data.

It is first checked whether the data is found to lie in a subspace of dimension lower than \( p \). If so, a warning is given, as well as the dimension of the subspace and a direction which is orthogonal to it. See \textit{diroutl} for more details on the computation of the DO. To visualize the depth of bivariate data one can apply the \textit{mrainbowplot} function. It plots the data colored according to their depth.

The output values of this function are based on the output of the \textit{diroutl} function. More details can be found there.

**Value**
A list with components:

- **\texttt{depthX}**: Vector of length \( n \) giving the directional projection depth of the observations in \( x \).
- **\texttt{depthZ}**: Vector of length \( m \) giving the directional projection depth of the points in \( z \).
- **\texttt{cutoff}**: Points whose directional projection depth is smaller than this cutoff can be considered as outliers.
- **\texttt{flagX}**: Observations of \( x \) whose directional outlyingness exceeds the cutoff receive a flag \texttt{FALSE}, regular observations receive a flag \texttt{TRUE}. 
flagZ Points of z whose directional outlyingness exceeds the cutoff receive a flag equal to FALSE, otherwise they receive a flag TRUE.

singularSubsets When the input parameter type is equal to "Affine", the number of $p$-subsets that span a subspace of dimension smaller than $p - 1$. In such a case the orthogonal direction can not be uniquely determined. This is an indication that the data are not in general position.

dimension When the data $x$ are lying in a lower dimensional subspace, the dimension of this subspace.

hyperplane When the data $x$ are lying in a lower dimensional subspace, a direction orthogonal to this subspace. When a direction $v$ is found such that the robust directional scale of $xv$ is equal to zero, this equals $v$.

inSubspace When a direction $v$ is found such that $DO(xv)$ is ill-defined, the observations from $x$ which belong to the hyperplane orthogonal to $v$ receive a value TRUE. The other observations receive a value FALSE.

Author(s)
J. Raymaekers

References

See Also
dirOutl, dprojmedian, mrainbowplot, adjOutl, outlyingness

Examples

```R
# Compute the directional projection depth
# of a simple two-dimensional dataset.
# Outliers are plotted in red.

data(bloodfat)
Result <- dprojdepth(x = bloodfat)
IndOutliers <- which(!Result$flagX)
plot(bloodfat)
points(bloodfat[IndOutliers,], col = "red")

# A multivariate rainbowplot may be obtained using mrainbowplot.
plot.options = list(legend.title = "DPD")
mrainbowplot(x = bloodfat,
            depths = Result$depthX, plot.options = plot.options)

# Options for the underlying outlyingness routine may be passed
# using the options argument.
Result <- dprojdepth(x = bloodfat,options = list(type = "Affine",ndir=100))
```
dprojmedian

Location estimates based on directional projection depth.

Description

Computes a directional projection depth based location estimate of a $p$-dimensional dataset $x$.

Usage

dprojmedian(x, dprojection.depths = NULL, options = NULL)

Arguments

- **x**: An $n$ by $p$ data matrix with observations in the rows and variables in the columns.
- **dprojection.depths**: Vector containing the directional projection depths of the points in $x$.
- **options**: A list of options to pass to the dprojdepth routine. See dprojdepth for more details.

Details

The algorithm depends on the function dprojdepth to calculate the directional projection depths of the dataset $x$. If the directional projection depths of the points $x_i$ have already been calculated they can be passed as an optional argument to save computation time. If not, directional projection depths will be calculated and the user can pass a list with options to the dprojdepth function.

It is first checked whether the data lie in a subspace of dimension smaller than $p$. If so, a warning is given, as well as the dimension of the subspace and a direction which is orthogonal to it.

Value

A list with components:

- **max**: The point of $x$ with maximal directional projection depth. If multiple points have maximum depth, their center of gravity is returned.
- **gravity**: The center of gravity for the 50 percent points with highest directional projection depth. Also called the center of gravity of the bag.

Author(s)

J. Raymaekers
fheatmap

fheatmap

References


See Also
dirOutl, dprojdepth, adjOutl, outlyingness

Examples

# Compute a location estimate of a simple two-dimensional dataset.
data(bloodfat)
result <- dprojmedian(x = bloodfat)
plot(bloodfat)
points(result$max, col = "red", pch = 15)
points(result$gravity, col = "blue", pch = 16)

# Options for the underlying sprojdepth routine may be passed
# using the options argument.
result <- dprojmedian(x = bloodfat, options = list(type = "Affine", ndir = 10))
plot(bloodfat)
points(result$max, col = "red", pch = 15)
points(result$gravity, col = "blue", pch = 16)

# One may also calculate the depths of the points in the data
# separately. This avoids having to recompute the depths when these
# are previously calculated.
depth.result <- dprojdepth(x = bloodfat)
result <- dprojmedian(x = bloodfat, dprojection.depths = depth.result$depthX)

fheatmap

Create MFOD heatmap

Description

Creates the heatmap in paper either for depth or for distances.

Usage

fheatmap(rowValues, cellValues, type, scalename = "")
Arguments

rowValues a cellValues a
type One of "depth" or "distance". Determines whether a depth or a distance heatmap is made.
scalename a

Details

fg

Author(s)

P. Segaert

References

Hubert M., Rousseeuw P.J., Segaert P. (2015). Multivariate functional outlier detection (with re-

Examples

library("ggplot2")
data(octane)
Result <- mfd(octane, diagnostic = TRUE, type = "sprojdepth")
Plot <- fheatmap(rowValues = Result$MFdepthZ,
cellValues = Result$crossdepthZ,
type = "depth",
scalename = "SPD")

Result <- foutl(octane, diagnostic = TRUE, type = "fAO")
Plot <- fheatmap(rowValues = Result$fOutlyingnessZ,
cellValues = Result$crossDistsZ,
type = "distance",
scalename = "AO")

#Customize the look of the plot
#Plot <- Plot + xlab("time point")
#Plot

---

fom Draws the Functional Outlier Map (FOM)

Description

Creates the Functional Outlier Map, a graphical tool to detect outliers in multivariate functional
data. Depending on the position of a multivariate curve in the FOM, different types of outliers may
be distinguished.
Usage

```r
fom(fOutlResult, cutoff = FALSE)
```

Arguments

- `fOutlResult`: The return of a call to `fOutl`
- `cutoff`: Boolean indicating whether the cutoff should be drawn.

Details

The `fom` is only applicable when `fOutl` is called with the following options: `diagnostic = TRUE` and `type` equaling either `fAO` or `fDO`.

The functional outlier map was proposed by Hubert et al. (2015) and subsequently improved by Rousseeuw et al. (2018). It consists of graphical tool to detect outliers in multivariate functional data based on functional outlyingness measures such as the `fAO` or `fDO` (see `fOutl`).

The coordinates of the points in the FOM correspond to two outlyingness indicators. On the horizontal axis, the functional outlyingness as obtained by the routine `fOutl` is plotted. On the vertical axis the scaled standard deviation of the cross-sectional outlyingness measures across time are plotted. The FOM thus consists of the following points representing the curve `i`: `(fOutl_i; std_j(fOutl_i(t_j))/(1+fOutl_i))`. The scaling of the standard deviation is added to ensure that curves with a different location, measured from the center, but with the same relative variability have a similar `y`-coordinate.

For some underlying multivariate outlyingness measures, the return of `fOutl` contains an additional argument signalling whether curve `i` is outlying as measured in the multivariate space at time point `j`. This information is incorporated in the FOM by plotting the points with a different degree of outlyingness with a different symbol. Curves that are outlying in the multivariate space determined by at least 75% of the total number of time points in the domain are plotted using filled diamonds. Similary curves that are outlying in at least 50% or 25% of the time points are plotted in filled squares and filled triangles respectively. Curves that are outlyingn in fewer than 25% of the time points plotted using filled circles.

The user may opt to draw a cut off line for the detection of outliers on the FOM. This cutoff was introduced in Rousseeuw et al. (2018) and is based on the euclidean distance between the points on the FOM and the origin, after scaling with the median. More specifically, let \( (fOutl_i; vOutl_i) = (fOutl_i; std_j(fOutl_i(t_j))/(1+fOutl_i)) \) and let \( cOutl_i^2 = (fOutl_i/\text{median}(fOutl))^2+(vOutl_i/\text{median}(vOutl))^2 \). Finally, with \( lcOutl_i = \log(0.1 + cOutl_i) \), an observation lies outside of the cutoff when \( (lcOutl_i - \text{median}(lcOutl))/\text{mad}(lcOutl) > \Phi^{-1}(0.995) \).

This FOM may be read in a way similar to the outlier map of robust regression (Rousseeuw and van Zomeren 1990) and the outlier map of robust principal components (Hubert et al. 2005). Points in the lower left part of the FOM represent regular curves which hold a central position in the dataset. Points in the lower right part are curves with a high `fOutl` but a low variability of their cross-sectional `fOutl` values. This happens for shift outliers, i.e. curves which have the same shape as the majority but are shifted on the whole domain. Points in the upper left part have a low `fOutl` but a high variability in cross-sectional `fOutl`. Typical examples are isolated outliers, i.e. curves which only display outlyingness over a small part of their domain. The points in the upper right part of the FOM have both a high `fOutl` and a high cross-sectional `fOutl`. These correspond to curves which are strongly outlying on a substantial part of their domain.
Author(s)

P. Segaert

References

Hubert M., Rousseeuw P.J., Segaert P. (2015). Multivariate functional outlier detection (with re-

Rousseeuw, P.J., Raymaekers, J., Hubert, M., (2018), A Measure of Directional Outlyingness with
Applications to Image Data and Video. Journal of Computational and Graphical Statistics, 27,
345–359.

Examples

data(octane)

# To construct the FOM, one first need to calculate
# the functional outlyingness.
# Note that the option diagnostic in foutl must be
# set to TRUE. If not calling fom will result in an
# error
Result <- foutl(octane, alpha = 0, type = "fAO", diagnostic = TRUE);
fom(Result)

# The user may opt to draw a cut off line seperating the outliers.
# which will be plotted in red
fom(Result, cutoff = TRUE)

# Six observations are flagged as outliers. These correspond to
# the sample with added ethanol.

---

fOutl

Function outlyingness measures for functional data

Description

Computes several measures of functional outlyingness for multivariate functional data.

Usage

fOutl(x, z = NULL, type = "fAO", alpha = 0, time = NULL,
diagnostic = FALSE, distOptions = NULL)
Arguments

- **x**: A three-dimensional $t$ by $n$ by $p$ array, with $t$ the number of observed time points, $n$ the number of functional observations and $p$ the number of measurements for every functional observation at every time point.

- **z**: An optional three-dimensional $t$ by $m$ by $p$ array, containing the observations for which to compute the functional outlyingness with respect to $x$. If $z$ is not specified, it is set equal to $x$. The time points of $z$ should correspond to those of $x$.

- **type**: The depth used in the computations. The outlyingness measure used in the computations. One of the following options: "fSDO", "fAO", "fDO" or "fbd". Defaults to "fAO".

- **alpha**: Specifies the weights at every cross-section. When $\alpha = 0$, uniform weights are used. Otherwise $\alpha$ should be a weight vector of length $t$. Defaults to 0.

- **time**: If the measurements are not equidistant, a sorted numeric vector containing a set of time points. Defaults to 1:t.

- **diagnostic**: If set to TRUE, the output contains some additional components:
  - **crossDists**: an $n$ by $t$ matrix containing the multivariate outlyingness of each observation at each time point.
  - **locOutl**: output containing flags for local outlyingness (see "Value" for more details).

- **distOptions**: A list of options to pass to the function calculating the cross-sectional distances. See outlyingness, adjOutl, or bagdistance.

Details

The functional outlyingness of a multivariate curve with respect to a given set of multivariate curves is defined as the weighted average of its multivariate outlyingness at each time point (Hubert et al., 2015). The functional outlyingness can be computed in all dimensions $p$ using bagdistance, projection depth and skewness-adjusted projection depth.

When the data array $z$ is specified, the functional outlyingness and diagnostic information for the data array $x$ is also returned whenever the underlying outlyingness routine allows it. For more information see the specific routines listed in the section "See Also".

In some situations, additional diagnostics are available to flag outlying time points. At each time point, observations from the data array $x$ are marked if they are flagged as outliers. The observations from the data array $x$ are marked if their scaled outlyingness is larger than a prescribed cut-off value from the chi-square distribution. For more details see the respective outlyingness routines.

It is possible that at certain time points a part of the algorithm can not be executed due to e.g. exact fits. In that case the weight of that particular time point is set to zero. A warning is issued at the end of the algorithm to signal these time points. Furthermore the output contains an extra argument giving the indices of the time points where problems occured.
Value

A list with the following components:

- `fOutlyingnessX`: Vector of length $n$ containing the functional outlyingness of every curve from $x$.
- `fOutlyingnessZ`: Vector of length $m$ containing the functional outlyingness of every curve from $z$.
- `weights`: Vector of weights according to the input parameter $\alpha$.
- `crossDistsX`: An $n$ by $t$ matrix containing the multivariate outlyingness of each observation of $x$ at each point. Only provided if the input parameter `diagnostic` is set to `TRUE`.
- `crossDistsZ`: An $m$ by $t$ matrix containing the multivariate outlyingness of each observation of $z$ at each point. Only provided if the input parameter `diagnostic` is set to `TRUE`.
- `locOutlX`: An $n$ by $t$ matrix flagging local outlyingness for $x$. Only provided if the input parameter `diagnostic` is set to `TRUE`. The $(i,j)$th element takes value 1 if curve $x_i$ is outlying at time point $j$.
- `locOutlZ`: An $m$ by $t$ matrix flagging local outlyingness for $z$. Only provided if the input parameter `diagnostic` is set to `TRUE`. The $(i,j)$th element takes value 1 if curve $z_i$ is outlying at time point $j$.
- `IndFlagExactFit`: Vector containing the indices of the time points for which an exact fit is detected.

Author(s)

P. Segaert

References


See Also

`bagdistance`, `outlyingness`, `adjOutl`

Examples

```r
# We will illustrate the function using a univariate functional sample.
data(octane)
Data <- octane

# When the option diagnostic is set to TRUE, a crude diagnostic
# to detect outliers can be extracted from the local outlyingness
# indicators.
Result <- fOutl(x = Data, type = "FAO", diagnostic = TRUE)
```
matplot(Data[,1], type = "l", col = "black", lty = 1)
for (i in 1:dim(Data)[2]) {
  if(sum(Result$locOutlZ > 0) {
    obsData <- matrix(Data[,i,1], nrow = 1)
    obsData[1,Result$locOutlZ[1,]] <- NA
    obsData <- rbind(obsData, obsData)
    matpoints(t(obsData), col = "red", pch = 15)
  }
}
# For more advance outlier detection techniques, see the
# from routine.

---

**geological**

*Composition of elements in soil samples around the Baltic Sea*

**Description**

This data originates from a geological survey on the composition in agricultural soils from 10 countries surrounding the Baltic Sea. Top soil (0-25 cm) and bottom soil (50-75 cm) samples from 768 sites were analysed. This data frame contains the measurements corresponding to the total concentration of four elements in the top soil samples.

**Usage**

```r
data(geological)
```

**Format**

A dataframe containing following variables:

- **Fe2O3** Iron Oxide
- **MgO** Magnesium oxide
- **MnO** Manganese oxide
- **TiO2** Titanium dioxide

**Source**


**Examples**

```r
data(geological)
plot(geological)
```
**glass**

*EXPMA spectra of glass samples*

### Description

The glass data set studied by Lemberge et al. (2000) consisting of 180 different 16-17th century archaeological glass samples. Electron Probe X-ray Microanalysis (EPXMA) intensities across 750 different wavelengths are recorded using a Jeol JSM 6300 scanning electron microscope equipped with an energy-dispersive Si(Li) X-ray detection system (SEM-EDX).

### Usage

```r
data("glass")
```

### Format

A three dimensional $t = 750$ by $n = 180$ by $p = 1$ array.

### Source


### References


### Examples

```r
data(glass)
matplot(glass[,1], type="l", lty=1, col = "black")
```

---

**hdepth**

*Halfspace depth of points relative to a dataset*

### Description

Computes the halfspace depth of $p$-dimensional points $z$ relative to a $p$-dimensional dataset $x$. Computation is exact for $p \leq 3$ and approximate when $p > 3$. For the approximate algorithm the halfspace depth is computed as the minimal univariate halfspace depth over many directions. To obtain the univariate halfspace depth in the direction $v$, the dataset $x$ is projected on $v$, and the univariate location depth of the points of $v'z_i$ to $xv$ is computed.
Usage

hdepth(x, z = NULL, options = list())

Arguments

x
An n by p data matrix with observations in the rows and variables in the columns.

z
An optional m by p matrix containing rowwise the points $z_i$ for which to compute the halfspace depth. If z is not specified, it is set equal to x.

options
A list of available options:

- type
  Determines the desired type of invariance for the approximate algorithm and should be one of "Affine", "Rotation" or "Shift". When the option "Affine" is used, the directions $v$ are orthogonal to hyperplanes spanned by $p$ observations from x. When the option "Rotation" is used, the directions pass by two randomly selected observations from x. With the option "Shift", directions are randomly generated. Defaults to "Affine".

- ndir
  Determines the number of directions $v$ by setting ndir to a specific number or to "all". In the latter case, an exhaustive search over all possible directions (according to type) is performed. If ndir is larger than the number of possible directions, the algorithm will automatically use this setting. Defaults to 250p when type="Affine", to 5000 when type="Rotation" and to 12500 when type="Shift".

- approx
  The user may force approximate calculation in two or three dimensions by setting this option to TRUE. Defaults to FALSE.

- seed
  A strictly positive integer specifying the seed to be used by the C++ code. Defaults to 10.

Details

Halfspace depth has been introduced by Tukey (1975). The halfspace depth of a point $z_i$ is defined as the minimal number of observations from x that are contained in any closed halfspace with boundary through $z_i$.

In dimensions $p = 2$ and $p = 3$ the computations are by default carried out exactly using the algorithms described in Rousseeuw and Ruts (1996) and Rousseeuw and Struyf (1998). This yields an affine invariant measure of depth. Approximate algorithms are also implemented which depending on the value chosen for type are affine, rotation or shift invariant. They can be used in any dimension. The shift invariant algorithm coincides with the random Tukey depth (Cuesta-Albertos and Nieto-Reyes, 2008).

The resulting halfspace depth values are invariant to affine transformations when the exact algorithm is used and invariant to affine transformations, rotations and shifts depending on the choice for type provided that the seed is kept fixed at different runs of the algorithm. Note that the halfspace depth
values values are guaranteed to decrease when more directions are considered provided the seed is
kept fixed, as this ensures that the random directions are generated in a fixed order.

If the halfspace depth needs to be computed for $m$ points $z_i$, it is recommended to apply the function
once with the matrix $z$ as input, instead of applying it $m$ times with input vectors $z_i$, as numerous
computations can be saved. The approximate algorithms automatically then also compute the depth
values of the observations in $x$. When only the halfspace depth of the observations in $x$ is required,
the call to the function should be 'hdepth(x)' or equivalently 'hdepth(x,x)'. In that case the depth
values will be stored in the 'depthZ' output field. For bivariate data these will be the exact values by
default. To visualize the depth of bivariate data one can apply the mrainbowplot function. It plots
the data colored according to their depth.

It is first checked whether the data lie in a subspace of dimension smaller than $p$. If so, a warning is
given, as well as the dimension of the subspace and a direction which is orthogonal to it.

Value

A list with components:

- **depthX**: Vector of length $n$ giving the halfspace depth of the observations in $x$.
  By default exact if $p \leq 3$ and approximate if $p > 3$ or the option approx is set
to TRUE.

- **depthZ**: Vector of length $m$ giving the halfspace depth of the points in $z$ relative to $x$.
  By default exact if $p \leq 3$ and approximate if $p > 3$ or the option approx is set
to TRUE.

- **singularSubsets**: When the input parameter type is equal to "Affine", the number of $p$-subsets
  that span a subspace of dimension smaller than $p-1$. In that case the orthogonal
direction can not be uniquely determined. This is an indication that the data are
not in general position. When the input parameter type is equal to "Rotation"
it is possible that two randomly selected points of the data coincide due to ties
in the data. In this case this value signals how many times this happens.

- **dimension**: When the data $x$ are lying in a lower dimensional subspace, the dimension of
  this subspace.

- **hyperplane**: When the data $x$ are lying in a lower dimensional subspace, a direction orthog-
  onal to this subspace.

Author(s)

P. Segaert based on Fortran code by P.J. Rousseeuw, I. Ruts and A. Struyf, and C++ code by P.
Segaert and K. Vakili.

References

Mathematicians, 2*, 523–531, Vancouver.


**See Also**

hdepthmedian, mrainbowplot, bagdistance, bagplot

**Examples**

```r
# Compute the halfspace depth of a simple
data(cardata90)
Result <- hdepth(x = cardata90)
mrainbowplot(cardata90, depths = Result$depth)

# In two dimensions we may also opt to use the
# approximate algorithm. The number of directions
# may be specified through the option list.
options <- list(type = "Rotation",
                ndir = 750,
                approx = TRUE)
Result <- hdepth(x = cardata90, options = options)
# The resulting halfspace depth is monotone decreasing
# in the number of directions.
options <- list(type = "Rotation",
                ndir = 10,
                approx = TRUE)
Result1 <- hdepth(x = cardata90, options = options)
options <- list(type = "Rotation",
                ndir = 500,
                approx = TRUE)
Result2 <- hdepth(x = cardata90, options = options)
which(Result1$depth < Result2$depth)
# This is however not the case when the seed is changed
options <- list(type = "Rotation",
                ndir = 10,
                approx = TRUE)
Result1 <- hdepth(x = cardata90, options = options)
options <- list(type = "Rotation",
                ndir = 50,
                approx = TRUE,
                seed = 897)
Result2 <- hdepth(x = cardata90, options = options)
which(Result1$depth < Result2$depth)
plot(Result1$depth, -Result2$depth,
     xlab = "Index", ylab = "Difference in halfspace depth")

# We can also consider directions through two data
# points. If the sample is small enough one may opt
# to search over all choose(n,2) directions.
# Note that the computational load increases dramatically
# as n becomes larger.
options <- list(type = "Rotation",
                ndir = "all",
                approx = TRUE)
Result1 <- hdepth(x = cardata90, options = options)

# Alternatively one may consider randomly generated directions.
options <- list(type = "shift",
                ndir = 250,
                approx = TRUE)
Result1 <- hdepth(x = cardata90, options = options)

---

hdepthmedian  Location estimates based on halfspace depth.

Description

Computes the halfspace median and its corresponding halfspace depth for a \( p \)-dimensional data set \( x \). Computation is exact for \( p \leq 2 \) and approximate for \( p > 2 \).

Usage

hdepthmedian(x, maxdir = NULL)

Arguments

- **x**  
  An \( n \) by \( p \) data matrix.

- **maxdir**  
  The number of projections used in the approximate algorithm.  
  Defaults to 250\( p \).

Details

The halfspace median, or Tukey median, is the multivariate point with largest halfspace depth with respect to the data \( x \). This point is not always unique. In that case the halfspace median corresponds to the center of gravity of the convex set of deepest points.

It is first checked whether the data is found to lie in a subspace of dimension lower than \( p \). If so, the routine will give a warning, giving back the dimension of the subspace together with a direction describing a hyperplane containing this subspace.

For bivariate data the exact algorithm of Rousseeuw and Ruts (1998) is applied. When the data are not in general position (i.e. when there is a line containing more than two observations) dithering is performed by adding random Gaussian noise to the data. In this the output argument dithered will contain a flag.

When \( p > 2 \) the approximate algorithm of Struyf and Rousseeuw (2000) is applied. It is an iterative procedure based on projections. Their number can be chosen by the input parameter maxdir.
**Value**

A list containing:

- **median**: The coordinates of the halfspace median. Approximate when \( p > 2 \).
- **depth**: The halfspace depth of the halfspace median. Approximate when \( p > 2 \).
- **dithered**: Logical indicating whether dithering has been applied in the exact algorithm. FALSE indicates no dithering has been applied. TRUE indicates dithering has been applied.
- **ndir**: The number of projections used by the approximate algorithm. Due to the possibility of singularity of certain \( p \) subsamples it is possible that not all \( \text{maxdir} \) directions are evaluated.

- **AlgStopFlag**: Indicates which stopping rule is used by the approximate algorithm. 0 indicates the maximum number of projections was reached, 1 indicates no improvement of the location estimate was made after \( 10(p + 1) \) steps.
- **dimension**: If the data are lying in a lower dimensional subspace, the dimension of this subspace.
- **hyperplane**: If the data are lying in a lower dimensional subspace, a direction orthogonal to this subspace.

**Author(s)**

P. Segaert based on Fortran code by P.J. Rousseeuw, I. Ruts and A. Struyf

**References**


**Examples**

```r
# Compute a location estimate of a simple two-dimensional dataset.

data(cardata90)
Result <- hdepthmedian(x=cardata90)
```
medcouple  

A robust measure of skewness for univariate data

Description
Computes the medcouple, a robust measure of skewness for univariate data. For multivariate data the medcouple is calculated on each column of the data matrix.

Usage
```r
medcouple(x, do.reflect = NULL)
```

Arguments
- `x`: An `n` by `p` data matrix.
- `do.reflect`: Logical indicating whether the medcouple should also be computed on the reflected sample `−x`, with final result `(mc(x)−mc(−x))/2`. Defaults TRUE when `n <= 100` and to FALSE otherwise.

Details
The medcouple is a robust measure of skewness yielding values between −1 and 1. For left- and right-skewed data the medcouple is negative and positive respectively.

The medcouple is defined as the median of the kernel function

\[ h(x_i, x_j) = \frac{(x_j - \text{med}(x)) - (\text{med}(x) - x_i)}{x_j - x_i} \]

evaluated over all couples \((x_i, x_j)\) where \(x_i\) is smaller than the median of \(x\) and \(x_j\) larger than the median of \(x\). When there are multiple observations tied to the median, the kernel is defined separately as the denominator is not defined for these observations. Let \(m_1 < \ldots < m_k\) denote the indices of the observations which are tied to the median. Then \(h(x_{m_i}, x_{m_j})\) is defined to equal −1 if \(i + j - 1 < k\), 0 when \(i + j - 1 = k\) and +1 if \(i + j - 1 > k\). To compute the medcouple an algorithm with time complexity \(O(n\log(n))\) is applied. For details, see https://en.wikipedia.org/wiki/Medcouple.

For numerical accuracy it is advised, for small data sets, to compute the medcouple on both \(x\) and \(−x\). The final value of the medcouple may then be obtained as a linear combination of both calculations. This procedure is warranted by the properties of the medcouple. Indeed the medcouple of the distribution \(X\) equals minus the medcouple of the reflected distribution \(−X\). Moreover the medcouple is location and scale invariant.

Note that missing values are not allowed.

Value
- `mc`: A `p`-vector containing the medcouple of each column of the data matrix `x`.

Author(s)
P. Segaert with original code from M. Maechler and G. Brys.
References


Examples

# Calculate the medcouple of a bivariate data set.
# Note that the medcouple of each variable is returned.
data(bloodfat)
medcouple(bloodfat)

# For smaller data sets it is advisable to calculate
# the medcouple on both the sample and the reflected sample.
small.data <- bloodfat[1:25,]
medcouple(small.data, do.reflect = FALSE)
-medcouple(-small.data, do.reflect = FALSE)
# Small difference are due to numerical instabilities.
# Use the option do.reflect to increase expected accuracy.
medcouple(small.data, do.reflect = TRUE)

mfd

Multivariate functional depth for functional data

Description

Computes the multivariate functional depth for multivariate functional data.

Usage

mfd(x, z = NULL, type = "hdepth", alpha = 0, time = NULL, diagnostic = FALSE,
    depthOptions = NULL)

Arguments

x

A three dimensional $t$ by $n$ by $p$ array, with $t$ the number of observed time points,
$n$ the number of functional observations and $p$ the number of measurements for
every functional observation at every time point.

z

An optional three-dimensional $t$ by $m$ by $p$ array, containing the observations
for which to compute the multivariate functional depth with respect to $x$. If $z$
is not specified, it is set equal to $x$. The time points of $z$ should correspond to
those of $x$.

type

The depth used in the computations. One of the following options: "hdepth",
"projdepth", "sprojdepth", "dprojdepth", "sdepth". Defaults to "hdepth".
alpha Specifies the weights at every cross-section. When alpha = 0, uniform weights are used. Weights following equation (2) in Claeskens et al. (2014) are obtained by setting alpha to a number smaller than the maximal depth at any time point. The weights are then proportional to the volume of the alpha-depth regions at each cross-section. Otherwise alpha should be a weight vector of length t.
Defaults to 0.

time If the measurements are not equidistant, a sorted numeric vector containing a set of time points.
Defaults to 1:t.
diagnostic If set to TRUE, the output contains some additional components:
crossDepths: an n by t matrix containing the multivariate depth of each observation at each time point
locOutl: output containing flags for local outlyingness (see "Value" for more details)
Defaults to FALSE.
depthOptions A list of options to pass to the function calculating the cross-sectional depths.
See hdepth, projdepth, sprojdepth, dprojdepth or sdepth.

Details

The multivariate functional depth of a multivariate curve with respect to a given set of multivariate curves is defined as the weighted average of its multivariate depth at each time point (Claeskens et al., 2014). The MFD can be computed in all dimensions p using halfspace depth, projection depth and skewness-adjusted projection depth. For p <= 2 also simplicial depth is available.

When the data array z is specified, the MFD depth and diagnostic information for the data array x is also returned whenever the underlying depth routine allows it. For more information see the specific depth routines listed in the section "See Also"

For the weight vector, three options are available: uniform weights, user-defined weights or weights proportional to the volume of the alpha-depth contour at each time point. The alpha-depth contours are computed using the depthContour function.

In some situations, additional diagnostics are available to flag outlying time points, as described in Hubert et al. (2012). At each time point, observations from the data array x are marked if they are flagged as outliers. When using any of the projection depth measures, this flag is automatically returned by the corresponding functions. When using halfspace depth, the diagnostic is only available for bivariate curves. The observations from the data array x are marked if they are flagged as outliers by the bagplot, or similarly if their bagdistance is larger than 3 at that time point. This can be seen as a measure of local outlyingness. The option is not available for simplicial depth.

It is possible that at certain time points a part of the algorithm can not be executed due to e.g. exact fits. In that case the weight of that particular time point is set to zero. A warning is issued at the end of the algorithm to signal these time points. Furthermore the output contains an extra argument giving the indices of the time points where problems occurred.

Value

A list with the following components:

MFDdepthX Vector of length n containing the MFD depth of every curve from x.
MFDdepthZ  Vector of length \( m \) containing the MFD depth of every curve from \( z \).
weights  Vector of weights according to the input parameter \( \alpha \).
crossDepthsX  An \( n \) by \( t \) matrix containing the multivariate depth of each observation of \( x \) at each point. Only provided if the input parameter \( \text{diagnostic} \) is set to \text{TRUE}.
crossDepthsZ  An \( m \) by \( t \) matrix containing the multivariate depth of each observation of \( z \) at each point. Only provided if the input parameter \( \text{diagnostic} \) is set to \text{TRUE}.
locOut1X  An \( n \) by \( t \) matrix flagging local outlyingness for \( x \). Only provided if the input parameter \( \text{diagnostic} \) is set to \text{TRUE}.
The \((i,j)\)th element takes value 1 if curve \( x_i \) is outlying at time point \( j \).
locOut1Z  An \( m \) by \( t \) matrix flagging local outlyingness for \( z \). Only provided if the input parameter \( \text{diagnostic} \) is set to \text{TRUE}.
The \((i,j)\)th element takes value 1 if curve \( z_i \) is outlying at time point \( j \).
IndFlagExactFit  Vector containing the indices of the time points for which an exact fit is detected.
IndFlagBag  Vector containing the indices of the time points for which the bagplot could not be computed.
IndFlagIso  Vector containing the indices of the time points for which the cross-sectional \( \alpha \)-depth contours could not be computed.
IndFlagAlpha  Vector containing the indices of the time points for which the volume of the cross-sectional \( \alpha \)-depth contours could not be computed.

Author(s)

P. Segaert and M. Hubert

References


See Also

depthContour, hdepth, projdepth, sprojdepth, dprojdepth, sdepth

Examples

data(octane)
Result <- mfd(x = octane, alpha = 0.125, diagnostic = \text{TRUE})
Multivariate functional median for functional data.

Description

Computes the multivariate functional median, an estimate for the central tendency for multivariate functional data.

Usage

```r
mfdmedian(x, type="hdepth", crossDepthsX = NULL,
          depthOptions = NULL, centerOption = "maxdepth")
```

Arguments

- `x`: A three dimensional `t` by `n` by `p` array, with `t` the number of observed time points, `n` the number of functional observations and `p` the number of measurements for every functional observation at every time point.
- `type`: The depth used in the computations. One of the following options: "hdepth", "projdepth", "sprojdepth", "dprojdepth", "sdepth". Defaults to "hdepth".
- `crossDepthsX`: Depths at each time point. Can be used to save time.
- `depthOptions`: A list of options to pass to the function calculating the cross-sectional depths. See `hdepth`, `projdepth`, `sprojdepth`, `dprojdepth` or `sdepth`.
- `centerOption`: By default the 'maxdepth' center estimate is used for the projmedian and the sprojmedian. This can be changed by setting `centerOption` to 'gravity' or 'Huber' depending on whether projmedian or sprojmedian is used.

Details

The multivariate functional median a multivariate functional data set is defined as the multivariate curve connecting the cross-sectional multivariate depth medians (Claeskens et al., 2014). The MFD median can be computed in all dimensions `p` using halfspace depth, projection depth and skewness-adjusted projection depth.

It is possible that at certain time points a part of the algorithm cannot be executed due to e.g. exact fits. In that case the estimate for the center will be set to NaN. A warning is issued at the end of the algorithm to signal these time points. Furthermore the output contains an extra argument giving the indices of the time points where problems occurred.

Value

A list with the following component:

- `MFDmedian`: An (t,x,p) matrix containing the estimated central curve.
- `IndFlagExactFit`: Vector containing the indices of the time points for which an exact fit is detected.
Author(s)

P. Segaert

References


Examples

```r
# We will illustrate the function using a bivariate functional sample.
data(characterA)[,1:50,]
Result <- mfdmedian(Data)

par(mfrow = c(1,2))
matplot(Data[,1], type = "l", col = "black", lty = 1, ylab = "x-coordinate")
matlines(Result$MFDmedian[,1], type = "l", col = "red", lty = 1)
matpoints(Result$MFDmedian[,1], col = "red", pch = 15)
matplot(Data[,2], type = "l", col = "black", lty = 1, ylab = "y-coordinate")
matlines(Result$MFDmedian[,2], type = "l", col = "red", lty = 1)
matpoints(Result$MFDmedian[,2], col = "red", pch = 15)
par(mfrow = c(1,1))

# Other depth function such as adjusted outlyingness may also
# used to determine the cross-sectional depth median.
# In this case the depth median is calculated by the
# sprojmedian routine.
# When different depth median are available, a specific depth
# median can be selected using the centerOption. Optional
# arguments used by the sprojmedian routine may be specified
# using the depthOptions. For example one might choose the
# "Rotation" equivariance for 300 directions.
Result <- mfdmedian(Data, type = "sprojdepth",
                     depthOptions = list(type = "Rotation",
                                          ndir = 300),
                     centerOption = "gravity")

par(mfrow = c(1,2))
matplot(Data[,1], type = "l", col = "black", lty = 1, ylab = "x-coordinate")
matlines(Result$MFDmedian[,1], type = "l", col = "red", lty = 1)
matpoints(Result$MFDmedian[,1], col = "red", pch = 15)
matplot(Data[,2], type = "l", col = "black", lty = 1, ylab = "y-coordinate")
matlines(Result$MFDmedian[,2], type = "l", col = "red", lty = 1)
matpoints(Result$MFDmedian[,2], col = "red", pch = 15)
par(mfrow = c(1,1))

# If the user already placed a call to the mdf routine
# with the diagnostic options set to TRUE, the
# mfdmedian can easily be obtained by passing the cross-sectional
# depths. This drastically saves computing time.
result <- mfd(x = Data, type = "sprojdepth", diagnostic = TRUE)
Result <- mfdmedian(Data, type = "sprojdepth",
                     depthOptions = list(type = "Rotation",
                                          ndir = 300),
                     centerOption = "gravity")
par(mfrow = c(1,1))
```

```
`mrainbowplot`

Rainbow plot for bivariate data

Description

Makes a scatterplot of bivariate data and colors the observations according to their depth value.

Usage

```r
mrainbowplot(x, depths, col = NULL, plot.options = list())
```

Arguments

- `x`: An \( n \) by 2 data matrix.
- `depths`: A column vector of length \( n \).
  The depth values of the observations in \( x \). The coloring is based on these depth values.
- `col`: An \( m \times 3 \) matrix.
  Colors in rgb format. The user may use this argument to set the colorscale of the depth range. The first row should contain the rgb values for the lowest depth value, the last row the rgb values of the color for the deepest depth value. Colors for other depth values are interpolated. When more then two rows are provided the color range will be equidistantly divided over the different colors.
- `plot.options`: A list of available options:
  - `legend.title` Title of the legend.
  - `point.size` Numeric defining the size of the points in the plot.

Details

The plot is made using ggplot2. The plot itself is returned by the function and is fully customisable using standard ggplot2 commands. Similar plots for multivariate data with \( p > 2 \) can be made using the ggpairs function in the library GGally.

Author(s)

P. Segaert

```r
crossDepthsx = tResult$crossdepthX,
centerOption = "gravity")
```
Examples

```r
data(cardata90)
Result <- projdepth(x = cardata90)
plot.options <- list(legend.title = "PD")
plot <- mrainbowplot(cardata90, 
  depths = Result$depthZ, plot.options = plot.options)
library("ggplot2")
plot + ggtitle("Rainbowplot of the cardata using projection depth.")

#The default color range may be adjusted using the col argument.
RGBmatrix <- c(1, 0, 0, #Red
              1, 1, 1, #White
              0, 1, 0) #Green
RGBmatrix <- matrix(RGBmatrix, ncol = 3, byrow = TRUE)
plot <- mrainbowplot(cardata90, 
  depths = Result$depthZ, col = RGBmatrix, 
  plot.options = plot.options)
plot + ggtitle("Rainbowplot of the cardata using projection depth.")
```

---

**mri**

**Intensities of MRI images**

**Description**

Felipe et al. (2005) obtained intensities of MRI images of 9 different parts of the human body (plus a group consisting of all remaining body regions, which was of course very heterogeneous). They then transformed their data to univariate curves.

**Usage**

```r
data("plane")
```

**Format**

A list of arrays corresponding to each bodypart. For each bodypart, a three-dimensional \( t = 99 \) by \( n \) by \( p = 1 \) array is available. The index \( t \) corresponds to the different points of measurement, the index \( n \) to the different observations.

**Details**

When using this data set please cite both Felipe et al. (2005) and Hubert et al. (2017).

**Source**


References


Examples
data(octane)
par(mfrow = c(2,1))
matplot(octane[,1], ylab = "x-coordinate", main = "plane 1")
par(mfrow = c(1,1))

octane  
Near infrared spectra of gasoline samples.

Description

The last six observations contain added ethanol.

Usage
data(octane)

Format

Matrix observations correspond to rows.

Source


References


Examples
data(octane)
matplot(octane[,1], type="l", lty=1, col = "black")
Stahel-Donoho outlyingness of points relative to a dataset

Description

Computes the Stahel-Donoho outlyingness (SDO) of $p$-dimensional points $z$ relative to a $p$-dimensional dataset $x$. For each multivariate point $z_i$, its outlyingness relative to $x$ is defined as its maximal univariate Stahel-Donoho outlyingness measured over all directions. To obtain the univariate Stahel-Donoho outlyingness in the direction $v$, the dataset $x$ is projected on $v$, and the robustly standardized distance of $v'z_i$ to the robust center of the projected data points $xv$ is computed.

Usage

```r
outlyingness(x, z = NULL, options = list())
```

Arguments

- **x**: An $n$ by $p$ data matrix.
- **z**: An optional $m$ by $p$ matrix containing rowwise the points $z_i$ for which to compute the outlyingness. If $z$ is not specified, it is set equal to $x$.
- **options**: A list of available options:
  - **type**: Determines the desired type of invariance and should be one of "Affine", "Rotation" or "Shift". When the option "Affine" is used, the directions $v$ are orthogonal to hyperplanes spanned by $p$ observations from $x$. When the option "Rotation" is used, the directions pass by two randomly selected observations from $x$. With the option "Shift", directions are randomly generated. Defaults to "Affine".
  - **ndir**: Determines the number of directions $v$ by setting $\text{ndir}$ to a specific number or to "all". In the latter case, an exhaustive search over all possible directions (according to type) is performed. If $\text{ndir}$ is larger than the number of possible directions, the algorithm will automatically use this setting. Defaults to $250p$ when type="Affine", to 5000 when type="Rotation" and to 12500 when type="Shift".
  - **stand**: Determines how to robustly standardize the projected data: "MedMad" uses the median and the MAD, "unimcd" uses the univariate MCD of location and scale. Defaults to "MedMad".
  - **centered**: When the data matrix $x$ is already centered, no robust center should be computed in each direction. In that case, centered should be set to TRUE. Defaults to FALSE.
When the input argument \texttt{stand} is equal to \texttt{unimcd}, the parameter $h$ controls the number of data points that define the MCD estimator (see \texttt{covMcd} in the \texttt{robustbase} package). This value should lie between $\left\lfloor n/2 \right\rfloor + 1$ and $n$. Defaults to $\left\lfloor n/2 \right\rfloor + 1$.

- \texttt{seed}
  - A strictly positive integer specifying the seed to be used by the C++ code. Defaults to 10.

### Details

The Stahel-Donoho outlyingness has been introduced by Stahel (1981) and Donoho (1982). It is mostly suited to measure the degree of outlyingness of multivariate points with respect to a data cloud from an elliptical distribution.

Depending on the dimension $p$, different approximate algorithms are implemented. The affine invariant algorithm can only be used when $n > p$. It draws $\texttt{ndir}$ times at random $p$ observations from $x$ and considers the direction orthogonal to the hyperplane spanned by these $p$ observations. At most $p$ out of $n$ directions can be considered. The orthogonal invariant version can be applied to high-dimensional data. It draws $\texttt{ndir}$ times at random 2 observations from $x$ and considers the direction through these observations. Here, at most 2 out of $n$ directions can be considered. Finally, the shift invariant version randomly draws $\texttt{ndir}$ vectors from the unit sphere. The resulting Stahel-Donoho outlyingness values are invariant to affine transformations, rotations and shifts respectively provided that the \texttt{seed} is kept fixed at different runs of the algorithm. Note that the SDO values are guaranteed to increase when more directions are considered provided the seed is kept fixed, as this ensures that the random directions are generated in a fixed order.

An observation from $x$ and $z$ is flagged as an outlier if its SDO exceeds a cutoff value. This cutoff value is determined using the procedure in Rousseeuw et al. (2018). First, the logarithm of the SDO values is taken to render their distribution more symmetric, after which a normal approximation yields a cutoff on these values. The cutoff is then transformed back by applying the exponential function.

It is first checked whether the data lie in a subspace of dimension smaller than $p$. If so, a warning is given, as well as the dimension of the subspace and a direction which is orthogonal to it. Moreover, from the definition of the Stahel-Donoho outlyingness it follows that the outlyingness is ill-defined when the robust scale of the data projected on the direction $v$ equals zero. In this case the algorithm will stop and give a warning. The returned values then include the direction $v$ as well as an indicator specifying which of the observations of $x$ belong to the hyperplane orthogonal to $v$.

### Value

A list with components:

- \texttt{outlyingnessX} Vector of length $n$ giving the outlyingness of the observations in $x$.
- \texttt{outlyingnessZ} Vector of length $m$ giving the outlyingness of the points in $z$.
- \texttt{cutoff} Points whose outlyingness exceeds this cutoff can be considered as outliers with respect to $x$. 
Observations of \( x \) whose outlyingness exceeds the cutoff value receive a flag \textit{FALSE}, regular observations receive a flag \textit{TRUE}.

Points of \( z \) whose outlyingness exceeds the cutoff value receive a flag equal to \textit{FALSE}, otherwise they receive a flag \textit{TRUE}.

When the input parameter type is equal to \textit{"Affine"}, the number of \( p \)-subsets that span a subspace of dimension smaller than \( p - 1 \). In such a case the orthogonal direction can not be uniquely determined. This is an indication that the data are not in general position. When the input parameter type is equal to \textit{"Rotation"} it is possible that two randomly selected points of the data coincide due to ties in the data. In such a case this value signals how many times this happens.

When the data \( x \) are lying in a lower dimensional subspace, the dimension of this subspace.

When the data \( x \) are lying in a lower dimensional subspace, a direction orthogonal to this subspace. When a direction \( v \) is found such that the robust scale of \( xv \) is equal to zero, this equals \( v \).

When a direction \( v \) is found such that the robust scale of \( xv \) is zero, the observations from \( x \) which belong to the hyperplane orthogonal to \( v \) receive a value \textit{TRUE}. The other observations receive a value \textit{FALSE}.

P. Segaert using C++ code by K. Vakili and P. Segaert.


\textbf{See Also}

projdepth, projmedian, adjOutl, dirOutl

\textbf{Examples}

```r
# Compute the outlyingness of a simple two-dimensional dataset.
# Outliers are plotted in red.
```
if (requireNamespace("robustbase", quietly = TRUE)) {
  BivData <- log(robustbase::Animals2)
} else {
  BivData <- matrix(rnorm(120), ncol = 2)
  BivData <- rbind(BivData, matrix(c(6, 6, -2), ncol = 2))
}

Result <- outlyingness(x = BivData)
IndOutliers <- which(!Result$flagX)
plot(BivData)
points(BivData[IndOutliers,], col = "red")

# The number of directions may be specified through
# the option list. The resulting outlyingness is
# monotone increasing in the number of directions.
Result1 <- outlyingness(x = BivData,
  options = list(ndir = 50))
Result2 <- outlyingness(x = BivData,
  options = list(ndir = 100))
which(Result2$outlyingnessX - Result1$outlyingnessX < 0)
# This is however not the case when the seed is changed
Result1 <- outlyingness(x = BivData,
  options = list(ndir = 50))
Result2 <- outlyingness(x = BivData,
  options = list(ndir = 100,
               seed = 950))
plot(Result2$outlyingnessX - Result1$outlyingnessX,
     xlab = "Index", ylab = "Difference in outlyingness")

# We can also consider directions through two data
# points. If the sample is small enough one may opt
# to search over all choose(n,2) directions.
# Note that the computational load increases dramatically
# as n becomes larger.
Result <- outlyingness(x = BivData,
  options = list(type = "Rotation",
               ndir = "all"))

IndOutliers <- which(!Result$flagX)
plot(BivData)
points(BivData[IndOutliers,], col = "red")

# Alternatively one may consider randomly generated directions.
Result <- outlyingness(x = BivData,
  options = list(type = "Shift",
               ndir = 1000))

IndOutliers <- which(!Result$flagX)
plot(BivData)
Description

The fighter plane dataset of Thakoor and Gao (2005) describes 7 shapes: of the Mirage, Eurofighter, F-14 with wings closed, F-14 with wings opened, Harrier, F-22 and F-15. Each class contains 30 shape samples obtained from digital pictures, which Thakoor and Gao (2005) then reduced to the univariate functions.

Usage

data("plane")

Format

A list of arrays corresponding to each plane. For each plane, a three-dimensional \( t \) by \( n \) by \( p = 1 \) array is available. The index \( t \) corresponds to the different points of measurement, the index \( n \) to the different observation.

Details

When using this data set please cite both Thakoor et al. (2005) and Hubert et al. (2017).

Source


References


Examples

data(plane)
par(mfrow = c(2,1))
matplot(y = plane$plane1[ ,1],
       type = "l", col = "black", lty = 1, xlab = "", ylab="x-coordinate", main = "plane 1")
matplot(y = plane$plane2[ ,1],
       type = "l", col = "black", lty = 1, xlab = "", ylab="x-coordinate", main = "plane 2")
par(mfrow = c(1,1))

plotContours  

*Draws depth contours of bivariate data*

Description

Draws the depth contours of bivariate data computed with depthContour.

Usage

plotContours(x, depthContour, data = TRUE)

Arguments

- **x**: An \( n \) by 2 data matrix.
- **depthContour**: The result of a call to depthContour.
- **data**: Logical value indicating whether the data \( x \) should be plotted.

Details

The plot is made using gglot2. The plot itself is returned by the function and is fully customisable using standard gglot2 commands.

Author(s)

P. Segaert

References


See Also

depthContour
**projdepth**

*Projection depth of points relative to a dataset*

**Description**

Computes the projection depth of \( p \)-dimensional points \( z \) relative to a \( p \)-dimensional dataset \( x \).

**Usage**

\[
\text{projdepth}(x, z = \text{NULL}, \text{options} = \text{list()})
\]

**Arguments**

- \( x \): An \( n \) by \( p \) data matrix with observations in the rows and variables in the columns.
- \( z \): An optional \( m \) by \( p \) matrix containing rowwise the points \( z_i \) for which to compute the projection depth. If \( z \) is not specified, it is set equal to \( x \).
- \( \text{options} \): A list of options to pass to the underlying \text{outlyingness} routine. See \text{outlyingness} for the full list of options.

**Details**

Projection depth is based on the Stahel-Donoho outlyingness (SDO) and is computed as \( 1/(1 + SDO) \). It is mostly suited to measure the degree of outlyingness of multivariate points with respect to a data cloud from an elliptical distribution.

It is first checked whether the data lie in a subspace of dimension smaller than \( p \). If so, a warning is given, as well as the dimension of the subspace and a direction which is orthogonal to it.

See \text{outlyingness} for more details on the computation of the SDO. To visualize the depth of bivariate data one can apply the \text{mrainbowplot} function. It plots the data colored according to their depth.

The output values of this function are based on the output of the \text{outlyingness} function. More details can be found there.
Value

A list with components:

- **depthX**: Vector of length \( n \) giving the projection depth of the observations in \( x \).
- **depthZ**: Vector of length \( m \) giving the projection depth of the points in \( z \).
- **cutoff**: Points whose projection depth is smaller than this cutoff can be considered as outliers. Equivalently the outliers are those points whose Stahel-Donoho outlyingness exceed the corresponding cutoff.
- **flagX**: Observations of \( x \) whose projection depth is smaller than the cutoff receive a flag **FALSE**, regular observations receive a flag **TRUE**.
- **flagZ**: Points of \( z \) whose projection depth is smaller than the cutoff receive a flag equal to **FALSE**, otherwise they receive a flag **TRUE**.
- **singularSubsets**: When the input parameter type is equal to "Affine", the number of \( p \)-subsets that span a subspace of dimension smaller than \( p - 1 \). In that case the orthogonal direction can not be uniquely determined. This is an indication that the data are not in general position. When the input parameter type is equal to "Rotation" it is possible that two randomly selected points of the data coincide due to ties in the data. In this case this value signals how many times this happens.
- **dimension**: When the data \( x \) are lying in a lower dimensional subspace, the dimension of this subspace.
- **hyperplane**: When the data \( x \) are lying in a lower dimensional subspace, a direction orthogonal to this subspace. When a direction \( v \) is found such that the robust scale of \( xv \) is equal to zero, this equals \( v \).
- **inSubspace**: When a direction \( v \) is found such that the robust scale of \( xv \) is zero, the observations from \( x \) which belong to the hyperplane orthogonal to \( v \) receive a value **TRUE**. The other observations receive a value **FALSE**.

Author(s)

P. Segaert

References


See Also

[outlyingness, projmedian, mrainbowplot, adjOutl, dirOutl]

Examples

```r
# Compute the projection depth of a simple two-dimensional dataset.
# Outliers are plotted in red.
```
if (requireNamespace("robustbase", quietly = TRUE)) {
  BivData <- log(robustbase::Animals2)
} else {
  BivData <- matrix(rnorm(120), ncol = 2)
  BivData <- rbind(BivData, matrix(c(6, 6, 6, -2), ncol = 2))
}
Result <- projdepth(x = BivData)
IndOutliers <- which(!Result$flagX)
plot(BivData)
points(BivData[IndOutliers,], col = "red")

# A multivariate rainbowplot may be obtained using mrainbowplot.
plot.options = list(legend.title = "PD")
mrainbowplot(x = BivData,
  depths = Result$depthX, plot.options = plot.options)

# Options for the underlying outlyingness routine may be passed
# using the options argument.
Result <- projdepth(x = BivData,
  options = list(type = "Affine",
                 ndir = 1000,
                 stand = "MedMad",
                 h = nrow(BivData)
                 )
)

projmedian

Location estimates based on projection depth.

Description
Computes a projection depth base location estimate of a \( p \)-dimensional dataset \( x \).

Usage
projmedian(x, projection.depths = NULL, options = NULL)

Arguments
- \( x \) An \( n \) by \( p \) data matrix with observations in the rows and variables in the columns.
- \( \text{projection.depths} \) Vector containing the projection depths of the points in \( x \).
- \( \text{options} \) A list of options to pass to the projdepth routine. See projdepth for more details.
Details

The algorithm depends on the function `projdepth` to calculate the projection depths of the dataset \( x \). If the projection depths of the points \( x_i \) have already been calculated they can be passed as an optional argument to save computation time. If not, projections depths will be calculated and the user can pass a list with options to the `projdepth` function.

It is first checked whether the data lie in a subspace of dimension smaller than \( p \). If so, a warning is given, as well as the dimension of the subspace and a direction which is orthogonal to it.

Value

A list with components:

- **max**: The point of \( x \) with maximal projection depth. If multiple points have maximum depth, their center of gravity is returned.
- **gravity**: The center of gravity for the 50 percent points with highest projection depth. Also called the center of gravity of the bag.
- **Huber**: The weighted center of gravity of all points. The weights are defined by the Huber function with parameter \( \alpha = 1 / (1 + \sqrt{qchisq(0.95, p)}) \). Observations for which the projection depth is more than \( \alpha \) receive weight 1, other points receive weight \( \sqrt{qchisq(0.95, p)} / \text{outlyingness} \)^2.

Author(s)

P. Segaert

References


See Also

`outlyingness`, `projdepth`, `adjOutl`, `dirOutl`

Examples

```r
# Compute a location estimate of a simple two-dimensional dataset.
if (requireNamespace("robustbase", quietly = TRUE)) {
  BivData <- log(robustbase::Animals2)
} else {
  BivData <- matrix(rnorm(120), ncol = 2)
  BivData <- rbind(BivData, matrix(c(6,6, 6, -2), ncol = 2))
}

result <- projmedian(x = BivData)
plot(BivData)
points(result$max, col = "red", pch = 15)
points(result$gravity, col = "blue", pch = 16)
```
rdepth

points(result$Huber, col = "orange", pch = 17)

# Options for the underlying projdepth routine may be passed
# using the options argument.
result <- projmedian(x = BivData,
  options = list(type = "Affine",
    ndir = 10,
    stand = "MedMad",
    h = nrow(BivData)
  )
)

plot(BivData)
points(result$max, col = "red", pch = 15)
points(result$gravity, col = "blue", pch = 16)
points(result$Huber, col = "orange", pch = 17)

# One may also calculate the depths of the points in the data
# separately. This avoids having to recompute the depths when these
# are previously calculated.
depth.result <- projdepth(x = BivData)
result <- projmedian(x = BivData,
  projection.depts = depth.result$depthX)

rdepth

Regression depth of hyperplanes

Description
Computes the regression depth of several hyperplanes with respect to a multiple regression dataset with \( p \) explanatory variables. The calculation is exact for \( p \leq 3 \). An approximate algorithm is used for \( p > 3 \).

Usage

rdepth(x, z = NULL, ndir = NULL)

Arguments

\( x \)  
An \( n \) by \( p + 1 \) regression dataset. The first \( p \) columns are the explanatory variables. The last column corresponds to the response variable.

\( z \)  
An \( m \) by \( p + 1 \) matrix containing rowwise the hyperplanes for which to compute the regression depth. The first column should contain the intercepts. If \( z \) is not specified, it is set equal to \( x \).

\( ndir \)  
Controls the number of directions when the approximate algorithm is used. Defaults to \( 250p \).
Details

Regression depth has been introduced in Rousseeuw and Hubert (1999). To compute the regression depth of a hyperplane, different algorithms are used. When \( p \leq 3 \) it can be computed exactly. In higher dimensions an approximate algorithm is used (Rousseeuw and Struyf 1998).

It is first checked whether the data \( x \) lie in a subspace of dimension smaller than \( p + 1 \). If so, a warning is given, as well as the dimension of the subspace and a direction which is orthogonal to it.

Value

A list with components:

- `depthZ` A vector containing the regression depth of the hyperplanes in \( z \).
- `singularSubsets` A vector of length \( m \).
  For each hyperplane, it contains the number of singular subsamples when the approximate algorithm is used. The actual number of directions used in the algorithm therefore equals `ndir`singularSubsets`.
- `dimension` When the data \( x \) are lying in a lower dimensional subspace, the dimension of this subspace.
- `hyperplane` When the data \( x \) are lying in a lower dimensional subspace, a direction orthogonal to this subspace.

Author(s)

P. Segaert using Fortran code by P.J. Rousseeuw, I. Ruts and A. Struyf

References


See Also

`rdepthmedian, cmltest`

Examples

```r
# Illustrate the concept of regression depth in the case of simple
# linear regression.

data("stars")

# Compute the least squares fit. Due to outliers
# this fit will be bad and thus should result in a small
# regression depth.
temp <- lsfit(x = stars[,1], y = stars[,2])$coefficients
```
intercept <- temp[1]
slope <- temp[2]
plot(stars, ylim = c(4, 7))
abline(a = intercept, b = slope)
abline(a = -9.2, b = 3.2, col = "red")

# Let us compare the regression depth of the least squares fit
# with the depth of the better fitting red regression line.
z <- rbind(cbind(intercept, slope),
            cbind(-9.2, 3.2))
result <- rdepth(x = stars, z = z)
result$depthZ
text(x = 3.8, y = 5.3, labels = round(result$depthZ[1], digits = 2))
text(x = 4.45, y = 4.8, labels = round(result$depthZ[2], digits = 2), col = "red")

# Compute the depth of some other regression lines to illustrate the concept.
# Note that the stars data set has 47 observations and 1/47 = 0.0212766.
z <- rbind(cbind(6.2, 0),
            cbind(6.5, 0))
result <- rdepth(x = stars, z = z)
abline(a = 6.2, b = 0, col = "blue")
abline(a = 6.5, b = 0, col = "darkgreen")
text(x = 3.8, y = 6.25, labels = round(result$depthZ[1], digits = 2), col = "blue")
text(x = 4, y = 6.55, labels = round(result$depthZ[2], digits = 2), col = "darkgreen")

# One point needs to removed to make the blue line a nonfit. The green line is clearly
# a nonfit.

rdepthmedian

Hyperplane of maximal regression depth

Description
Computes the deepest regression, i.e. the hyperplane with maximal regression depth given a regression dataset with \( p \) explanatory variables. The computation is exact for simple regression, and approximate otherwise.

Usage
rdepthmedian(x, maxit = NULL)

Arguments
x
An \( n \) by \( p + 1 \) regression data matrix.
The first \( p \) columns are the explanatory variables. The last column corresponds to the response variable.

maxit
The maximum number of iterations.
Defaults to 100.
Details

In simple regression the deepest regression fit can be computed exactly by considering all lines through two data points and taking the one with maximal regression depth. If several lines have the same maximal regression depth, their average is taken.

When $p > 1$, the approximate MEDSWEEP algorithm is applied (Van Aelst et al, 2002).

It is first checked whether the data lie in a subspace of dimension smaller than $p + 1$. If so, a warning is given, as well as the dimension of the subspace and a direction which is orthogonal to it.

Value

A list with components:

- **deepest**: A $(p + 1)$ vector containing the estimates of the deepest regression fit. The last $p$ values are the slopes, the first value corresponds to the intercept.
- **depth**: The depth of the deepest regression hyperplane.
- **ndir**: The number of performed iterations used in the medsweep algorithm.
- **dimension**: When the data $x$ are lying in a lower dimensional subspace, the dimension of this subspace.
- **hyperplane**: When the data $x$ are lying in a lower dimensional subspace, a direction orthogonal to this subspace.

Author(s)

P. Segaert using Fortran code by S. Van Aelst

References


See Also

rdepth, cmltest

Examples

```r
# Illustrate the concept of deepest regression line in the case of simple linear regression.
data("stars")
plot(stars)
result <- rdepthmedian(x = stars)
abline(result$deepest)

x <- matrix(rnorm(3000), ncol = 3) + 10
rdepthmedian(x = x)
```
sdepth

Simplicial depth of points relative to a dataset

Description

Computes the simplicial depth of \( p \)-dimensional points \( z \) relative to a \( p \)-dimensional dataset \( x \). Only dimension \( p \leq 2 \) is supported.

Usage

sdepth(x, z = NULL)

Arguments

\( x \)  
An \( n \) by \( p \) data matrix with observations in the rows and variables in the columns.

\( z \)  
An optional \( m \) by \( p \) matrix containing rowwise the points \( z_i \) for which to compute the simplicial depth. If \( z \) is not specified, it is set equal to \( x \).

Details

The simplicial depth has been introduced by Liu (1990). The simplicial depth of a point \( z_i \) is defined as the number of simplices with vertices in \( x \) that contain \( z_i \). Exact computation of the simplicial depth for bivariate data is performed by means of the algorithm described in Rousseeuw and Ruts (1996). To visualize the depth of bivariate data one can apply the \texttt{mrainbowplot} function. It plots the data with coloring according to their depth.

It is first checked whether the data lie in a subspace of dimension smaller than \( p \). If so, a warning is given, as well as the dimension of the subspace and a direction which is orthogonal to it.

Value

A list with components:

- \texttt{depthZ}  
Vector of length \( m \) giving the simplicial depth of the points in \( z \).

- \texttt{dimension}  
When the data \( x \) are lying in a lower dimensional subspace, the dimension of this subspace.

- \texttt{hyperplane}  
When the data \( x \) are lying in a lower dimensional subspace, a direction orthogonal to this subspace.

Author(s)

P. Segaert, based on Fortran code by P.J. Rousseeuw, I. Ruts and A. Struyf.

References


See Also

mrainbowplot

Examples

data(bloodfat)
Result <- sdepth(x = bloodfat)
mrainbowplot(bloodfat, depth = Result$depthz)

sprojdepth

Skewness-adjusted projection depth of points relative to a dataset

Description

Computes the skewness-adjusted projection depth of \( p \)-dimensional points \( z \) relative to a \( p \)-dimensional dataset \( x \).

Usage

sprojdepth(x, z = NULL, options = NULL)

Arguments

x  
An \( n \) by \( p \) data matrix with observations in the rows and variables in the columns.

z  
An optional \( m \) by \( p \) matrix containing rowwise the points \( z_i \) for which to compute the projection depth. If \( z \) is not specified, it is set equal to \( x \).

options  
A list of options to pass to the underlying adjoutl routine. See adjoutl for the full list of options.

Details

Skewness-adjusted projection depth is based on the adjusted outlyingness and is computed as \( 1/(1 + AO) \). As adjusted outlyingness extends the Stahel-Donoho outlyingness towards skewed distributions, the skewness-adjusted projection depth is suited for both elliptical distributions and skewed multivariate data.

It is first checked whether the data is found to lie in a subspace of dimension lower than \( p \). If so, a warning is given, as well as the dimension of the subspace and a direction which is orthogonal to it.

See adjoutl for more details on the computation of the AO. To visualize the depth of bivariate data one can apply the mrainbowplot function. It plots the data colored according to their depth.

The output values of this function are based on the output of the adjoutl function. More details can be found there.
Value

A list with components:

- **depthX**: Vector of length \( n \) giving the skewness-adjusted projection depth of the observations in \( x \).
- **depthZ**: Vector of length \( m \) giving the skewness-adjusted projection depth of the points in \( z \).
- **cutoff**: Points whose skew-adjusted projection depth is smaller than this cutoff can be considered as outliers.
- **flagX**: Observations of \( x \) whose adjusted outlyingness exceeds the cutoff receive a flag FALSE, regular observations receive a flag TRUE.
- **flagZ**: Points of \( z \) whose adjusted outlyingness exceeds the cutoff receive a flag equal to FALSE, otherwise they receive a flag TRUE.
- **singularSubsets**: When the input parameter type is equal to "Affine", the number of \( p \)-subsets that span a subspace of dimension smaller than \( p-1 \). In that case the orthogonal direction can not be uniquely determined. This is an indication that the data are not in general position. When the input parameter type is equal to "Rotation" it is possible that two randomly selected points of the data coincide due to ties in the data. In this case this value signals how many times this is the case.
- **dimension**: When the data \( x \) are lying in a lower dimensional subspace, the dimension of this subspace.
- **hyperplane**: When the data \( x \) are lying in a lower dimensional subspace, a direction orthogonal to this subspace. When a direction \( v \) is found such that the robust skew-adjusted scale of \( xv \) is equal to zero, this equals \( v \).
- **inSubspace**: When a direction \( v \) is found such that \( AO(xv) \) is ill-defined, the observations from \( x \) which belong to the hyperplane orthogonal to \( v \) receive a value TRUE. The other observations receive a value FALSE.

Author(s)

P. Segaert with original code from M. Maechler, G. Brys, K. Vakili

References


See Also

adjOutl, sprojmedian, mrainbowplot, dirOutl, outlyingness
Examples

# Compute the skewness-adjusted projection depth
# of a simple two-dimensional dataset.
# Outliers are plotted in red.

data(bloodfat)
Result <- sprojdepth(x = bloodfat)
IndOutliers <- which(!Result$flagX)
plot(bloodfat)
points(bloodfat[IndOutliers,], col = "red")

# A multivariate rainbowplot may be obtained using mrainbowplot.
plot.options = list(legend.title = "SPD")
mrainbowplot(x = bloodfat,
   depths = Result$depthX, plot.options = plot.options)

# Options for the underlying outlyingness routine may be passed
# using the options argument.
Result <- sprojdepth(x = bloodfat,
   options = list(type = "Affine",
       ndir = 1000,
       seed = 12345
   )
)

sprojmedian

Location estimates based on skewness-adjusted projection depth.

Description

Computes a skewness-adjusted projection depth based location estimate of a \( p \)-dimensional dataset \( x \).

Usage

sprojmedian(x, sprojection.depths = NULL, options = NULL)

Arguments

x

An \( n \) by \( p \) data matrix with observations in the rows and variables in the columns.

sprojection.depths

Vector containing the skewness-adjusted projection depths of the points in \( x \).

options

A list of options to pass to the sprojdepth routine. See sprojdepth for more details.
Details

The algorithm depends on the function `sprojdepth` to calculate the skewness-adjusted projection depths of the dataset $x$. If the skewness-adjusted projection depths of the points $x_i$ have already been calculated they can be passed as an optional argument to save computation time. If not, skewness-adjusted projection depths will be calculated and the user can pass a list with options to the `sprojdepth` function.

It is first checked whether the data lie in a subspace of dimension smaller than $p$. If so, a warning is given, as well as the dimension of the subspace and a direction which is orthogonal to it.

Value

A list with components:

- `max`: The point of $x$ with maximal skewness-adjusted projection depth. If multiple points have maximum depth, their center of gravity is returned.
- `gravity`: The center of gravity for the 50 percent points with highest skewness-adjusted projection depth. Also called the center of gravity of the bag.

Author(s)

P. Segaert

References


See Also

`adjOutl`, `sprojdepth`, `dirOutl`, `outlyingness`

Examples

```r
# Compute a location estimate of a simple two-dimensional dataset.
data(bloodfat)
result <- sprojmedian(x = bloodfat)
plot(bloodfat)
points(result$max, col = "red", pch = 15)
points(result$gravity, col = "blue", pch = 16)
```

# Options for the underlying sprojdepth routine may be passed
# using the options argument.
result <- sprojmedian(x = bloodfat,
    options = list(type = "Affine",
                   ndir = 10)
```
stars

plot(bloodfat)
points(result$max, col = "red", pch = 15)
points(result$gravity, col = "blue", pch = 16)

# One may also calculate the depths of the points in the data
# separately. This avoids having to recompute the depths when these
# are previously calculated.
depth.result <- sprojdepth(x = bloodfat)
result <- sprojmedian(x = bloodfat,
  sprojection.depths = depth.result$depthX)

---

stars data

Description

Data corresponding to a Hertzsprung-Russel diagram of the star cluster CYG OB1 containing 47 stars in the direction of Cygnus. A typical Hertzsprung-Russel diagram shows the logarithm of the temperature in reverse order from high to low.

The data has been taken from Humphreys (1978) by C. Doom who calibrated them according to Vansina en De Greve (1982).

Usage

data(stars)

Format

Following variables are included:

- LogTemp  Logarithm of the effective temperature at the star’s surface.
- LogLight  Logarithm of a star’s light intensity.

Source


References


symtest

Examples

data(stars)
plot(stars,xlim=rev(range(stars[,1])))

summary

symtest x z options=list()

Arguments

x An n by 2 data matrix.
z A vector of length 2.
options A list of options to pass to hdepth.
See hdepth for more information.

Details

The following hypothesis test is performed:

H0: The data come from a continuous distribution P which is angularly symmetric about z.
The test statistic being used is the halfspace depth of z with respect to x. Under the null hypothesis
the halfspace depth of z equals 1/2. The distribution of the test statistic under H0 is

\[ F_n(k) = \frac{1}{2} \int_{-\infty}^{k} h_{P,2}(y) \, dy \]

The p-value of the test is \[ F_n(k) \] with \[ k = h_{P,2}(x, z) \].

Value

pval The p-value of the hypothesis test.

Author(s)

P. Segaert

References

Model Validity, Birkhäuser Boston, pp.401–412.
Examples

```r
# Perform the test on a simple data example.
data(cardata90)
syntes(x = cardata90, z = hdepthmedian(cardata90)$median)
plot(cardata90)
```

---

tables $Near$ Infrared Spectroscopy responses for a batch of pills

Description

The original data set consists of Near Infrared Spectroscopy (NIR) data of a batch of pills with different levels of the active compound. This data set considers 70 observations weighing 90 mg and 20 observations weighing 250 mg and also differ in the amount of active compound.

The data corresponds to the form discussed in Hubert et al. (2015), see below.

Usage

```r
data("wine")
```

Format

A three dimensional $t = 404$ by $n = 90$ by $p = 3$ array, with $t$ the number of observed time points, $n$ the number of functional observations and $p$ the number of measurements for every functional observation at every wavelength.

Details

For each wavelength and each curve, a three-dimensional vector of observations is available. The second coordinate corresponds to the original data, the first coordinate is corresponds to the mean of the corresponding curve. The last component corresponds to the derivative of the original curve data.

When using this data, please cite both of the references listed below.

Source


References

Proton Nuclear Magnetic Resonance spectra of 40 different wine samples

Description
The original data set consists of Proton Nuclear Magnetic Resonance (NMR) spectra of 40 different wine samples in the spectral region from 6.00 to 0.50. This data set corresponds to the region between wavelengths 5.62 and 5.37 only for which \( t = 397 \) measurements are available for each curve. The data has been analyzed in Hubert et al. (2015), see below.

Usage
```
data("wine")
```

Format
A three dimensional \( t = 397 \) by \( n = 40 \) by \( p = 1 \) array, with \( t \) the number of observed time points, \( n \) the number of functional observations and \( p \) the number of measurements for every functional observation at every wavelength.

Details
When using this data set, please cite both of the references below.

Source

References

Examples
```
data(wine)
matplot(wine[,1], type="l", lty=1, col = "black")
```
Index

*Topic **Graphical**
  bagplot, 9
  fheatmap, 31
  fom, 32
  mrainbowplot, 50
  plotContours, 58

*Topic **datasets**
  bloodfat, 11
  cardata90, 12
  characterA, 13
  characterI, 14
  geological, 37
  mri, 51
  octane, 52
  plane, 57
  stars, 72
  tablets, 74
  wine, 75

*Topic **functional**
  distSpace, 25
  fOutl, 34
  mfd, 45
  mfdmedian, 48

*Topic **multivariate**
  adjOutl, 3
  bagdistance, 6
  compBagplot, 16
  depthContour, 19
  dirOutl, 22
  distSpace, 25
  dprojdepth, 28
  dprojmedian, 30
  hdepth, 38
  hdepthmedian, 42
  medcouple, 44
  outlyingness, 53
  projdepth, 59
  projmedian, 61
  sdepth, 67
  sprojdepth, 68
  sprojmedian, 70
  symtest, 73

*Topic **regression**
  cmltest, 15
  rdepth, 63
  rdepthmedian, 65
  adjbox, 5
  adjOutl, 3, 25, 29, 31, 36, 55, 60, 62, 69, 71
  adjOutlyingness, 5

  bagdistance, 6, 21, 36, 41
  bagplot, 8, 9, 18, 41
  bloodfat, 11
  cardata90, 12
  characterA, 13
  characterI, 14
  cmltest, 15, 64, 66
  compBagplot, 10, 16
  covMcd, 54
  depthContour, 8, 19, 47, 58
  dirOutl, 5, 22, 29, 31, 55, 60, 62, 69, 71
  distSpace, 25
  dprojdepth, 25, 28, 31, 47
  dprojmedian, 25, 29, 30
  fheatmap, 31
  fom, 32
  fOutl, 33, 34
  geological, 37
  glass, 38
  hdepth, 8, 10, 18, 38, 47
  hdepthmedian, 41, 42
  medcouple, 44
  mfd, 45

76
INDEX

mfdmedian, 48
mrainbowplot, 29, 41, 50, 60, 68, 69
mri, 51

octane, 52
outlyingness, 5, 25, 29, 31, 36, 53, 60, 62, 69, 71

plane, 57
plotContours, 21, 58
projdepth, 10, 18, 47, 55, 59, 62
projmedian, 55, 60, 61

rdepth, 15, 63, 66
rdepthmedian, 15, 64, 65

sdepth, 47, 67
sprojdepth, 5, 10, 18, 47, 68, 71
sprojmedian, 5, 69, 70
stars, 72
symtest, 73

tablets, 74

wine, 75