Package ‘penppml’

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Title Penalized Poisson Pseudo Maximum Likelihood Regression

Version 0.1.0

Description A set of tools that enables efficient estimation of penalized Poisson Pseudo Maximum Likelihood regressions, using lasso or ridge penalties, for models that feature one or more sets of high-dimensional fixed effects. The methodology is based on Breinlich, Corradi, Rocha, Ruta, Santos Silva, and Zylkin (2021) <http://hdl.handle.net/10986/35451> and takes advantage of the method of alternating projections of Gaure (2013) <doi:10.1016/j.csda.2013.03.024> for dealing with HDFE, as well as the coordinate descent algorithm of Friedman, Hastie and Tibshirani (2010) <doi:10.18637/jss.v033.i01> for fitting lasso regressions. The package is also able to carry out cross-validation and to implement the plugin lasso of Belloni, Chernozhukov, Hansen and Kozbur (2016) <doi:10.1080/07350015.2015.1102733>.

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**AtA**

*Computing A'A*

**Description**

Computes A'A using C++.

**Usage**

AtA(A)

**Arguments**

- **A**  
  A matrix.

---

**cluster_matrix**

*Cluster-robust Standard Error Estimation*

**Description**

cluster_matrix is a helper for computation of cluster-robust standard errors.

**Usage**

cluster_matrix(e, cluster, x)

**Arguments**

- **e**  
  Vector of residuals.
- **cluster**  
  Vector of clusters.
- **x**  
  Regressor matrix.

**Value**

Gives the XeeX matrix.
**collinearity_check**  
*Checking for Perfect Multicollinearity*

**Description**

collinearity_check checks for perfect multicollinearity in a model with high-dimensional fixed effects. It calls \texttt{lfe::demeanlist} in order to partial out the fixed effects, and then uses \texttt{stats::lm.wfit} to discard linearly dependent variables.

**Usage**

collinearity_check(y, x, fes, hdfetol)

**Arguments**

- **y**: Dependent variable (a numeric vector).
- **x**: Regressor matrix.
- **fes**: List of fixed effects.
- **hdfetol**: Tolerance for the centering, passed on to \texttt{lfe::demeanlist}.

**Value**

A numeric vector containing the variables that pass the collinearity check.

---

**compute_fes**  
*Fixed Effects Computation*

**Description**

This function is a helper for \texttt{xvalidate} that computes FEs using PPML First Order Conditions (FOCs).

**Usage**

compute_fes(
    y,
    fes,
    x,
    b,
    insample_obs = rep(1, n),
    onlymus = FALSE,
    tol = 1e-08,
    verbose = FALSE
)
countries

Arguments

- **y**: Dependent variable (a vector).
- **fes**: List of fixed effects.
- **x**: Regressor matrix.
- **b**: A vector of coefficient estimates.
- **insample_obs**: Vector of observations used to estimate the b coefficients.
- **onlymus**: Logical. If TRUE, returns only the conditional means.
- **tol**: A tolerance parameter.
- **verbose**: Logical. If TRUE, prints messages to the console while evaluating.

Value

If onlymus = TRUE, the vector of conditional means. Otherwise, a list with two elements:

- **mu**: conditional means.
- **fe_values**: fixed effects.

<table>
<thead>
<tr>
<th>countries</th>
<th>Country ISO Codes</th>
</tr>
</thead>
</table>

Description

An auxiliary data set with basic geographic information about country ISO 3166 codes included in the trade data set.

Usage

```r
countries
```

Format

A data frame with 249 rows and 4 variables.

- **iso**: Country ISO 3166 code.
- **name**: Country name.
- **region**: Continent.
- **subregion**: sub-continental region.

Source

eigenMatMult

**Faster Matrix Multiplication**

**Description**

Faster matrix multiplication using C++.

**Usage**

```c
eigenMatMult(A, B)
eigenMapMatMult(A, B)
```

**Arguments**

- `A`, `B` Matrices.

---

fastolsCpp

**Faster Least Squares Estimation**

**Description**

Finds Least Squares solutions using C++.

**Usage**

```c
fastolsCpp(X, y)
```

**Arguments**

- `X` Regressor matrix.
- `y` Dependent variable (a vector).

**Value**

The vector of parameter (beta) estimates.
fastridge

Finding Ridge Regression Solutions

Description
A wrapper around fastridgeCpp, for faster computation of the analytical solution for ridge regression.

Usage
fastridge(x, y, weights = rep(1/n, n), lambda, standardize = TRUE)

Arguments
- x: Regressor matrix.
- y: Dependent variable (a numeric vector).
- weights: Vector of weights.
- lambda: Penalty parameter.
- standardize: Logical. If TRUE, x is standardized using the weights.

Value
A vector of coefficient (beta) estimates.

fastridgeCpp

Faster Ridge Regression

Description
Finds Ridge solutions using C++.

Usage
fastridgeCpp(X, y, lambda)

Arguments
- X: Regressor matrix.
- y: Dependent variable (a vector).
- lambda: Penalty parameter (a number).

Value
The vector of parameter (beta) estimates.
**faststddev**  
*Faster Standard Deviation*

**Description**  
Computes standard deviation using C++.

**Usage**  
`faststddev(X, w)`

**Arguments**  
- `X`: Regressor matrix.
- `w`: Weights.

**Value**  
Vector of standard deviations of the parameter estimates.

---

**fastwmean**  
*Faster Weighted Mean*

**Description**  
Computes weighted mean using C++.

**Usage**  
`fastwmean(X, w)`

**Arguments**  
- `X`: Regressor matrix.
- `w`: Weights.

**Value**  
Weighted mean.
Generating a List of Fixed Effects

**Description**

genfes generates a list of fixed effects by creating interactions of paired factors.

**Usage**

genfes(data, inter)

**Arguments**

data A data frame including the factors.
inter A list: each element includes the variables to be interacted (both names and column

**Value**

A list containing the desired interactions of vars, with the same length as inter.

Generating Model Structure

**Description**

genmodel transforms a data frame into the needed components for our main functions (a y vector, a x matrix and a fes list).

**Usage**

genmodel(
  data,
  dep = 1,
  indep = NULL,
  fixed = NULL,
  cluster = NULL,
  selectobs = NULL
)
Arguments

- **data**: A data frame containing all relevant variables.
- **dep**: A string with the name of the independent variable or a column number.
- **indep**: A vector with the names or column numbers of the regressors. If left unspecified, all remaining variables (excluding fixed effects) are included in the regressor matrix.
- **fixed**: A vector with the names or column numbers of factor variables identifying the fixed effects, or a list with the desired interactions between variables in `data`.
- **cluster**: Optional. A string with the name of the clustering variable or a column number. It’s also possible to input a vector with several variables, in which case the interaction of all of them is taken as the clustering variable.
- **selectobs**: Optional. A vector indicating which observations to use.

Value

A list with four elements:

- **y**: y vector.
- **x**: x matrix.
- **fes**: list of fixed effects.
- **cluster**: cluster vector.

Description

**hdfeppml** fits an (unpenalized) Poisson Pseudo Maximum Likelihood (PPML) model with high-dimensional fixed effects (HDFE).

Usage

```r
hdfeppml(
  data,
  dep = 1,
  indep = NULL,
  fixed = NULL,
  cluster = NULL,
  selectobs = NULL,
  ...
)
```
Arguments

data       A data frame containing all relevant variables.
dep        A string with the name of the independent variable or a column number.
indep      A vector with the names or column numbers of the regressors. If left unspecified, all remaining variables (excluding fixed effects) are included in the regressor matrix.
fixed      A vector with the names or column numbers of factor variables identifying the fixed effects, or a list with the desired interactions between variables in data.
cluster    Optional. A string with the name of the clustering variable or a column number. It’s also possible to input a vector with several variables, in which case the interaction of all of them is taken as the clustering variable.
selectobs  Optional. A vector indicating which observations to use (either a logical vector or a numeric vector with row numbers, as usual when subsetting in R).
...
Further options. For a full list, see hdfeppml_int.

Details

This function is a thin wrapper around hdfeppml_int, providing a more convenient interface for data frames. Whereas the internal function requires some preliminary handling of data sets (y must be a vector, x must be a matrix and fixed effects fes must be provided in a list), the wrapper takes a full data frame in the data argument, and users can simply specify which variables correspond to y, x and the fixed effects, using either variable names or column numbers.

More formally, hdfeppml_int performs iteratively re-weighted least squares (IRLS) on a transformed model, as described in Correia, Guimarães and Zylkin (2020) and similar to the ppmlhdfe package in Stata. In each iteration, the function calculates the transformed dependent variable, partials out the fixed effects (calling lfe::demeanlist) and then solves a weighted least squares problem (using fast C++ implementation).

Value

A list with the following elements:

- coefficients: a 1 x ncol(x) matrix with coefficient (beta) estimates.
- residuals: a 1 x length(y) matrix with the residuals of the model.
- mu: a 1 x length(y) matrix with the final values of the conditional mean µ.
- deviance:
- bic: Bayesian Information Criterion.
- x_resid: matrix of demeaned regressors.
- z_resid: vector of demeaned (transformed) dependent variable.
- se: standard errors of the coefficients.
References


Examples

```r
# To reduce run time, we keep only countries in the Americas:
americas <- countries$iso[countries$region == "Americas"]
test <- hdfeppml(data = trade[, -(5:6)],
                   dep = "export",
                   fixed = list(c("exp", "time"),
                                 c("imp", "time"),
                                 c("exp", "imp")),
                   selectobs = (trade$imp %in% americas) & (trade$exp %in% americas))
```

**Description**

`hdfeppml_int` is the internal algorithm called by `hdfeppml` to fit an (unpenalized) Poisson Pseudo Maximum Likelihood (PPML) regression with high-dimensional fixed effects (HDFE). It takes a vector with the dependent variable, a regressor matrix and a set of fixed effects (in list form: each element in the list should be a separate HDFE).

**Usage**

```r
hdfeppml_int(
  y,
  x,
  fes,
  tol = 1e-08,
  hdfetol = 1e-04,
  colcheck = TRUE,
  mu = NULL,
)```

saveX = TRUE,
init_z = NULL,
verbose = FALSE,
maxiter = 1000,
cluster = NULL,
vcv = TRUE)

Arguments

y Dependent variable (a vector)
x Regressor matrix.
fes List of fixed effects.
tol Tolerance parameter for convergence of the IRLS algorithm.
hdfe_tol Tolerance parameter for the within-transformation step, passed on to lfe::demeanlist.
colcheck Logical. If TRUE, checks for perfect multicollinearity in x.
mu Optional: initial values of the conditional mean µ, to be used as weights in the first iteration of the algorithm.
saveX Logical. If TRUE, it returns the values of x and z after partialling out the fixed effects.
init_z Optional: initial values of the transformed dependent variable, to be used in the first iteration of the algorithm.
verbose Logical. If TRUE, it prints information to the screen while evaluating.
maxiter Maximum number of iterations (a number).
cluster Optional: a vector classifying observations into clusters (to use when calculating SEs).
vcv Logical. If TRUE (the default), it returns standard errors.

Details

More formally, hdeviceppml_int performs iteratively re-weighted least squares (IRLS) on a transformed model, as described in Correia, Guimarães and Zylkin (2020) and similar to the ppmlhdfe package in Stata. In each iteration, the function calculates the transformed dependent variable, partials out the fixed effects (calling lfe::demeanlist, which uses the algorithm in Gaure (2013)) and then solves a weighted least squares problem (using fast C++ implementation).

Value

A list with the following elements:

- coefficients: a 1 x ncol(x) matrix with coefficient (beta) estimates.
- residuals: a 1 x length(y) matrix with the residuals of the model.
- mu: a 1 x length(y) matrix with the final values of the conditional mean µ.
- deviance:
- bic: Bayesian Information Criterion.
• x_resid: matrix of demeaned regressors.
• z_resid: vector of demeaned (transformed) dependent variable.
• se: standard errors of the coefficients.

References

Examples
# To reduce run time, we keep only countries in the Americas:
americas <- countries$iso[countries$region == "Americas"]
trade <- trade[(trade$imp %in% americas) & (trade$exp %in% americas), ]
# Now generate the needed x, y and fes objects:
y <- trade$export
x <- data.matrix(trade[, -1:-6])
imp_time = interaction(trade$imp, trade$time),
imp_time = interaction(trade$imp, trade$time),
pair = interaction(trade$exp, trade$imp))
# Finally, the call to hdfeppml_int:
reg <- hdfeppml_int(y = y, x = x, fes = fes)

iceberg

Iceberg Lasso Implementation (in development)

Description
A function performs standard plugin lasso PPML estimation (without fixed effects) for several dependent variables in a single step. This is still IN DEVELOPMENT: at the current stage, only coefficient estimates are are provided and there is no support for clustered errors.

Usage
iceberg(data, dep, indep = NULL, selectobs = NULL, ...)

**Arguments**

- **data**: A data frame containing all relevant variables.
- **dep**: A string with the names of the independent variables or their column numbers.
- **indep**: A vector with the names or column numbers of the regressors. If left unspecified, all remaining variables (excluding fixed effects) are included in the regressor matrix.
- **selectobs**: Optional. A vector indicating which observations to use (either a logical vector or a numeric vector with row numbers, as usual when subsetting in R).

... Further arguments, including:
- **tol**: Tolerance parameter for convergence of the IRLS algorithm.
- **glmnet tol**: Tolerance parameter to be passed on to glmnet::glmnet.
- **penweights**: Optional: a vector of coefficient-specific penalties to use in plugin lasso.
- **colcheck**: Logical. If TRUE, checks for perfect multicollinearity in x.
- **K**: Maximum number of iterations.
- **verbose**: Logical. If TRUE, prints information to the screen while evaluating.
- **lambda**: Penalty parameter (a number).
- **phipost**: Logical. If TRUE, it carries out a post-lasso estimation with just the selected variables and reports the coefficients from this regression.

**Details**

This function enables users to implement the "iceberg" step in the two-step procedure described in Breinlich, Corradi, Rocha, Ruta, Santos Silva and Zylkin (2020). To do this after using the plugin method in mlfitppml, just select all the variables with non-zero coefficients in dep and the remaining regressors in indep. The function will then perform separate lasso estimation on each of the selected dependent variables and report the coefficients.

**Value**

A matrix with coefficient estimates for all dependent variables.

**References**


Examples

```r
iceberg_results <- iceberg(data = trade[, -(1:6)],
  dep = c("ad_prov_14", "cp_prov_23", "tbt_prov_07",
    "tbt_prov_33", "tf_prov_41", "tf_prov_45"),
  selectobs = (trade$time == "2016"))
```

---

**manyouter**  
*Many Outer Products*

**Description**

Compute a large number of outer products (useful for clustered SEs) using C++.

**Usage**

```r
manyouter(A, B, c)
```

**Arguments**

- **A, B**  
  Numeric vectors.

- **c**  
  Integer.

---

**mlfitppml**  
*General Penalized PPML Estimation*

**Description**

*mlfitppml* is a general-purpose wrapper function for penalized PPML estimation. This is a flexible tool that allows users to select:

- Penalty type: either lasso or ridge.
- Penalty parameter: users can provide a single global value for lambda (a single regression is estimated), a vector of lambda values (the function estimates the regression using each of them, sequentially) or even coefficient-specific penalty weights.
- Method: plugin lasso estimates can be obtained directly from this function too.
- Cross-validation: if this option is enabled, the function uses IDs provided by the user to perform k-fold cross-validation and reports the resulting RMSE for all lambda values.
Usage

```r
mlfitppml(
  data,
  dep = 1,
  indep = NULL,
  fixed = NULL,
  cluster = NULL,
  selectobs = NULL,
  ...
)
```

Arguments

data
A data frame containing all relevant variables.

dep
A string with the name of the independent variable or a column number.

indep
A vector with the names or column numbers of the regressors. If left unspecified, all remaining variables (excluding fixed effects) are included in the regressor matrix.

fixed
A vector with the names or column numbers of factor variables identifying the fixed effects, or a list with the desired interactions between variables in `data`.

cluster
Optional. A string with the name of the clustering variable or a column number. It’s also possible to input a vector with several variables, in which case the interaction of all of them is taken as the clustering variable.

selectobs
Optional. A vector indicating which observations to use (either a logical vector or a numeric vector with row numbers, as usual when subsetting in R).

... Further arguments, including:

- **penalty**: A string indicating the penalty type. Currently supported: "lasso" and "ridge".
- **method**: The user can set this equal to "plugin" to perform the plugin algorithm with coefficient-specific penalty weights (see details). Otherwise, a single global penalty is used.
- **post**: Logical. If TRUE, estimates a post-penalty regression with the selected variables.
- **xval**: Logical. If TRUE, cross-validation is performed using the IDs provided in the IDs argument as folds. Note that, by default, observations are assigned individual IDs, which makes the cross-validation algorithm very time-consuming.

For a full list of options, see `mlfitppml_int`.

Details

This function is a thin wrapper around `mlfitppml_int`, providing a more convenient interface for data frames. Whereas the internal function requires some preliminary handling of data sets (y must be a vector, x must be a matrix and fes must be provided in a list), the wrapper takes a full data frame in the data argument, and users can simply specify which variables correspond to y, x and the fixed effects, using either variable names or column numbers.
For technical details on the algorithms used, see hdefpml (post-lasso regression), penhdefpml (standard penalized regression), penhdefpml_cluster (plugin lasso), and xvalidate (cross-validation).

Value

A list with the following elements:

- **beta**: if post = FALSE, a length(lambdas) x ncol(x) matrix with coefficient (beta) estimates from the penalized regressions. If post = TRUE, this is the matrix of coefficients from the post-penalty regressions.
- **beta_pre**: if post = TRUE, a length(lambdas) x ncol(x) matrix with coefficient (beta) estimates from the penalized regressions.
- **bic**: Bayesian Information Criterion.
- **lambdas**: vector of penalty parameters.
- **ses**: standard errors of the coefficients of the post-penalty regression. Note that these are only provided when post = TRUE.
- **rmse**: if xval = TRUE, a matrix with the root mean squared error (RMSE - column 2) for each value of lambda (column 1), obtained by cross-validation.
- **phi**: coefficient-specific penalty weights (only if method == "plugin").

References


Examples

# To reduce run time, we keep only countries in the Americas:
americas <- countries$iso[countries$region == "Americas"]
# Now we can use our main functions on the reduced trade data set:
test <- mlfitppml(data = trade[, -(5:6)],
  dep = "export",
  fixed = list(c("exp", "time"),
              c("imp", "time"),
              c("exp", "imp")),
  selectobs = (trade$imp %in% americas) & (trade$exp %in% americas),
  lambdas = c(0.01, 0.001),
  post = TRUE)
**mlfitppml_int**

Description

`mlfitppml_int` is the internal wrapper called by `mlfitppml` for penalized PPML estimation. This in turn calls `penhdfeppml_int`, `penhdfeppml_cluster_int` and `hdfeppml_int` as needed. It takes a vector with the dependent variable, a regressor matrix and a set of fixed effects (in list form: each element in the list should be a separate HDFE). This is a flexible tool that allows users to select:

- Penalty type: either lasso or ridge.
- Penalty parameter: users can provide a single global value for lambda (a single regression is estimated), a vector of lambda values (the function estimates the regression using each of them, sequentially) or even coefficient-specific penalty weights.
- Method: plugin lasso estimates can be obtained directly from this function too.
- Cross-validation: if this option is enabled, the function uses IDs provided by the user to perform k-fold cross-validation and reports the resulting RMSE for all lambda values.

Usage

```r
mlfitppml_int(
  y,
  x,
  fes,
  lambdas,
  penalty = "lasso",
  tol = 1e-08,
  hdfetol = 1e-04,
  colcheck = TRUE,
  post = TRUE,
  cluster = NULL,
  method = "bic",
  IDs = 1:n,
  verbose = FALSE,
  xval = FALSE,
  standardize = TRUE,
  vcv = TRUE,
  penweights = NULL,
  K = 15
)
```
Arguments

- **y**: Dependent variable (a vector)
- **x**: Regressor matrix.
- **fes**: List of fixed effects.
- **lambdas**: Vector of penalty parameters.
- **penalty**: A string indicating the penalty type. Currently supported: "lasso" and "ridge".
- **tol**: Tolerance parameter for convergence of the IRLS algorithm.
- **hdfetol**: Tolerance parameter for the within-transformation step, passed on to \texttt{lfe::demeanlist}.
- **colcheck**: Logical. If \texttt{TRUE}, checks for perfect multicollinearity in \texttt{x}.
- **post**: Logical. If \texttt{TRUE}, estimates a post-penalty regression with the selected variables.
- **cluster**: Optional: a vector classifying observations into clusters (to use when calculating SEs).
- **method**: The user can set this equal to "plugin" to perform the plugin algorithm with coefficient-specific penalty weights (see details). Otherwise, a single global penalty is used.
- **IDs**: A vector of fold IDs for k-fold cross validation. If left unspecified, each observation is assigned to a different fold (warning: this is likely to be very resource-intensive).
- **verbose**: Logical. If \texttt{TRUE}, it prints information to the screen while evaluating.
- **xval**: Logical. If \texttt{TRUE}, it carries out cross-validation.
- **standardize**: Logical. If \texttt{TRUE}, x variables are standardized before estimation.
- **vcv**: Logical. If \texttt{TRUE} (the default), the post-estimation model includes standard errors.
- **penweights**: Optional: a vector of coefficient-specific penalties to use in plugin lasso when \texttt{method} == "plugin".
- **K**: Maximum number of iterations for the plugin algorithm to converge.

Details

For technical details on the algorithms used, see \texttt{hdfeppml_int} (post-lasso regression), \texttt{penhdfeppml_int} (standard penalized regression), \texttt{penhdfeppml_cluster_int} (plugin lasso), and \texttt{xvalidate} (cross-validation).

Value

A list with the following elements:

- **beta**: if \texttt{post = FALSE}, a length(lambdas) x ncol(x) matrix with coefficient (beta) estimates from the penalized regressions. If \texttt{post = TRUE}, this is the matrix of coefficients from the post-penalty regressions.
- **beta_pre**: if \texttt{post = TRUE}, a length(lambdas) x ncol(x) matrix with coefficient (beta) estimates from the penalized regressions.
- **bic**: Bayesian Information Criterion.
• lambdas: vector of penalty parameters.
• ses: standard errors of the coefficients of the post-penalty regression. Note that these are only provided when post = TRUE.
• rmse: if xval = TRUE, a matrix with the root mean squared error (RMSE - column 2) for each value of lambda (column 1), obtained by cross-validation.
• phi: coefficient-specific penalty weights (only if method == "plugin").

References


Examples

# First, we need to transform the data (this is what mlfitppml handles internally). Start by
# filtering the data set to keep only countries in the Americas:
americas <- countries$iso[countries$region == "Americas"]
trade <- trade[(trade$imp %in% americas) & (trade$exp %in% americas), ]
# Now generate the needed x, y and fes objects:
y <- trade$export
x <- data.matrix(trade[, -1:-6])
fes <- list(exp_time = interaction(trade$exp, trade$time),
             imp_time = interaction(trade$imp, trade$time),
             pair = interaction(trade$exp, trade$imp))
# Finally, we try mlfitppml_int with a lasso penalty (the default) and two lambda values:
reg <- mlfitppml_int(y = y, x = x, fes = fes, lambdas = c(0.1, 0.01))
# We can also try plugin lasso:
reg <- mlfitppml_int(y = y, x = x, fes = fes, cluster = fes$pair, method = "plugin")
# For an example with cross-validation, please see the vignette.
penhdfeppml

One-Shot Penalized PPML Estimation with HDFE

Description

penhdfeppml fits a penalized PPML regression for a given type of penalty and a given value of the penalty parameter. The penalty can be either lasso or ridge, and the plugin method can be enabled via the method argument.

Usage

penhdfeppml(
  data,
  dep = 1,
  indep = NULL,
  fixed = NULL,
  cluster = NULL,
  selectobs = NULL,
  ...
)

Arguments

data A data frame containing all relevant variables.
dep A string with the name of the independent variable or a column number.
indep A vector with the names or column numbers of the regressors. If left unspecified, all remaining variables (excluding fixed effects) are included in the regressor matrix.
fixed A vector with the names or column numbers of factor variables identifying the fixed effects, or a list with the desired interactions between variables in data.
cluster Optional. A string with the name of the clustering variable or a column number. It’s also possible to input a vector with several variables, in which case the interaction of all of them is taken as the clustering variable.
selectobs Optional. A vector indicating which observations to use (either a logical vector or a numeric vector with row numbers, as usual when subsetting in R).
...

Further options, including:

- penalty: A string indicating the penalty type. Currently supported: "lasso" and "ridge".
- method: The user can set this equal to "plugin" to perform the plugin algorithm with coefficient-specific penalty weights (see details). Otherwise, a single global penalty is used.

For a full list of options, see penhdfeppml_int.
Details

This function is a thin wrapper around `penhdfeppml_int`, providing a more convenient interface for data frames. Whereas the internal function requires some preliminary handling of data sets (\( y \) must be a vector, \( x \) must be a matrix and \( fes \) must be provided in a list), the wrapper takes a full data frame in the `data` argument, and users can simply specify which variables correspond to \( y \), \( x \) and the fixed effects, using either variable names or column numbers.

More formally, `penhdfeppml_int` performs iteratively re-weighted least squares (IRLS) on a transformed model, as described in Breinlich, Corradi, Rocha, Ruta, Santos Silva and Zylkin (2021). In each iteration, the function calculates the transformed dependent variable, partials out the fixed effects (calling \texttt{lfe::demeanlist}) and then calls \texttt{glmnet::glmnet} if the selected penalty is lasso (the default). If the user has selected ridge, the analytical solution is instead computed directly using fast C++ implementation.

For information on how the plugin lasso method works, see `penhdfeppml_cluster`.

Value

If `method == \"lasso\"` (the default), an object of class \texttt{elnet} with the elements described in \texttt{glmnet}, as well as:

- \texttt{mu}: a 1 x \texttt{length(y)} matrix with the final values of the conditional mean \( \mu \).
- \texttt{deviance}.
- \texttt{bic}: Bayesian Information Criterion.
- \texttt{phi}: coefficient-specific penalty weights (only if `method == \"plugin\"`).
- \texttt{x_resid}: matrix of demeaned regressors.
- \texttt{z_resid}: vector of demeaned (transformed) dependent variable.

If `method == \"ridge\"`, a list with the following elements:

- \texttt{beta}: a 1 x \texttt{ncol(x)} matrix with coefficient (beta) estimates.
- \texttt{mu}: a 1 x \texttt{length(y)} matrix with the final values of the conditional mean \( \mu \).
- \texttt{deviance}.
- \texttt{bic}: Bayesian Information Criterion.
- \texttt{x_resid}: matrix of demeaned regressors.
- \texttt{z_resid}: vector of demeaned (transformed) dependent variable.

References


Examples

```r
# To reduce run time, we keep only countries in the Americas:
americas <- countries$iso[countries$region == "Americas"]
test <- penhdfeppml(data = trade[, -(5:6)],
   dep = "export",
   fixed = list(c("exp", "time"),
               c("imp", "time"),
               c("exp", "imp")),
   lambda = 0.05,
   selectobs = (trade$imp %in% americas) & (trade$exp %in% americas))
```

---

**penhdfeppml_cluster**  
*Plugin Lasso Estimation*

**Description**

Performs plugin lasso - PPML estimation with HDFE. This is an internal function, called by `mlfitppml` and `penhdfeppml` when users select the `method = "plugin"` option, but it's made available as a stand-alone option for advanced users who may prefer to avoid some overhead imposed by the wrappers.

**Usage**

```r
penhdfeppml_cluster(
   data,
   dep = 1,
   indep = NULL,
   fixed = NULL,
   cluster = NULL,
   selectobs = NULL,
   ...
)
```

**Arguments**

- `data`  
  A data frame containing all relevant variables.

- `dep`  
  A string with the name of the independent variable or a column number.

- `indep`  
  A vector with the names or column numbers of the regressors. If left unspecified, all remaining variables (excluding fixed effects) are included in the regressor matrix.
**penhdfepm_cluster**

**fixed**  
A vector with the names or column numbers of factor variables identifying the fixed effects, or a list with the desired interactions between variables in `data`.

**cluster**  
A string with the name of the clustering variable or a column number. It’s also possible to input a vector with several variables, in which case the interaction of all of them is taken as the clustering variable. Note that this is NOT OPTIONAL in this case: our plugin algorithm requires clusters to be specified.

**selectobs**  
Optional. A vector indicating which observations to use (either a logical vector or a numeric vector with row numbers, as usual when subsetting in R).

...  
Further options. For a full list of options, see `penhdfepm_cluster_int`.

**Details**

This function is a thin wrapper around `penppm_cluster_int`, providing a more convenient interface for data frames. Whereas the internal function requires some preliminary handling of data sets (y must be a vector, x must be a matrix and fes must be provided in a list), the wrapper takes a full data frame in the `data` argument, and users can simply specify which variables correspond to y, x and the fixed effects, using either variable names or column numbers.

The plugin method uses coefficient-specific penalty weights that account for heteroskedasticity. The penalty parameters are calculated automatically by the function using statistical theory - for a brief discussion of this, see Breinlich, Corradi, Rocha, Ruta, Santos Silva and Zylkin (2021), and for a more in-depth analysis, check Belloni, Chernozhukov, Hansen, and Kozbur (2016), which introduced the specific implementation used in this package. Heuristically, the penalty parameters are set at a level high enough so that the absolute value of the score for each regressor must be statistically large relative to its standard error in order for the regressors to be selected.

**Value**

An object of class `elnet` with the elements described in `glmnet`, as well as the following:

- `mu`: a 1 x length(y) matrix with the final values of the conditional mean $\mu$.
- `deviance`.
- `bic`: Bayesian Information Criterion.
- `phi`: coefficient-specific penalty weights.
- `x_resid`: matrix of demeaned regressors.
- `z_resid`: vector of demeaned (transformed) dependent variable.

**References**


**Examples**

```r
# To reduce run time, we keep only countries in the Americas:
americas <- countries$iso[countries$region == "Americas"]
test <- penhdfeppml_cluster(data = trade[, -(5:6)],
                           dep = "export",
                           fixed = list(c("exp", "time"),
                                        c("imp", "time"),
                                        c("exp", "imp")),
                           cluster = c("exp", "imp"),
                           selectobs = (trade$imp %in% americas) & (trade$exp %in% americas),
                           tol = 1e-5, hdfetol = 1e-1)
```

---

**penhdfeppml_cluster_int**

*Plugin Lasso Estimation*

**Description**

Performs plugin lasso - PPML estimation with HDFE. This is an internal function, called by `mlfitppml_int` and `penhdfeppml_int` when users select the method = "plugin" option, but it’s made available as a stand-alone option for advanced users who may prefer to avoid some overhead imposed by the wrappers.

**Usage**

```r
penhdfeppml_cluster_int(y,
                        x,
                        fes,
                        cluster,
                        tol = 1e-08,
                        hdfetol = 1e-04,
                        glmnettol = 1e-12,
                        penalty = "lasso",
                        penweights = NULL,
                        saveX = TRUE,
                        mu = NULL,
                        colcheck = TRUE,
                        K = 15,
                        init_z = NULL,
                        ...)```

Arguments

- **y**: Dependent variable (a vector)
- **x**: Regressor matrix.
- **fes**: List of fixed effects.
- **cluster**: Optional: a vector classifying observations into clusters (to use when calculating SEs).
- **tol**: Tolerance parameter for convergence of the IRLS algorithm.
- **hdfetol**: Tolerance parameter for the within-transformation step, passed on to lfe::demeanlist.
- **glmnettol**: Tolerance parameter to be passed on to glmnet::glmnet.
- **penalty**: Only "lasso" is supported at the present stage.
- **penweights**: Optional: a vector of coefficient-specific penalties to use in plugin lasso when method == "plugin".
- **saveX**: Logical. If TRUE, it returns the values of x and z after partialling out the fixed effects.
- **mu**: Optional: initial values of the conditional mean µ, to be used as weights in the first iteration of the algorithm.
- **colcheck**: Logical. If TRUE, checks for perfect multicollinearity in x.
- **K**: Maximum number of iterations.
- **init_z**: Optional: initial values of the transformed dependent variable, to be used in the first iteration of the algorithm.
- **post**: Logical. If TRUE, estimates a post-penalty regression with the selected variables.
- **verbose**: Logical. If TRUE, it prints information to the screen while evaluating.
- **lambda**: Penalty parameter (a number).

Details

The plugin method uses coefficient-specific penalty weights that account for heteroskedasticity. The penalty parameters are calculated automatically by the function using statistical theory - for a brief discussion of this, see Breinlich, Corradi, Rocha, Ruta, Santos Silva and Zylkin (2021), and for a more in-depth analysis, check Belloni, Chernozhukov, Hansen, and Kozbur (2016), which introduced the specific implementation used in this package. Heuristically, the penalty parameters are set at a level high enough so that the absolute value of the score for each regressor must be statistically large relative to its standard error in order for the regressors to be selected.
Value

An object of class `elnet` with the elements described in `glmnet`, as well as the following:

- **mu**: a 1 x length(y) matrix with the final values of the conditional mean $\mu$.
- **deviance**.
- **bic**: Bayesian Information Criterion.
- **phi**: coefficient-specific penalty weights.
- **x_resid**: matrix of demeaned regressors.
- **z_resid**: vector of demeaned (transformed) dependent variable.

References


Examples

```r
# To reduce run time, we keep only countries in the Americas:
Americas <- countries$iso[countries$region == "Americas"]
trade <- trade[(trade$imp %in% Americas) & (trade$exp %in% Americas),]
# Now generate the needed x, y and fes objects:
y <- trade$export
x <- data.matrix(trade[, -1:-6])
fes <- list(exp_time = interaction(trade$exp, trade$time),
            imp_time = interaction(trade$imp, trade$time),
            pair = interaction(trade$exp, trade$imp))
# Finally, we try penhdfeppml_cluster_int:
reg <- penhdfeppml_cluster_int(y = y, x = x, fes = fes, cluster = fes$pair)
```
**Description**

`penhdfeppml_int` is the internal algorithm called by `penhdfeppml` to fit a penalized PPML regression for a given type of penalty and a given value of the penalty parameter. It takes a vector with the dependent variable, a regressor matrix and a set of fixed effects (in list form: each element in the list should be a separate HDFE). The penalty can be either lasso or ridge, and the plugin method can be enabled via the `method` argument.

**Usage**

```r
penhdfeppml_int(
  y, x, fes, lambda, tol = 1e-08, hdfetol = 1e-04, glmnettol = 1e-12,
  penalty = "lasso", penweights = NULL, saveX = TRUE, mu = NULL,
  colcheck = TRUE, init_z = NULL, post = FALSE, verbose = FALSE,
  standardize = TRUE, method = "placeholder", cluster = NULL,
  debug = FALSE
)
```

**Arguments**

- `y` Dependent variable (a vector)
- `x` Regressor matrix.
- `fes` List of fixed effects.
- `lambda` Penalty parameter (a number).
- `tol` Tolerance parameter for convergence of the IRLS algorithm.
- `hdfetol` Tolerance parameter for the within-transformation step, passed on to `lfe::demeanlist`.
- `glmnettol` Tolerance parameter to be passed on to `glmnet::glmnet`.
- `penalty` A string indicating the penalty type. Currently supported: "lasso" and "ridge".
penweights Optional: a vector of coefficient-specific penalties to use in plugin lasso when method == "plugin".

saveX Logical. If TRUE, it returns the values of x and z after partialling out the fixed effects.

mu Optional: initial values of the conditional mean \( \mu \), to be used as weights in the first iteration of the algorithm.

colcheck Logical. If TRUE, checks for perfect multicollinearity in x.

init_z Optional: initial values of the transformed dependent variable, to be used in the first iteration of the algorithm.

post Logical. If TRUE, estimates a post-penalty regression with the selected variables.

verbose Logical. If TRUE, it prints information to the screen while evaluating.

standardize Logical. If TRUE, x variables are standardized before estimation.

method The user can set this equal to "plugin" to perform the plugin algorithm with coefficient-specific penalty weights (see details). Otherwise, a single global penalty is used.

cluster Optional: a vector classifying observations into clusters (to use when calculating SEs).

dead Logical. If TRUE, this helps with debugging penalty weights by printing output of the first iteration to the console and stopping the estimation algorithm.

Details

More formally, penhdfeppml_int performs iteratively re-weighted least squares (IRLS) on a transformed model, as described in Breinlich, Corradi, Rocha, Ruta, Santos Silva and Zylkin (2020). In each iteration, the function calculates the transformed dependent variable, partials out the fixed effects (calling lfe::demeanlist) and then and then calls glmnet::glmnet if the selected penalty is lasso (the default). If the user selects ridge, the analytical solution is instead computed directly using fast C++ implementation.

For information on the plugin lasso method, see penhdfeppml_cluster_int.

Value

If method == "lasso" (the default), an object of class elnet with the elements described in glmnet, as well as:

- mu: a 1 x length(y) matrix with the final values of the conditional mean \( \mu \).
- deviance.
- bic: Bayesian Information Criterion.
- phi: coefficient-specific penalty weights (only if method == "plugin").
- x_resid: matrix of demeaned regressors.
- z_resid: vector of demeaned (transformed) dependent variable.

If method == "ridge", a list with the following elements:

- beta: a 1 x ncol(x) matrix with coefficient (beta) estimates.
plugin_lasso_int

- **mu**: a \(1 \times \text{length}(y)\) matrix with the final values of the conditional mean \(\mu\).
- **deviance**.
- **bic**: Bayesian Information Criterion.
- **x_resid**: matrix of demeaned regressors.
- **z_resid**: vector of demeaned (transformed) dependent variable.

**References**


**Examples**

```r
# To reduce run time, we keep only countries in the Americas:
americas <- countries$iso[countries$region == "Americas"]
trade <- trade[(trade$imp %in% americas) & (trade$exp %in% americas), ]
# Now generate the needed x, y and fes objects:
y <- trade$export
x <- data.matrix(trade[, -1:-6])
fes <- list(exp_time = interaction(trade$exp, trade$time),
            imp_time = interaction(trade$imp, trade$time),
            pair = interaction(trade$exp, trade$imp))
# Finally, we try penhdfeppml_int with a lasso penalty (the default):
reg <- penhdfeppml_int(y = y, x = x, fes = fes, lambda = 0.1)

# We can also try ridge:
reg <- penhdfeppml_int(y = y, x = x, fes = fes, lambda = 0.1, penalty = "ridge")
```

---

**Description**

This is the internal function upon which the iceberg wrapper is built. It performs standard plugin lasso PPML estimation without fixed effects, relying on glmnet::glmnet. As the other internals in the package, it needs a y vector and an x matrix.
Usage

plugin_lasso_int(
    y,
    x,
    tol = 1e-08,
    glmnettol = 1e-12,
    penweights = NULL,
    colcheck = FALSE,
    K = 50,
    verbose = FALSE,
    lambda = NULL,
    phipost = FALSE
)

Arguments

y Dependent variable (a vector).
x Regressor matrix.
tol Tolerance parameter for convergence of the IRLS algorithm.
glmnettol Tolerance parameter to be passed on to glmnet::glmnet.
penweights Optional: a vector of coefficient-specific penalties to use in plugin lasso.
colcheck Logical. If TRUE, checks for perfect multicollinearity in x.
K Maximum number of iterations.
verbose Logical. If TRUE, prints information to the screen while evaluating.
lambda Penalty parameter (a number).
phipost Logical. If TRUE, it carries out a post-lasso estimation with just the selected variables and reports the coefficients from this regression.

Value

A list with 14 elements, including beta, which is the only one we use in the wrapper. For a full list, see glmnet.

select_fes Filtering fixed effect lists

Description

A helper function for xvalidate that filters a list of fixed effects and returns the modified list. Used to split the fixed effects for cross-validation.

Usage

select_fes(fe_list, select_obs, list = TRUE)
standardize_wt

**Arguments**

- `fe_list` A list of fixed effects.
- `select_obs` A vector of selected observations / rows.
- `list` Logical. If TRUE, it returns a list. Otherwise, a data frame.

**Value**

A modified list of fixed effects.

---

**standardize_wt**  
*Weighted Standardization*

**Description**

Performs weighted standardization of x variables. Used in fastridge.

**Usage**

`standardize_wt(x, weights = rep(1/n, n), intercept = TRUE, return.sd = FALSE)`

**Arguments**

- `x` Regressor matrix.
- `weights` Weights.
- `intercept` Logical. If TRUE, adds an intercept.
- `return.sd` Logical. If TRUE, it returns standard errors for the means.

**Value**

If `return.sd == FALSE`, it gives the matrix of standardized regressors. If `return.sd == TRUE`, then it returns the vector of standard errors of the means of the variables.

---

**trade**  
*International trade agreements data set*

**Description**

A panel data set containing bilateral trade flows between 210 exporters and 262 importers between 1964 and 2016. The data set also contains information about trade agreements in force between country pairs, as well as 16 dummies for specific provisions in those agreements (a small selection from a broader data set).

**Usage**

`trade`
Format

A data frame with 194,092 rows and 22 variables:

- **exp** Exporter country (ISO 3166 code)
- **imp** Importer country (ISO 3166 code).
- **time** Year.
- **export** Merchandise trade exports in USD.
- **id** Agreement ID code.
- **agreement** Agreement name.
- **ad_prov_14** Anti-dumping actions allowed and with specific provisions for material injury.
- **cp_prov_23** Does the agreement contain provisions that promote transparency?
- **tbt_prov_07** Technical Regulations - Is the use of international standards promoted?
- **tbt_prov_33** Does the agreement go beyond the TBT (Technical Barriers to Trade) Agreement?
- **tf_prov_41** Harmonization and common legal framework
- **tf_prov_45** Issuance of proof of origin
- **ser_prov_47** Does the agreement contain a standstill provision?
- **inv_prov_22** Does the agreement grant Fair and Equitable Treatment (FET)?
- **et_prov_38** Prohibits export-related performance requirements, subject to exemptions.
- **ipr_prov_44** Stipulates that GIs can be registered and protected through a TM system
- **env_prov_18** Does the agreement require states to control ozone-depleting substances?
- **ipr_prov_15** Incorporates/reaffirms all multilateral agreements to which both parties are a party (general obligation)
- **moc_prov_21** Does the transfer provision explicitly exclude “good faith and non-discriminatory application of its laws” related to bankruptcy, insolvency or creditor rights protection?
- **ste_prov_30** Does the agreement regulate subsidization to state enterprises?
- **lm_prov_10** Does the agreement include reference to internationally recognized labor standards?
- **cp_prov_26** Does the agreement regulate consumer protection?

Source

**xex**

* **XeeX Matrix Computation**

**Description**

Given matrix $ee'$ and matrix $X$, compute $X(k)'ee'X(k)$ for each regressor $X$.

**Usage**

```r
xex(X, e, S)
```

**Arguments**

- **X**: Regressor matrix.
- **e**: Residuals.
- **S**: Cluster sizes.

**Value**

The matrix product $X(k)'ee'X(k)$.

---

**xvalidate**

* **Implementing Cross Validation**

**Description**

This is the internal function called by `mlfitppml_int` to perform cross-validation, if the option is enabled. It is available also on a stand-alone basis in case it is needed, but generally users will be better served by using the wrapper `mlfitppml`.

**Usage**

```r
xvalidate(
  y,
  x,
  fes,
  IDs,
  testID = NULL,
  tol = 1e-08,
  hdfetol = 1e-04,
  colcheck = TRUE,
  init_mu = NULL,
  init_x = NULL,
  init_z = NULL,
  verbose = FALSE,
)```

cluster = NULL,
penalty = "lasso",
method = "placeholder",
standardize = TRUE,
penweights = rep(1, ncol(x_reg)),
lambda = 0
)

Arguments

y Dependent variable (a vector)
x Regressor matrix.
fes List of fixed effects.
IDs A vector of fold IDs for k-fold cross validation. If left unspecified, each observation is assigned to a different fold (warning: this is likely to be very resource-intensive).
testID Optional. A number indicating which ID to hold out during cross-validation. If left unspecified, the function cycles through all IDs and reports the average RMSE.
tol Tolerance parameter for convergence of the IRLS algorithm.
hdfetol Tolerance parameter for the within-transformation step, passed on to lfe::demeanlist.
colcheck Logical. If TRUE, checks for perfect multicollinearity in x.
init_mu Optional: initial values of the conditional mean $\mu$, to be used as weights in the first iteration of the algorithm.
init_x Optional: initial values of the independent variables.
init_z Optional: initial values of the transformed dependent variable, to be used in the first iteration of the algorithm.
verbose Logical. If TRUE, it prints information to the screen while evaluating.
cluster Optional: a vector classifying observations into clusters (to use when calculating SEs).
penalty A string indicating the penalty type. Currently supported: "lasso" and "ridge".
method The user can set this equal to "plugin" to perform the plugin algorithm with coefficient-specific penalty weights (see details). Otherwise, a single global penalty is used.
standardize Logical. If TRUE, x variables are standardized before estimation.
penweights Optional: a vector of coefficient-specific penalties to use in plugin lasso when method == "plugin".
lambda Penalty parameter, to be passed on to penhdfeppml_int or penhdfeppml_cluster_int.

Details

xvalidate carries out cross-validation with the user-provided IDs by holding out each one of them, sequentially, as in the k-fold procedure (unless testID is specified, in which case it just uses this ID for validation). After filtering out the holdout sample, the function simply calls penhdfeppml_int and penhdfeppml_cluster_int to estimate the coefficients, it predicts the conditional means for the held-out observations and finally it calculates the root mean squared error (RMSE).
Value

A list with two elements:

- `rmse`: root mean squared error (RMSE).
- `mu`: conditional means.

References


Examples

```r
# First, we need to transform the data. Start by filtering the data set to keep only countries in
# the Americas:
americas <- countries$iso[countries$region == "Americas"]
trade <- trade[(trade$imp %in% americas) & (trade$exp %in% americas),]

# Now generate the needed x, y and fes objects:
y <- trade$export
x <- data.matrix(trade[, -1:-6])
fes <- list(exp_time = interaction(trade$exp, trade$time),
            imp_time = interaction(trade$imp, trade$time),
            pair = interaction(trade$exp, trade$imp))

# We also need to create the IDs. We split the data set by agreement, not observation:
id <- unique(trade[, 5])
nfolds <- 10
unique_ids <- data.frame(id = id, fold = sample(1:nfolds, size = length(id), replace = TRUE))
cross_ids <- merge(trade[, 5], unique_ids, by = "id", all.x = TRUE)

# Finally, we try xvalidate with a lasso penalty (the default) and two lambda values:
reg <- xvalidate(y = y, x = x, fes = fes, lambda = 0.001,
                 IDs = cross_ids$fold, verbose = TRUE)
```
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