

# Package ‘powerLaw’

January 25, 2024

**Type** Package

**Title** Analysis of Heavy Tailed Distributions

**Version** 0.80.0

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**Description** An implementation of maximum likelihood estimators for a variety of heavy tailed distributions, including both the discrete and continuous power law distributions. Additionally, a goodness-of-fit based approach is used to estimate the lower cut-off for the scaling region.

**License** GPL-2 | GPL-3

**URL** <https://github.com/csgillespie/powerLaw>

**BugReports** <https://github.com/csgillespie/powerLaw/issues>

**Depends** R (>= 3.4.0)

**Imports** methods, parallel, pracma, stats, utils

**Suggests** covr, knitr, testthat

**VignetteBuilder** knitr

**ByteCompile** true

**Encoding** UTF-8

**Collate** 'aaa\_all\_classes.R' 'all\_generics.R' 'bootstrap.R'  
'bootstrap\_p.R' 'checks.R' 'compare\_distributions.R'  
'conlnorm.R' 'data\_help\_files.R' 'def\_conexp.R' 'def\_conpl.R'  
'def\_conweibull.R' 'def\_disexp.R' 'def\_dislnorm.R'  
'def\_displ.R' 'def\_dispois.R' 'def\_template.R'  
'dist\_data\_cdf\_methods.R' 'estimate\_pars.R' 'estimate\_xmin.R'  
'get\_n.R' 'get\_ntail.R' 'lines\_methods.R' 'plcon.R' 'pldis.R'  
'plot\_methods.R' 'points\_methods.R' 'powerlaw-package.R'  
'show\_methods.R' 'timer.R'

**RoxygenNote** 7.2.3

**NeedsCompilation** no

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**Repository** CRAN

**Date/Publication** 2024-01-25 18:00:02 UTC

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powerLaw-package	<i>The powerLaw package</i>
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## Description

The powerLaw package aims to make fitting power laws and other heavy-tailed distributions straightforward. This package contains R functions for fitting, comparing and visualising heavy tailed distributions. Overall, it provides a principled approach to fitting power laws to data.

## Details

The code developed in this package has been heavily influenced by the R code of Laurent Dubroca and Cosma Shalizi.

## Author(s)

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## References

A. Clauset, C.R. Shalizi, and M.E.J. Newman, "Power-law distributions in empirical data" SIAM Review 51(4), 661-703 (2009).

## See Also

<https://github.com/csgillespie/poweRlaw>

---

bootstrap\_moby

*Example bootstrap results for the full Moby Dick data set*

---

## Description

To explore the uncertainty in the model fit, this package provides a bootstrap function.

**bootstrap\_moby** The output from running 5000 bootstraps on the full Moby Dick data set (for a discrete power law) using the bootstrap function.

**bootstrap\_p\_moby** The output from running 5000 bootstraps on the full Moby Dick data set (for a discrete power law) using the bootstrap\_p function.

The bootstrap\_moby values correspond to the first row of table 6.1 in the Clauset et al paper:

bootstrap\_moby\$gof the K-S statistic

bootstrap\_moby\$bootstraps a data frame for the optimal values from the bootstrapping procedure. Column 1: K-S, Column 2: xmin, Column 3: alpha. So standard deviation of column 2 and 3 is 2.2 and 0.033 (the paper gives 2 and 0.02 respectively).

The bootstrap\_p\_moby gives the p-value for the hypothesis test of whether the data follows a power-law. For this simulation study, we get a value of 0.43 (the paper gives 0.49).

## Format

A list

## Source

M. E. J. Newman, "Power laws, Pareto distributions and Zipf's law." Contemporary Physics 46, 323 (2005).

## See Also

moby, bootstrap, bootstrap\_p

## Examples

```
## Generate the bootstrap_moby data set
## Not run:
data(moby)
m = displ$new(moby)
bs = bootstrap(m, no_of_sims=5000, threads=4, seed=1)

## End(Not run)

#' ## Generate the bootstrap_p_moby data set
## Not run:
bs_p = bootstrap_p(m, no_of_sims=5000, threads=4, seed=1)

## End(Not run)
```

---

compare\_distributions *Vuong's test for non-nested models*

---

## Description

Since it is possible to fit power law models to any data set, it is recommended that alternative distributions are considered. A standard technique is to use Vuong's test. This is a likelihood ratio test for model selection using the Kullback-Leibler criteria. The test statistic,  $R$ , is the ratio of the log-likelihoods of the data between the two competing models. The sign of  $R$  indicates which model is better. Since the value of  $R$  is estimated, we use the method proposed by Vuong, 1989 to select the model.

## Usage

```
compare_distributions(d1, d2)
```

## Arguments

d1	A distribution object
d2	A distribution object

## Details

This function compares two models. The null hypothesis is that both classes of distributions are equally far from the true distribution. If this is true, the log-likelihood ratio should (asymptotically) have a Normal distribution with mean zero. The test statistic is the sample average of the log-likelihood ratio, standardized by a consistent estimate of its standard deviation. If the null hypothesis is false, and one class of distributions is closer to the "truth", the test statistic goes to  $\pm$ -infinity with probability 1, indicating the better-fitting class of distributions.

**Value**

This function returns

`test_statistic` The test statistic.

`p_one_sided` A one-sided p-value, which is an upper limit on getting that small a log-likelihood ratio if the first distribution, `d1`, is actually true.

`p_two_sided` A two-sided p-value, which is the probability of getting a log-likelihood ratio which deviates that much from zero in either direction, if the two distributions are actually equally good.

`ratio` A data frame with two columns. The first column is the `x` value and second column is the difference in log-likelihoods.

**Note**

Code initially based on R code developed by Cosma Rohilla Shalizi (<http://bactra.org/>). Also see Appendix C in Clauset et al, 2009.

**References**

Vuong, Quang H. (1989): "Likelihood Ratio Tests for Model Selection and Non-Nested Hypotheses", *Econometrica* 57: 307–333.

**Examples**

```
#####
# Example data #
#####
x = rpldis(100, xmin=2, alpha=3)

#####
##Continuous power law #
#####
m1 = conpl$new(x)
m1$setXmin(estimate_xmin(m1))

#####
##Exponential
#####
m2 = conexp$new(x)
m2$setXmin(m1$getXmin())
est2 = estimate_pars(m2)
m2$setPars(est2$pars)

#####
##Vuong's test #
#####
comp = compare_distributions(m1, m2)
plot(comp)
```

---

 conlnorm-class

*Heavy-tailed distributions*


---

### Description

The **powerLaw** package supports a number of distributions:

**displ** Discrete power-law

**dislnorm** Discrete log-normal

**dispois** Discrete Poisson

**disexp** Discrete Exponential

**conpl** Continuous power-law

**conlnorm** Continuous log-normal

**conexp** Continuous exponential

Each object inherits the `discrete_distribution` or the `ctn_distribution` class.

### Arguments

... The object is typically created by passing data using the `dat` field. Each field has standard setters and getters.

### Value

a reference object

### Fields

Each distribution object has four fields. However, the object is typically created by passing data, to the `dat` field. Each field has standard setters and getters. See examples below

**dat** The data set.

**xmin** The lower threshold, `xmin`. Typically set after initialisation. For the continuous power-law, `xmin >= 0` for the discrete distributions, `xmin > 0`

**pars** A parameter vector. Typically set after initialisation. Note the lognormal distribution has two parameters.

**internal** A list. This list differs between objects and shouldn't be altered.

### Copying objects

Distribution objects are reference classes. This means that when we copy objects, we need to use the `copy` method, i.e. `obj$copy()`. See the examples below for further details.

**Examples**

```
#####
#Load data and create distribution object          #
#####
data(moby)
m = displ$new(moby)

#####
#Xmin is initially the smallest x value          #
#####
m$getXmin()
m$getPars()

#####
#Set Xmin and parameter                          #
#####
m$setXmin(2)
m$setPars(2)

#####
#Plot the data and fitted distribution           #
#####
plot(m)
lines(m)
#####
#Copying                                         #
#####
## Shallow copy
m_cpy = m
m_cpy$setXmin(5)
m$getXmin()
## Instead
m_cpy = m$copy()
```

---

dist\_all\_cdf

*The data cumulative distribution function*


---

**Description**

This is generic function for distribution objects. This function calculates the data or empirical cdf. The functions `dist_data_all_cdf` and `dist_all_cdf` are only available for discrete distributions. Their main purpose is to optimise the bootstrap procedure, where generating a vector `xmin:xmax` is very quick. Also, when bootstrapping very large values can be generated.

**Usage**

```
dist_all_cdf(m, lower_tail = TRUE, xmax = 1e+05)
```

```

dist_data_cdf(m, lower_tail = TRUE, xmax = 1e+05)

dist_data_all_cdf(m, lower_tail = TRUE, xmax = 1e+05)

## S4 method for signature 'discrete_distribution'
dist_data_cdf(m, lower_tail = TRUE, xmax = 1e+05)

## S4 method for signature 'discrete_distribution'
dist_data_all_cdf(m, lower_tail = TRUE, xmax = 1e+05)

## S4 method for signature 'ctn_distribution'
dist_data_cdf(m, lower_tail = TRUE, xmax = 1e+05)

```

### Arguments

m	a distribution object.
lower_tail	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
xmax	default 1e5. The maximum x value calculated when working out the CDF.

### Note

This method does *not* alter the internal state of the distribution objects.

### Examples

```

#####
#Load data and create distribution object#
#####
data(moby_sample)
m = displ$new(moby_sample)
m$setXmin(7);m$setPars(2)

#####
# The data cdf #
#####
dist_data_cdf(m)

```

---

dist\_cdf

*The cumulative distribution function (cdf)*

---

### Description

This is a generic function for calculating the cumulative distribution function (cdf) of distribution objects. This is similar to base R's `pnorm` for the normal distribution. The `dist_cdf` function calculates the cumulative probability distribution for the current parameters and `xmin` value.



**Usage**

```
dist_cdf(m, q = NULL, lower_tail = FALSE)

## S4 method for signature 'conlnorm'
dist_cdf(m, q = NULL, lower_tail = TRUE)

## S4 method for signature 'conlnorm'
dist_all_cdf(m, lower_tail = TRUE, xmax = 1e+05)

## S4 method for signature 'conexp'
dist_cdf(m, q = NULL, lower_tail = TRUE)

## S4 method for signature 'conexp'
dist_all_cdf(m, lower_tail = TRUE, xmax = 1e+05)

## S4 method for signature 'conpl'
dist_cdf(m, q = NULL, lower_tail = TRUE)

## S4 method for signature 'conpl'
dist_all_cdf(m, lower_tail = TRUE, xmax = 1e+05)

## S4 method for signature 'conweibull'
dist_cdf(m, q = NULL, lower_tail = TRUE)

## S4 method for signature 'conweibull'
dist_all_cdf(m, lower_tail = TRUE, xmax = 1e+05)

## S4 method for signature 'disexp'
dist_cdf(m, q = NULL, lower_tail = TRUE)

## S4 method for signature 'disexp'
dist_all_cdf(m, lower_tail = TRUE, xmax = 1e+05)

## S4 method for signature 'dislnorm'
dist_cdf(m, q = NULL, lower_tail = TRUE)

## S4 method for signature 'dislnorm'
dist_all_cdf(m, lower_tail = TRUE, xmax = 1e+05)

## S4 method for signature 'displ'
dist_cdf(m, q = NULL, lower_tail = TRUE)

## S4 method for signature 'displ'
dist_all_cdf(m, lower_tail = TRUE, xmax = 1e+05)

## S4 method for signature 'dispois'
dist_cdf(m, q = NULL, lower_tail = TRUE)
```

```
## S4 method for signature 'dispois'
dist_all_cdf(m, lower_tail = TRUE, xmax = 1e+05)
```

### Arguments

m	a distribution object.
q	a vector values where the function will be evaluated. If q is NULL (default), then the data values will be used.
lower_tail	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
xmax	default 1e5. The maximum x value calculated when working out the CDF.

### Note

This method does *not* alter the internal state of the distribution objects.

### Examples

```
#####
#Load data and create distribution object#
#####
data(moby_sample)
m = displ$new(moby_sample)
m$setXmin(7); m$setPars(2)

#####
#Calculate the CDF at a particular values#
#####
dist_cdf(m, 10:15)

#####
#Calculate the CDF at the data values #
#####
dist_cdf(m)
```

---

dist\_ll

*The log-likelihood function*

---

### Description

This is generic function for distribution objects. This function calculates the log-likelihood for the current parameters and xmin value.

### Usage

```
dist_ll(m)

## S4 method for signature 'conlnorm'
dist_ll(m)
```

```
## S4 method for signature 'conexp'
dist_ll(m)

## S4 method for signature 'conpl'
dist_ll(m)

## S4 method for signature 'conweibull'
dist_ll(m)

## S4 method for signature 'disexp'
dist_ll(m)

## S4 method for signature 'dislnorm'
dist_ll(m)

## S4 method for signature 'displ'
dist_ll(m)

## S4 method for signature 'dispois'
dist_ll(m)
```

### Arguments

`m` a distribution object.

### Value

The log-likelihood

### Note

This method does *not* alter the internal state of the distribution objects.

### See Also

[dist\\_cdf\(\)](#), [dist\\_pdf\(\)](#) and [dist\\_rand\(\)](#)

### Examples

```
#####
#Load data and create distribution object#
#####
data(moby_sample)
m = displ$new(moby_sample)
m$setXmin(7); m$setPars(2)

#####
#Calculate the log-likelihood #
#####
dist_ll(m)
```

---

`dist_pdf`*The probability density function (pdf)*

---

**Description**

This is generic function for distribution objects. This function calculates the probability density function (pdf) for the current parameters and xmin value.

**Usage**

```
dist_pdf(m, q = NULL, log = FALSE)

## S4 method for signature 'conlnorm'
dist_pdf(m, q = NULL, log = FALSE)

## S4 method for signature 'conexp'
dist_pdf(m, q = NULL, log = FALSE)

## S4 method for signature 'conpl'
dist_pdf(m, q = NULL, log = FALSE)

## S4 method for signature 'conweibull'
dist_pdf(m, q = NULL, log = FALSE)

## S4 method for signature 'disexp'
dist_pdf(m, q = NULL, log = FALSE)

## S4 method for signature 'dislnorm'
dist_pdf(m, q = NULL, log = FALSE)

## S4 method for signature 'displ'
dist_pdf(m, q = NULL, log = FALSE)

## S4 method for signature 'dispois'
dist_pdf(m, q = NULL, log = FALSE)
```

**Arguments**

<code>m</code>	a distribution object.
<code>q</code>	a vector values where the function will be evaluated. If <code>q</code> is <code>NULL</code> (default), then the data values will be used.
<code>log</code>	default <code>FALSE</code> . If <code>TRUE</code> , probabilities are given as $\log(p)$ .

**Value**

The probability density (or mass) function

**Note**

This method does *not* alter the internal state of the distribution objects.

**See Also**

[dist\\_cdf\(\)](#), [dist\\_ll\(\)](#) and [dist\\_rand\(\)](#)

**Examples**

```
#####
#Create distribution object      #
#####
m = displ$new()
m$setXmin(7); m$setPars(2)

#####
#Calculate the pdf at particular values #
#####
dist_pdf(m, 7:10)
```

---

dist\_rand

*Random number generation for the distribution objects*

---

**Description**

This is generic function for generating random numbers from the underlying distribution of the distribution reference objects. This function generates n random numbers using the parameters and xmin values found in the associated reference object.

**Usage**

```
dist_rand(m, n)

## S4 method for signature 'conlnorm'
dist_rand(m, n = "numeric")

## S4 method for signature 'conexp'
dist_rand(m, n = "numeric")

## S4 method for signature 'conpl'
dist_rand(m, n = "numeric")

## S4 method for signature 'conweibull'
dist_rand(m, n = "numeric")

## S4 method for signature 'disexp'
dist_rand(m, n = "numeric")
```

```
## S4 method for signature 'dislnorm'
dist_rand(m, n = "numeric")

## S4 method for signature 'displ'
dist_rand(m, n = "numeric")

## S4 method for signature 'dispois'
dist_rand(m, n = "numeric")
```

### Arguments

`m` a distribution object.  
`n` number of observations to be generated.

### Value

`n` random numbers

### Note

This method does *not* alter the internal state of the distribution object.

### See Also

[dist\\_cdf\(\)](#), [dist\\_pdf\(\)](#) and [dist\\_ll\(\)](#)

### Examples

```
#####
#Create distribution object      #
#####
m = displ$new()
m$setXmin(7);m$setPars(2)

#####
#Generate five random numbers  #
#####
dist_rand(m, 5)
```

### Description

Density and distribution function of the continuous power-law distribution, with parameters `xmin` and `alpha`.

**Usage**

```
dplcon(x, xmin, alpha, log = FALSE)

pplcon(q, xmin, alpha, lower.tail = TRUE)

rplcon(n, xmin, alpha)
```

**Arguments**

x, q	vector of quantiles. The discrete power-law distribution is defined for $x > x_{\min}$
xmin	The lower bound of the power-law distribution. For the continuous power-law, $x_{\min} \geq 0$ . for the discrete distribution, $x_{\min} > 0$ .
alpha	The scaling parameter: $\alpha > 1$ .
log	logical (default FALSE) if TRUE, log values are returned.
lower.tail	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
n	Number of observations. If $\text{length}(n) > 1$ , the length is taken to be the number required.

**Value**

dplcon gives the density and pplcon gives the distribution function.

**Note**

The discrete random number generator is very inefficient

**Examples**

```
xmin = 1; alpha = 1.5
x = seq(xmin, 10, length.out=1000)
plot(x, dplcon(x, xmin, alpha), type="l")
plot(x, pplcon(x, xmin, alpha), type="l", main="Distribution function")
n = 1000
con_rns = rplcon(n, xmin, alpha)
con_rns = sort(con_rns)
p = rep(1/n, n)
#Zipfs plot
plot(con_rns, rev(cumsum(p)), log="xy", type="l")
```

---

dpldis

*Discrete power-law distribution*


---

**Description**

Density, distribution function and random number generation for the discrete power law distribution with parameters  $x_{\min}$  and  $\alpha$ .

**Usage**

```
dpldis(x, xmin, alpha, log = FALSE)

ppldis(q, xmin, alpha, lower.tail = TRUE)

rpldis(n, xmin, alpha, discrete_max = 10000)
```

**Arguments**

x, q	vector of quantiles. The discrete power-law distribution is defined for $x > \text{xmin}$ .
xmin	The lower bound of the power-law distribution. For the continuous power-law, $\text{xmin} \geq 0$ . for the discrete distribution, $\text{xmin} > 0$ .
alpha	The scaling parameter: $\text{alpha} > 1$ .
log	logical (default FALSE) if TRUE, log values are returned.
lower.tail	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
n	Number of observations. If $\text{length}(n) > 1$ , the length is taken to be the number required.
discrete_max	The value when we switch from the discrete random numbers to a CTN approximation.

**Details**

The Clauset, 2009 paper provides an algorithm for generating discrete random numbers. However, if this algorithm is implemented in R, it gives terrible performance. This is because the algorithm involves "growing vectors". Another problem is when alpha is close to 1, this can result in very large random number being generated (which means we need to calculate the discrete CDF for very large values).

The algorithm provided in this package generates true discrete random numbers up to 10,000 then switches to using continuous random numbers. This switching point can altered by changing the `discrete_max` argument.

In order to get a efficient power-law discrete random number generator, the algorithm needs to be implemented in C.

**Value**

dpldis returns the density, ppldis returns the distribution function and rpldis return random numbers.

**Note**

The naming of these functions mirrors standard R functions, i.e. `dnorm`. When alpha is close to one, generating random number can be very slow.

**References**

Clauset, Aaron, Cosma Rohilla Shalizi, and Mark EJ Newman. "Power-law distributions in empirical data." SIAM review 51.4 (2009): 661-703.



**Examples**

```

xmin = 1; alpha = 2
x = xmin:100

plot(x, dpldis(x, xmin, alpha), type="l")
plot(x, ppldis(x, xmin, alpha), type="l", main="Distribution function")
dpldis(1, xmin, alpha)

#####
## Random number generation #
#####
n = 1e5
x1 = rpldis(n, xmin, alpha)
## Compare with exact (dpldis(1, xmin, alpha))
sum(x1==1)/n
## Using only the approximation
x2 = rpldis(n, xmin, alpha, 0)
sum(x2==1)/n

```

---

estimate\_pars

*Estimates the distributions using mle.*


---

**Description**

estimate\_pars estimates the distribution's parameters using their maximum likelihood estimator. This estimate is conditional on the current xmin value.

**Usage**

```
estimate_pars(m, pars = NULL)
```

**Arguments**

m	A reference class object that contains the data.
pars	default NULL. A vector or matrix (number of columns equal to the number of parameters) of parameters used to #' optimise over. Otherwise, for each value of xmin, the mle will be used, i.e. estimate_pars(m). For small samples, the mle may be biased.

**Value**

returns list.

**Examples**

```

data(moby_sample)
m = displ$new(moby_sample)
estimate_xmin(m)
m$setXmin(7)
estimate_pars(m)

```

---

get\_bootstrap\_sims      *Estimating the lower bound (xmin)*

---

**Description**

When fitting heavy tailed distributions, sometimes it is necessary to estimate the lower threshold, `xmin`. The lower bound is estimated by minimising the Kolmogorov-Smirnoff statistic (as described in Clauset, Shalizi, Newman (2009)).

`get_KS_statistic` Calculates the KS statistic for a particular value of `xmin`.

`estimate_xmin` Estimates the optimal lower cutoff using a goodness-of-fit based approach. This function may issue warnings when fitting lognormal, Poisson or Exponential distributions. The warnings occur for large values of `xmin`. Essentially, we are discarding the bulk of the distribution and cannot calculate the tails to enough accuracy.

`bootstrap` Estimates the unncertainty in the `xmin` and parameter values via bootstrapping.

`bootstrap_p` Performs a bootstrapping hypothesis test to determine whether a suggested (typically power law) distribution is plausible. This is only available for distributions that have `dist_rand` methods available.

**Usage**

```
get_bootstrap_sims(m, no_of_sims, seed, threads = 1)
```

```

bootstrap(
  m,
  xmin = NULL,
  pars = NULL,
  xmax = 1e+05,
  no_of_sims = 100,
  threads = 1,
  seed = NULL,
  distance = "ks"
)

```

```
get_bootstrap_p_sims(m, no_of_sims, seed, threads = 1)
```

```

bootstrap_p(
  m,
  xmin = NULL,

```

```

    pars = NULL,
    xmax = 1e+05,
    no_of_sims = 100,
    threads = 1,
    seed = NULL,
    distance = "ks"
)

get_distance_statistic(m, xmax = 1e+05, distance = "ks")

estimate_xmin(m, xmin = NULL, pars = NULL, xmax = 1e+05, distance = "ks")

```

### Arguments

m	A reference class object that contains the data.
no_of_sims	number of bootstrap simulations. When no_of_sims is large, this can take a while to run.
seed	default NULL. An integer to be supplied to set.seed, or NULL not to set reproducible seeds. This argument is passed clusterSetRNGStream.
threads	number of concurrent threads used during the bootstrap.
xmins	default 1e5. A vector of possible values of xmin to explore. When a single value is passed, this represents the maximum value to search, i.e. by default we search from (1, 1e5). See details for further information.
pars	default NULL. A vector or matrix (number of columns equal to the number of parameters) of parameters used to #' optimise over. Otherwise, for each value of xmin, the mle will be used, i.e. estimate_pars(m). For small samples, the mle may be biased.
xmax	default 1e5. The maximum x value calculated when working out the CDF. See details for further information.
distance	A string containing the distance measure (or measures) to calculate. Possible values are ks or reweight. See details for further information.

### Details

When estimating `xmin` for discrete distributions, the search space when comparing the data-cdf (empirical cdf) and the `distribution_cdf` runs from `xmin` to `max(x)` where `x` is the data set. This **can** often be computationally brutal. In particular, when bootstrapping we generate random numbers from the power law distribution, which has a long tail.

To speed up computations for discrete distributions it is sensible to put an upper bound, i.e. `xmax` and/or explicitly give values of where to search, i.e. `xmin`.

Occasionally bootstrapping can generate strange situations. For example, all values in the simulated data set are less than `xmin`. In this case, the estimated distance measure will be `Inf` and the parameter values, `NA`.

There are other possible distance measures that can be calculated. The default is the Kolomogorov Smirnoff statistic (KS). This is equation 3.9 in the CSN paper. The other measure currently available is `reweight`, which is equation 3.11.

**Note**

Adapted from Laurent Dubroca's code

**Examples**

```
#####
# Load the data set and create distribution object#
#####
x = 1:10
m = displ$new(x)

#####
# Estimate xmin and pars                               #
#####
est = estimate_xmin(m)
m$setXmin(est)

#####
# Bootstrap examples                                   #
#####
## Not run:
bootstrap(m, no_of_sims=1, threads=1)
bootstrap_p(m, no_of_sims=1, threads=1)

## End(Not run)
```

---

get\_KS\_statistic      *Deprecated function*

---

**Description**

This function is now deprecated and may be removed in future versions.

**Usage**

```
get_KS_statistic(m, xmax = 1e+05, distance = "ks")
```

**Arguments**

m	A reference class object that contains the data.
xmax	default 1e5. The maximum x value calculated when working out the CDF. See details for further information.
distance	A string containing the distance measure (or measures) to calculate. Possible values are ks or reweight. See details for further information.

**See Also**

get\_distance\_statistic

---

get_n	<i>Sample size</i>
-------	--------------------

---

**Description**

Returns the sample size of the data set contained within the distribution object.

**Usage**

```
get_n(m)
```

**Arguments**

m                    a distribution object.

**Examples**

```
#####
# Load data and create example object
#####
data(moby_sample)
m = displ$new(moby_sample)
#####
# get_n and length should return the same value
#####
get_n(m)
length(moby_sample)
```

---

get_ntail	<i>Values greater than or equal to xmin</i>
-----------	---

---

**Description**

Returns the number of data points greater than or equal to current value of xmin. In the Clauset et al, paper this is called ntail.

**Usage**

```
get_ntail(m, prop = FALSE, lower = FALSE)
```

**Arguments**

m                    a distribution object.  
prop                 default FALSE. Return the value as a proportion of the total sample size  
lower                default FALSE. If TRUE returns sample size - ntail

**Examples**

```
#####
# Load data and create example object
#####
data(moby_sample)
m = displ$new(moby_sample)
m$setXmin(7)
#####
# Get ntail
#####
get_ntail(m)
sum(moby_sample >= 7)
```

---

lines,distribution-method

*Generic plotting functions*

---

**Description**

These are generic functions for distribution reference objects. Standard plotting functions, i.e. plot, points, and lines work with all distribution objects.

**Usage**

```
## S4 method for signature 'distribution'
lines(x, cut = FALSE, draw = TRUE, length.out = 100, ...)

## S4 method for signature 'distribution'
plot(x, cut = FALSE, draw = TRUE, ...)

## S4 method for signature 'distribution'
points(x, cut = FALSE, draw = TRUE, length.out = 100, ...)
```

**Arguments**

x	a distribution reference object.
cut	logical (default FALSE) - Where should the plot begin. If cut=FALSE, then the plot will start at the minimum data value. Otherwise, the plot will start from xmin
draw	logical (default TRUE). Should the plot/lines/points function plot or return the data (in a data frame object).
length.out	numeric, default 100. How many points should the distribution be evaluated at. This argument is only for plotting the fitted lines.
...	Further arguments passed to the lines functions.

**Note**

This method does *not* alter the internal state of the distribution objects.

---

`moby`*Moby Dick word count*

---

**Description**

The frequency of occurrence of unique words in the novel Moby Dick by Herman Melville.

The data set `moby_sample` is 2000 values sampled from the `moby` data set.

**Format**

A vector

**Source**

M. E. J. Newman, "Power laws, Pareto distributions and Zipf's law." *Contemporary Physics* 46, 323 (2005).

---

`native_american`*Casualties in the American Indian Wars (1776 and 1890)*

---

**Description**

These data files contain the observed casualties in the American Indian Wars. The data sets `native_american` and `us_american` contain the casualties on the Native American and US American sides respectively. Each data set is a data frame, with two columns: the number of casualties and the conflict date.

**Format**

Data frame

**Source**

Friedman, Jeffrey A. "Using Power Laws to Estimate Conflict Size." *The Journal of conflict resolution* (2014).

---

plot.bs_xmin	<i>Plot methods for bootstrap objects</i>
--------------	---

---

### Description

A simple wrapper around the plot function to aid with visualising the bootstrap results. The values plotted are returned as an invisible object.

### Usage

```
## S3 method for class 'bs_xmin'
plot(x, trim = 0.1, ...)

## S3 method for class 'bs_p_xmin'
plot(x, trim = 0.1, ...)

## S3 method for class 'compare_distributions'
plot(x, ...)
```

### Arguments

x	an object of class bs_xmin or bs_p_xmin
trim	When plotting the cumulative means and standard deviation, the first trim percentage of values are not displayed. default trim=0.1
...	graphics parameters to be passed to the plotting routines.

---

population	<i>City boundaries and the universality of scaling laws</i>
------------	---

---

### Description

This data set contains the population size of cities and towns in England. For further details on the algorithm used to determine city boundaries, see the referenced paper.

### Format

vector

### Source

Arcaute, Elsa, et al. "City boundaries and the universality of scaling laws." arXiv preprint arXiv:1301.1674 (2013).



---

`show,distribution-method`*Generic show method for distribution objects*

---

**Description**

The distribution objects have an internal structure that is used for caching purposes. Using the default show method gives the illusion of duplicate values. This show method aims to avoid this confusion.

**Usage**

```
## S4 method for signature 'distribution'  
show(object)
```

**Arguments**

`object`            A distribution object.

---

`swiss_prot`*Word frequency in the Swiss-Prot database*

---

**Description**

This dataset contains all the words extracted from the Swiss-Prot version 9 data (with the resulting frequency for each word). Other datasets for other database versions can be obtained by contacting Michael Bell

Full details in <http://arxiv.org/abs/arXiv:1208.2175v1>

**Format**

data frame

**Source**

Bell, MJ, Gillespie, CS, Swan, D, Lord, P. An approach to describing and analysing bulk biological annotation quality: A case study using UniProtKB. *Bioinformatics* 2012, 28, i562-i568.

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