

# Package ‘pttstability’

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**Title** Particle-Takens Stability

**Version** 1.1

**Description** Includes a collection of functions presented in ``Measuring stability in ecological systems without static equilibria'' by Clark et al. (2022) <[doi:10.1002/ecs2.4328](https://doi.org/10.1002/ecs2.4328)> in Ecosphere. These can be used to estimate the parameters of a stochastic state space model (i.e. a model where a time series is observed with error). The goal of this package is to estimate the variability around a deterministic process, both in terms of observation error - i.e. variability due to imperfect observations that does not influence system state - and in terms of process noise - i.e. stochastic variation in the actual state of the process. Unlike classical methods for estimating variability, this package does not necessarily assume that the deterministic state is fixed (i.e. a fixed-point equilibrium), meaning that variability around a dynamic trajectory can be estimated (e.g. stochastic fluctuations during predator-prey dynamics).

**Depends** R (>= 3.4)

**Imports** graphics, stats, rEDM (>= 1.7.0)

**License** GPL-3

**Encoding** UTF-8

**LazyData** true

**RoxygenNote** 7.2.1

**Collate** 'bayesfun.R' 'data.R' 'fake\_data.R' 'logit\_funs.R'  
  'particlefilter.R' 'pttstability\_man.R'

**NeedsCompilation** no

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## R topics documented:

colfun0	2
dat	3
density_fun0	3

detfun0 . . . . .	4
detfun0_sin . . . . .	5
EDMfun0 . . . . .	5
getcm . . . . .	6
ilogit . . . . .	6
indexsort . . . . .	7
inv_fun0 . . . . .	7
likelihood0 . . . . .	8
likelihood_EDM_piecewise . . . . .	9
logit . . . . .	10
logtnormal_imode . . . . .	10
lognormal_imode . . . . .	11
makedynamics_general . . . . .	11
obsfun0 . . . . .	13
parseparam0 . . . . .	14
particleFilterLL . . . . .	15
particleFilterLL_piecewise . . . . .	16
process_scof . . . . .	19
procfun0 . . . . .	20
procfun_ct . . . . .	21
pttstability . . . . .	21
sampler_fun0 . . . . .	26
sdproc_abstract . . . . .	26

**Index****28**


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<b>colfun0</b>	<i>default colonization function</i>
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**Description**

Simulates colonization events - events occur as a binomial random process with probability  $\text{ilogit}(p)$ , and populations are seeded with abundance  $\exp(A)$ .

**Usage**

```
colfun0(co, xt)
```

**Arguments**

- co        a numeric vector of length two ( $p, A$ ), specifying the logit-transformed colonization probability when abundance is zero, and the log-transformed abundance observed immediately after a colonization event
- xt        a number or numeric vector of abundances at time  $t$ , before colonization has occurred

**Value**

a numeric, including number or numeric vector of length xt, with predicted abundances after colonization has occurred

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dat	<i>Microcosm experimental data</i>
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**Description**

A dataset containing the abundances of *Chlamydomonas terricola* growing in a multi-species community. Includes 17 time steps covering 463 days in two treatments: LSA ('low' temperature, 'stable' oscillations, and 'absence' of predators) and LVA ('low' temperature, 'variable' oscillations, and 'absence' of predators).

**Usage**

```
dat
```

**Format**

A data frame with 271 rows and 4 variables:

- treatment** Experimental treatment
- number** Replicate number
- time** Day of experiment
- Chlamydomonas.terricola** Species abundance

**Source**

Burgmer & Hillebrand 2011, Oikos 120:922-933.

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density_fun0	<i>Default density function for prior</i>
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**Description**

Default density function, following the syntax for priors in the BayesianTools package. Uses flat priors for all parameters, within the given interval. Density function integrates to one.

**Usage**

```
density_fun0(param, minv, maxv)
```

**Arguments**

<code>param</code>	a vector model parameters
<code>minv</code>	a vector of minimum values for the interval
<code>maxv</code>	a vector of maximum values for the interval

**Value**

returns log likelihood of parameters given priors.

<code>detfun0</code>	<i>default deterministic function</i>
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**Description**

Simulates deterministic component of Ricker model, of the form  $xt+1 = xt \exp(\exp(sdet[1]) * (1 - xt / \exp(sdet[2])))$

**Usage**

```
detfun0(sdet, xt, time = NULL, ...)
```

**Arguments**

<code>sdet</code>	a numeric vector of length two, specifying growth rate and carrying capacity
<code>xt</code>	a number or numeric vector of abundances at time t
<code>time</code>	the timestep - defaults to NULL (i.e. not used)
<code>...</code>	additional arguments, for compatibility with other usages of the function - values are not used in this implementation

**Value**

a number or numeric vector of length `xt`, with predicted abundances at time `t+1`

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detfun0_sin	<i>deterministic function with time-varying carrying capacity</i>
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**Description**

Simulates deterministic component of Ricker model, of the form  $xt+1 = xt \exp(\exp(sdet[1])*(1-xt/K))$  where K varies with time as  $(\sin(time/2)+\exp(sdet[2])+0.5)*(2/3)$ . Function is calibrated such that for  $\exp(sdet[2]) = 1$ ,  $\text{mean}(K) = 1$ .

**Usage**

```
detfun0_sin(sdet, xt, time = NULL, ...)
```

**Arguments**

sdet	a numeric vector of length two, specifying growth rate and carrying capacity
xt	a number or numeric vector of abundances at time t
time	the timestep - defaults to NULL (i.e. not used)
...	additional arguments, for compatibility with other usages of the function - values are not used in this implementation

**Value**

a number or numeric vector of length xt, with predicted abundances at time t+1

---

EDMfun0	<i>REDM deterministic function</i>
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**Description**

Estimates future states of xt based on based behaviour

**Usage**

```
EDMfun0(smp_cf, yp, x, minest = 0, maxest = NULL, time)
```

**Arguments**

smp_cf	a matrix of s-map coefficients. Columns correspond to intercept and time lags, rows to observations. Final column corresponds to intercept term.
yp	a matrix of covariates to be multiplied by the smp_cf (typically time lags). Should have one fewer column than smp_cf.
x	observation at time-1, to be used to make the prediction.
minest	minimum value to return for prediction - defaults to 0.
maxest	maximum value to return for prediction - defaults to NULL (no maximum)
time	the time step (i.e. position in smp_cf) for the desired prediction. Prediction will be made based on observation in preceding time point (i.e. time-1).

**Value**

a number or numeric vector of length  $xt$ , with predicted abundances at time  $t+1$

**Source**

Adapted from Ye, Sugihara, et al. (2015), PNAS 112:E1569-E1576.

getcm

*Get rates***Description**

Calculates colonization rate, mortality rate, and expected mean occupancy time based on a time series

**Usage**

```
getcm(dat)
```

**Arguments**

dat	a numeric vector, including the timeseries
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**Value**

a list including colonization and mortality probability per time step (pc and pm, respectively), and pocc, the expected fraction of time that the species will be present

ilogit

*Inverse logit***Description**

Returns the inverse logit transformation of x

**Usage**

```
ilogit(x, ...)
```

**Arguments**

x	a number, vector, matrix, etc. to be transformed from $(-\inf, \inf)$ to $(0, 1)$ by the inverse logit transform
...	additional arguments to be passed to plogis

**Value**

transformed result

<code>indexsort</code>	<i>Sort output of particle filter</i>
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### Description

Sorts outputs of particle filter based on index - returns a sorted list of particles, based on the sampling trajectory through time. This is a somewhat more accurate estimate of the true posterior than are the stepwise samples provided by the filter.

### Usage

```
indexsort(fulltracemat, fulltraceindex, nsmp = NULL)
```

### Arguments

fulltracemat	full output of particles from the particleFilterLL function
fulltraceindex	full output of particle indices from the particleFilterLL function
nsmp	number of particle paths to sample - defaults to NULL, which samples all paths

### Value

an index-sorted matrix - each column shows the trajectory of a single particle

<code>inv_fun0</code>	<i>Default inverse transformation function</i>
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### Description

Takes in a matrix, where each column represents a parameter. Returns parameters in untransformed space. If length = 2, then in the order (obs1, proc1). If 3, then in the order (obs1, proc1, proc2). If 4, then in the order (obs1, obs2, proc1, proc2). If 6, then in the order (obs1, proc1, pcol1, pcol2, det1, det2) If 7, then in the order (obs1, proc1, proc2, pcol1, pcol2, det1, det2) If 8, then in the order (obs1, obs2, proc1, proc2, pcol1, pcol2, det1, det2)

### Usage

```
inv_fun0(x)
```

### Arguments

x	an nxm matrix with
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### Value

returns back-transformed values of parameters

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<i>likelihood0</i>	<i>Default likelihood function</i>
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---

## Description

Calculates likelihood of vector *y* given parameter values in *param*, based on the *particleFilterLL* function.

## Usage

```
likelihood0(
  param,
  y = y,
  parseparam = parseparam0,
  N = 1000,
  detfun = detfun0,
  edmdat = NULL,
  obsfun = obsfun0,
  procfun = procfun0,
  neff = FALSE,
  lowerbound = (-999)
)
```

## Arguments

<i>param</i>	An unformatted vector of parameters, to be passed to <i>parseparam</i> function.
<i>y</i>	A numeric vector of observed values, from which the likelihood of parameters and functions will be determined.
<i>parseparam</i>	A function for transforming the vector <i>param</i> into a form that can be read by <i>particleFilterLL</i> . See <i>particleFilterLL</i> for details.
<i>N</i>	Number of particles to simulate. Defaults to 1e3.
<i>detfun</i>	A function that simulates deterministic dynamics, which takes in arguments <i>sdet</i> (parameters for deterministic model, taken from <i>pars\$proc</i> ), and <i>xt</i> , observed abundances at time <i>t</i> . Returns estimated abundances at time <i>t+1</i> based on deterministic function (either a parametric function or an EDM function). Defaults to <i>detfun0</i> .
<i>edmdat</i>	A list including arguments to be passed to <i>block_lnlp</i> from <i>rEDM</i> package - see <i>block_lnlp</i> help file for details. Default for <i>edmdat</i> is <i>NULL</i> , which implies that EDM will not be applied - instead, a <i>detfun</i> and <i>pars\$det</i> must be included.
<i>obsfun</i>	The observation error function to be used: defaults to <i>obsfun0</i>
<i>procfun</i>	The process noise function to be used: defaults to <i>procfun0</i>
<i>neff</i>	Should effective sample size be used to scale likelihood? Defaults to FALSE. TRUE uses automatic sample size, based on correlations in <i>y</i> . Otherwise, can be any positive number.

lowerbound	Lower bound for log likelihood. Filter will be re-run if the value falls below this threshold. NOTE - this option may induce a bias in the resulting likelihood (and subsequent parameter) estimates. Should only be set if the lower limit is indicative of filter failure (e.g. if all particles) are degenerate. Defaults to (-Inf) - i.e. no lower limit.
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**Value**

Log likelihood generated by particleFilterLL function

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**likelihood\_EDM\_piecewise**

*calculate likelihood for piecewise data*

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**Description**

Calculates likelihoods across several segments of data - e.g. multiple plots from a single experiment. See documentation for particleFilterLL\_piecewise for examples of use.

**Usage**

```
likelihood_EDM_piecewise(
  param,
  y,
  libuse_y,
  smap_coefs,
  Euse,
  tuse,
  N,
  colpar = c(logit(1e-06), log(0.1))
)
```

**Arguments**

param	parameters to be passed to likelihood0 function
y	the time series to be analyzed
libuse_y	a matrix with two columns, specifying the start end end positions of segments within vector y
smap_coefs	a matrix of s-mapping coefficients
Euse	embedding dimension for the s-mapping analysis
tuse	theta for s-mapping analysis
N	number of particles
colpar	parameters to be passed to the colfun0 - defaults to c(logit(1e-6), log(0.1))

**Value**

summed log likelihood across all segments

logit	<i>Logit</i>
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### Description

Returns the logit transformation of x

### Usage

```
logit(x, ...)
```

### Arguments

- x a number, vector, matrix, etc. to be transformed from (0, 1) to (-inf inf) by the logit transform
- ... additional arguments to be passed to plogis

### Value

transformed result - impossible values are replaced with NA, without warnings

logitnormal_imode	<i>Get inverse logit-normal mode</i>
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### Description

Returns a mean for a logit normal such that the mode will be centered around mu

### Usage

```
logitnormal_imode(mu, sd)
```

### Arguments

- mu the value around which the mode should be centered (in logit space)
- sd the standard deviation of the logit distribution (in logit space)

### Value

the proposed mean for the distribution

<code>lognormal_imode</code>	<i>Get inverse log-normal mode</i>
------------------------------	------------------------------------

### Description

Returns a mean for a lognormal such that the mode will be centered around mu

### Usage

```
lognormal_imode(mu, sd)
```

### Arguments

<code>mu</code>	the value around which the mode should be centered (in log space)
<code>sd</code>	the standard deviation of the lognormal distribution (in log space)

### Value

the proposed mean for the distribution

<code>makedynamics_general</code>	<i>Simulate general time series</i>
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### Description

Simulates a time series following a user-defined deterministic function, observation function, process noise function, and colonization function.

### Usage

```
makedynamics_general(
  n = 1000,
  n0 = 0.1,
  pdet = c(log(3), log(1)),
  proc = c(log(1)),
  obs = c(log(1)),
  pcol = c(logit(0.2), log(1)),
  detfun = detfun0,
  procfun = procfun0,
  obsfun = obsfun0,
  colfun = colfun0,
  doplot = FALSE
)
```

### Arguments

n	number of timesteps to simulate
n0	starting population size
pdet	a numeric vector of parameters for the deterministic function
proc	a numeric vector of parameters for the process noise function
obs	a numeric vector of parameters for the observation error function
pcol	a numeric vector of parameters for the colonization function
detfun	A function that simulates deterministic dynamics, which takes in arguments sdet (parameters for deterministic model, taken from pars\$proc), and xt, observed abundances at time t. Returns estimated abundances at time t+1 based on deterministic function (either a parametric function or an EDM function). Defaults to detfun0.
procfun	A function that simulates process noise, which takes in arguments sp (parameters for process noise function, taken from pars\$proc) and xt (abundances prior to process noise). Returns abundances after process noise has occurred. Defaults to procfun0.
obsfun	An observation function, which takes in up to five variables, including so (a vector of parameter values, inherited from pars\$obs), yt (a number, showing observed abundance at time t), xt (predicted abundances), binary value "inverse", and number "N". If inverse = TRUE, then function should simulate N draws from the observation function, centered around value yt. If inverse = FALSE, then function should return log probability denisty of observed value yt given predicted values in xt. Defaults to obsfun0.
colfun	A function simulating colonization events, that takes in two arguments: co, a vector of parameter values taken from pars\$pcol, and xt, a number or numeric vector of abundances at time t, before colonization has occurred. Returns predicted abundances after colonization has occurred. Defaults to colfun0.
doplot	a logical specifying whether output should be plotted - defaults to FALSE

### Value

An n-by-3 dataframe of states, including obs (observed values), truth (true values), and noproc (values without process noise)

### Examples

```
#run function
datout<-makedynamics_general(n=2e4, proc = c(-2,log(1.2)))

#show regression of variance vs. mean for binned data
datout_ps<-datout[datout$true>0 & datout$noproc>0,]
#bins
sq<-seq(0, quantile(datout$true, 0.95), length=50)
ctd<-cut(datout_ps$noproc, sq)
#calculate mean and variance by bin
tdat<-data.frame(mu=(sq[-1]+sq[-length(sq)]) / 2,
```

```

var=tapply((datout_ps$true-datout_ps$noproc)^2, ctd, mean)
#plot result
plot(log(tdat$mu), log(tdat$var), xlab="mu", ylab="var")
#show regression
summary(mod<-lm(log(var)~log(mu), tdat)); abline(mod, col=2)

```

obsfun0

*default observation noise function*

## Description

Two options: If inverse=FALSE, calculates the log probability density of observation yt based on true state xt and observation error. Otherwise, simulates N random observations of yt. Observation error follows a Gaussian distribution truncated at zero, using a Tobit distribution. Note that probability density is calculated based on a Tobit distribution, with lower boundary zero.

## Usage

```

obsfun0(
  so,
  yt,
  xt = NULL,
  inverse = FALSE,
  N = NULL,
  minsd = 0.01,
  time = NULL
)

```

## Arguments

so	a numeric vector of length one, specifying either log-transformed standard deviation of the observation error as a fraction of the observation, or two log-transformed parameters of the form $sd=\exp(B0)+\exp(B1)*x$ .
yt	a number, representing a potential observed value of xt
xt	a number or numeric vector of "true" (or simulated) abundances at time t, from which the likelihood of yt will be calculated - defaults to NULL for inverse=TRUE
inverse	a logical specifying whether inverse (i.e. random number generator) function should be implemented - defaults to FALSE
N	number of draws from the random number generator, if inverse=TRUE - defaults to NULL
minsd	minimum observation error allowed (e.g. if observation = 0), to prevent log likelihoods of -infinity - defaults to 0.01
time	the timestep - defaults to NULL (i.e. not used)

**Value**

If inverse=FALSE, returns a list including LL, a number or numeric vector of length xt, with predicted log likelihoods of observation yt, and wts, a number or vector with weights corresponding to the relative likelihood of each observation (after accounting for variable continuous vs. discrete probability distributions). If inverse = FALSE, returns N random draws from the observation function.

parseparam0

*Parse parameters***Description**

Takes in a vector of 3 or 6 parameters, and puts them into a list of the format expected by the particleFilterLL function.

**Usage**

```
parseparam0(
  param,
  colparam = c(logit(0.2), log(0.1)),
  detparam = c(log(1.2), log(1))
)
```

**Arguments**

- |          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
|----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| param    | List of parameters, of length 2, 3, 4, 6, 7, or 8. If 2, then in the order (obs1, proc1). If 3, then in the order (obs1, proc1, proc2). If 4, then in the order (obs1, obs2, proc1, proc2). If 6, then in the order (obs1, proc1, pcol1, pcol2, det1, det2) If 7, then in the order (obs1, proc1, proc2, pcol1, pcol2, det1, det2) If 8, then in the order (obs1, obs2, proc1, proc2, pcol1, pcol2, det1, det2) Note that if param is of length 2 or 3, then detparam and colparam must be supplied. See obsfun0, procfun0, and detfun0 for more details. |
| colparam | Optional vector of length two, including parameters for the colonization function.                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| detparam | Optional vector of length two, including parameters for the deterministic function.                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |

**Value**

a formatted list of parameters

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<code>particleFilterLL</code>	<i>particle filter</i>
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## Description

General function for calculating the log-likelihood of a stochastic discrete-time model, based on a noisy observation of time-series  $y$ . Returns estimates of true values of  $y$ , as well as for process noise, observation error, colonization rates, and extinction rates. Function is adapted from the R code of Knape and Valpine (2012), Ecology 93:256-263.

## Usage

```
particleFilterLL(
  y,
  pars,
  N = 1000,
  detfun = detfun0,
  procfun = procfun0,
  obsfun = obsfun0,
  colfun = colfun0,
  edmdat = NULL,
  dotraceback = FALSE,
  fulltraceback = FALSE
)
```

## Arguments

<code>y</code>	A numeric vector of observed values, from which the likelihood of parameters and functions will be determined.
<code>pars</code>	A list of parameter values. Must include elements <code>obs</code> (observation error parameters), <code>proc</code> (process noise parameters), and <code>pcol</code> (colonization parameters), which are passed on to their respective functions, described below. If <code>edmdat=NULL</code> , then element <code>det</code> (deterministic process parameters) must be included.
<code>N</code>	Number of particles to simulate. Defaults to <code>1e3</code> .
<code>detfun</code>	A function that simulates deterministic dynamics, which takes in arguments <code>sdet</code> (parameters for deterministic model, taken from <code>pars\$proc</code> ), and <code>xt</code> , observed abundances at time $t$ . Returns estimated abundances at time $t+1$ based on deterministic function (either a parametric function or an EDM function). Defaults to <code>detfun0</code> .
<code>procfun</code>	A function that simulates process noise, which takes in arguments <code>sp</code> (parameters for process noise function, taken from <code>pars\$proc</code> ) and <code>xt</code> (abundances prior to process noise). Returns abundances after process noise has occurred. Defaults to <code>procfun0</code> .

<code>obsfun</code>	An observation function, which takes in up to five variables, including <code>so</code> (a vector of parameter values, inherited from <code>pars\$obs</code> ), <code>yt</code> (a number, showing observed abundance at time <code>t</code> ), <code>xt</code> (predicted abundances), binary value "inverse", and number "N". If <code>inverse = TRUE</code> , then function should simulate N draws from the observation function, centered around value <code>yt</code> . If <code>inverse = FALSE</code> , then function should return log probability denisty of observed value <code>yt</code> given predicted values in <code>xt</code> . Defaults to <code>obsfun0</code> .
<code>colfun</code>	A function simulating colonization events, that takes in two arguments: <code>co</code> , a vector of parameter values taken from <code>pars\$pcol</code> , and <code>xt</code> , a number or numeric vector of abundances at time <code>t</code> , before colonization has occurred. Returns predicted abundances after colonization has occurred. Defaults to <code>colful0</code> .
<code>edmdat</code>	A list including arguments to be passed to <code>block_lnlp</code> from rEDM package - see <code>block_lnlp</code> help file for details. Can also include optional matrix "extra_columns", a matrix with <code>length(y)</code> rows including extra covariates for attractor reconstruction, which defaults to <code>NULL</code> (i.e. no additional columns). Default for <code>edmdat</code> is <code>NULL</code> , which implies that EDM will not be applied - instead, a <code>detfun</code> and <code>pars\$det</code> must be included.
<code>dotraceback</code>	A logical, indicating whether estimated values and demographic rates should be reported - defaults to <code>FALSE</code>
<code>fulltraceback</code>	A logical, indicating whether full matrix of particles for all time steps should be returned.

### Value

`LL` (total log likelihood), `LLlst` (log likelihood for each time step), `Nest` (mean estimated state), `Nsd` (standard deviation of estimated state), `Nest_noproc` (mean estimated state at time `t+1` without process error), `Nsd_noproc` (standard deviation of estimated state at time `t+1` without process error), `fulltracemat` (full traceback of particle paths), `fulltracemat_noproc` (full traceback of particle paths at time `t+1` without process noise), and `fulltraceindex` (index positions for the particle traces over time)

### Source

Adapted from Knape and Valpine (2012), Ecology 93:256-263.

**particleFilterLL\_piecewise**  
*run particle filter across piecewise data*

### Description

Calculates likelihoods across several segments of data - e.g. multiple plots from a single experiment. Requires several implicitly defined variables to run:

**Usage**

```
particleFilterLL_piecewise(
  param,
  N,
  y,
  libuse_y,
  smap_coefs,
  Euse,
  tuse,
  colpar = c(logit(1e-06), log(0.1)),
  nsmp = 1,
  lowerbound = -999,
  maxNuse = 512000
)
```

**Arguments**

<code>param</code>	parameters to be passed to <code>parseparam0</code> function
<code>N</code>	number of particles
<code>y</code>	the time series to be analyzed
<code>libuse_y</code>	a matrix with two columns, specifying the start end end positions of segments within vector <code>y</code>
<code>smap_coefs</code>	a matrix of s-mapping coefficients
<code>Euse</code>	embedding dimension for the s-mapping analysis
<code>tuse</code>	theta for s-mapping analysis
<code>colpar</code>	parameters to be passed to the <code>colfun0</code> - defaults to <code>c(logit(1e-6), log(0.1))</code>
<code>nsmp</code>	number of sample particle trajectories to return - defaults to 1
<code>lowerbound</code>	minimum accepted likelihood - used to automatically select number of particles. Defaults to -999
<code>maxNuse</code>	maximum number of particles to simulate - defaults to 512000

**Value**

results from particle filter - including mean estimates (`Nest`) and standard deviations (`Nsd`), across particles, and sample particle trajectories with (`Nsmp`) and without (`Nsmp_noproc`) process noise

**Examples**

```
# load data
data(dat)

# make list of starting and ending positions for each replicate in the dat list
libmat<-NULL
trtmat<-data.frame(trt=as.character(sort(unique(dat$treatment))))
datnum<-1:nrow(dat)
```

```

for(i in 1:nrow(trtmat)) {
  ps1<-which(dat$treatment==trtmat$trt[i])
  replst<-sort(unique(dat$number[ps1]))

  for(j in 1:length(replst)) {
    ps2<-which(dat$number[ps1]==replst[j])
    libmat<-rbind(libmat, data.frame(trt=trtmat$trt[i], rep=replst[j],
      start=min(datnum[ps1][ps2]), end=max(datnum[ps1][ps2])))
  }
}

## run particle filter
# select treatment to analyse: enter either "LSA" or "LSP"
trtuse<-"HSP"
# extract library positions for treatment
libuse<-as.matrix(libmat[libmat$trt==trtuse,c("start", "end")])
# save abundance data to variable y
yps<-which(dat$treatment==trtuse)
y<-dat[, "Chlamydomonas.terricola"] [yps]
libuse_y<-libuse-min(libuse)+1 # translate positions in dat to positions in y vector
y<-y/sd(y) # standardize to mean of one
timesteps<-dat$time[yps]

# get EDM parameters
require(rEDM) # load rEDM package
sout<-NULL
for(E in 2:4) {
  sout<-rbind(sout, s_map(y, E=E, silent = TRUE, lib = libuse_y))
}
tuse<-sout$theta[which.max(sout$rho)] # find theta (nonlinearity) parameter
euse<-sout$E[which.max(sout$rho)] # find embedding dimension
spred<-s_map(y, E=euse, theta=tuse, silent = TRUE,
  lib = libuse_y, stats_only = FALSE, save_smap_coefficients = TRUE)

# set priors (log-transformed Beta_obs, Beta_proc1, and Beta_proc2)
minvUSE_edm<-c(log(0.001), log(0.001)) # lower limits
maxvUSE_edm<-c(log(2), log(2)) # upper limits

## Not run:
## Run filter
# density, sampler, and prior functions for EDM function
# Commented-out code: Install BayesianTools package from GitHub if needed
#require(devtools)
#install_github("florianhartig/BayesianTools/BayesianTools")
# see BayesianTools documentation for details
require(BayesianTools)
density_fun_USE_edm<-function(param) density_fun0(param = param,
  minv = minvUSE_edm, maxv=maxvUSE_edm)
sampler_fun_USE_edm<-function(x) sampler_fun0(n = 1, minv = minvUSE_edm, maxv=maxvUSE_edm)
prior_edm <- createPrior(density = density_fun_USE_edm, sampler = sampler_fun_USE_edm,
  lower = minvUSE_edm, upper = maxvUSE_edm)

```

```

niter<-5e3 # number of steps for the MCMC sampler
N<-1e3 # number of particles
smap_coefs<-spred$smap_coefficients[[1]] # coefficients from s-mapping routine

# likelihood and bayesian set-ups for EDM functions
likelihood_EDM_piecewise_use<-function(x) {
  # default values for filter - see likelihood_EDM_piecewise documentation for details
  # note that colpar are set near zero because we expect no colonisation into a closed microcosm.
  likelihood_EDM_piecewise(param=x, y, libuse_y, smap_coefs, euse, tuse, N,
                            colpar = c(logit(1e-06), log(0.1)))
}

bayesianSetup_EDM <- createBayesianSetup(likelihood = likelihood_EDM_piecewise_use,
                                         prior = prior_edm)

# run MCMC optimization (will take ~ 15-20 min)
out_EDM <- runMCMC(bayesianSetup = bayesianSetup_EDM,
                     settings = list(iterations=niter, consoleUpdates=20))
burnin<-floor(niter/5) # burnin period
plot(out_EDM, start=burnin) # plot MCMC chains
gelmanDiagnostics(out_EDM, start=burnin) # calculate Gelman statistic
summary(out_EDM, start=burnin) # coefficient summary

## extract abundance estimate from particle filter
# use final estimate from MCMC chain
smp_EDM<-getSample(out_EDM, start=floor(niter/5))
tmp<-particleFilterLL_piecewise(param = smp_EDM[nrow(smp_EDM),], N=N, y = y, libuse_y = libuse_y,
                                  smap_coefs = smap_coefs, Euse = euse, tuse = tuse)
# mean estimated abundance
simout<-tmp$Nest
# sd estimated abundance
sdout<-tmp$Nsds
# sample from true particle trajectory
simout_smp<-tmp$Nsmp
# sample from true particle trajectory pre-process noise
simout_smp_noproc<-tmp$Nsmp_noproc

plot(timesteps, simout, xlab="Time", ylab="Abundance")
abline(h=0, lty=3)

## End(Not run)

```

**Description**

Processes s-mapping coefficients from rEDM into a matrix of form C1, C2, C3, ... C0, where C0 is the intercept, C1 is the current time step t, C2 is timestep t-1, C3 is timestep t-2, and so on. Rows correspond to the time step used to produce the prediction, e.g. row 4 is used to calculate

predicted value for time step 5. This is the format expected by the EDMfun0 function. Note - the format produced by the rEDM package has changed substantially over time, and so if you find that predictions are no longer working in this package, it is likely that this is due to another change in reporting format, in which case you may need to update this function accordingly (e.g. to re-align columns or rows).

### Usage

```
process_scof(smap_coefs)
```

### Arguments

**smap\_coefs** a matrix of s-map coefficients, taken from the *s\_map* function.

### Value

a matrix of s-mapping coefficients

<b>procfun0</b>	<i>default process noise function</i>
-----------------	---------------------------------------

### Description

Simulates effects of process noise following a Gaussian perturbation. Note that process noise only influences positive abundances (i.e. process noise cannot contribute to colonization)

### Usage

```
procfun0(sp, xt, inverse = FALSE, time = NULL)
```

### Arguments

<b>sp</b>	a numeric vector of length one or two, specifying either the log-transformed standard deviation of the process noise function, or an intercept and slope for calculating variance of process noise based on a power function of x, of the form var=exp(B0)*x^exp(B1)
<b>xt</b>	a number or numeric vector of abundances at time t, before process noise has occurred
<b>inverse</b>	a logical specifying whether the inverse (i.e. probability of drawing a value of zero given xt and sp) should be calculated
<b>time</b>	the timestep - defaults to NULL (i.e. not used)

### Value

a number or numeric vector of length xt, with predicted abundances after process noise has occurred

---

procfun_ct	<i>continuous-time process noise function</i>
------------	-----------------------------------------------

---

## Description

Simulates effects of process noise following a Gaussian perturbation. Note that process noise only influences positive abundances (i.e. process noise cannot contribute to colonization)

## Usage

```
procfun_ct(sp, xt, waiting_time = 1, time = NULL)
```

## Arguments

sp	a numeric vector of length two or three, where terms 1-2 specify either the log-transformed standard deviation of the process noise function, or an intercept and slope for calculating variance of process noise based on a power function of x, of the form var=exp(B0)*x^exp(B1) The final term in the vector represents the recovery rate - i.e. the continuous time rate at which abundances recover from perturbation
xt	a number or numeric vector of abundances at time t, before process noise has occurred
waiting_time	average time between disturbance events: defaults to 1
time	the timestep - defaults to NULL (i.e. not used)

## Value

a number or numeric vector of length xt, with predicted abundances after process noise has occurred

---

pttstability	<i>pttstability: Methods for Measuring Stability in Systems Without Static Equilibria</i>
--------------	-------------------------------------------------------------------------------------------

---

## Description

The pttstability ("ParTicle-Takens Stability") package is a collection of functions that can be used to estimate the parameters of a stochastic state space model (i.e. a model where a time series is observed with error).

## Applications

The goal of this package is to estimate the variability around a deterministic process, both in terms of observation error - i.e. variability due to imperfect observations that does not influence system state - and in terms of process noise - i.e. stochastic variation in the actual state of the process. Unlike classical methods for estimating variability, this package does not necessarily assume that the deterministic state is fixed (i.e. a fixed-point equilibrium), meaning that variability around a dynamic trajectory can be estimated (e.g. stochastic fluctuations during predator-prey dynamics).

By combining information about both the estimated deterministic state of the system and the estimated effects of process noise, this package can be used to compute a dynamic analog of various stability metrics - e.g. coefficient of variation (CV) or invariability. Estimated extinction rates and colonization rates can also be estimated.

## Contents

This package builds on three existing toolkits. First, it applies an updated version of the "particle-FilterLL" particle filter function of Knape and Valpine (2012) to calculate likelihoods of observed time series given modeled dynamics. Second, it applies empirical dynamic modeling (EDM) methods from the rEDM package to estimate deterministic dynamics even in cases where the underlying equations governing system behavior are not known. These models are based on Takens delay-embedding theorem, from which this package takes its name. Finally, it (optionally) uses the MCMC fitting methods from the BayesianTools package to estimate parameter values for the observation error, process noise, and (optionally) deterministic functions underlying observed dynamics.

The default observation error and process noise functions in this package (`obsfun0` and `procfun0`) take advantage of the Taylor Power law to separate noise components for relatively short time series. Observation error is assumed to scale with the observed state as  $sd\_obs(x) = x * exp(obs)$ , Process noise is either a constant (i.e.  $sd\_proc(x) = exp(proc)$ ), or, if two variables are given, process noise scales as a power function of the observed value as  $sd\_proc(x) = sqrt(exp(proc1) * x^exp(proc2))$

Note that although we include default functions in this package, users are able (and encouraged!) to write their own (including for observation error, process noise, deterministic dynamics, priors, and likelihoods).

## Source

- Knape, J., and Valpine, P. (2012). Fitting complex population models by combining particle filters with Markov chain Monte Carlo. *Ecology* 93:256-263.
- Ye, H., Sugihara, G., et al. (2015). Equation-free ecosystem forecasting. *PNAS* 112:E1569-E1576.
- Ye, H., et al. (2019). rEDM: Applications of Empirical Dynamic Modeling from Time Series. R package version 0.7.2.
- Hartig, F., et al. (2019). BayesianTools: General-Purpose MCMC and SMC Samplers and Tools for Bayesian Statistics. R package version 0.1.6.

## Examples

```
#Load packages
require(rEDM)

#Set seed
```

```

set.seed(5826)

## Simulate data
pars_true<-list(obs=log(0.15),
                 proc=c(log(0.1)),
                 pcol=c(logit(0.2), log(0.1)),
                 det=c(log(1.2),log(1)))
#parameters for the filter
pars_filter<-pars_true

#generate random parameter values
datout<-makedynamics_general(n = 100, n0 = exp(pars_true$det[2]),
                               pdet=pars_true$det, proc = pars_true$proc,
                               obs = pars_true$obs, pcol = pars_true$pcol,
                               detfun = detfun0_sin, procfun = procfun0,
                               obsfun=obsfun0, colfun=colfun0)
y<-datout$obs
plot(y, type = "l", xlab="time", ylab="observed abundance")

#get theta paramter for s-mapping
# assume best E = 4
# alternatively, we could run e.g. s_map(y, E=(2:5), silent = TRUE)
# to test a range of potential E values
Euse = 4
#run leave-one-out cross validation
s<-s_map(y, E=Euse, silent = TRUE)
tuse<-s$theta[which.min(s$rmse)] # retain best theta
plot(s$theta, s$rho, type="b")

## Run filter with "correct" parameter values
N = 1e3 # number of particles
#based on detful0
filterout_det<-particleFilterLL(y, pars=pars_filter, N,
                                  detfun = detfun0_sin, procfun = procfun0,
                                  dotracing = TRUE, fulltraceback = TRUE)
#based on EDM
filterout_edm<-particleFilterLL(y, pars=pars_filter, N, detfun = EDMfun0,
                                  edmdat = list(E=Euse, theta=tuse),
                                  procfun = procfun0, dotracing = TRUE,
                                  fulltraceback = TRUE)

#plot filter output
op = par(mar=c(4,4,2,2), mfrow=c(3,1))
#plot 30 of the 1000 particles to show general trend
# correct deterministic function
matplot(1:length(y), filterout_det$fulltracemat[,1:30],
        col=adjustcolor(1,alpha.f = 0.5), lty=3,
        type="l", xlab="time", ylab="abund", main="detfun0")
lines(1:length(y), y, col=2, lwd=1.5) #observations
lines(1:length(y), datout$true, col="blue", lwd=1.5, lty=2) #true values
lines(1:length(y), filterout_det$Nest, col=3, lwd=1.5) #mean filter estimate

# EDM function

```

```

matplot(1:length(y), filterout_edm$fulltracemat[,1:30],
        col=adjustcolor(1,alpha.f = 0.5), lty=3,
        type="l", xlab="time", ylab="abund", main="EDM")
lines(1:length(y), y, col=2, lwd=1.5)
lines(1:length(y), datout$true, col="blue", lwd=1.5, lty=2)
lines(1:length(y), filterout_edm$Nest, col=3, lwd=1.5)

plot(filterout_det$Nest, datout$true, xlim=range(c(filterout_det$Nest,
        filterout_edm$Nest)),
        xlab="predicted", ylab="true value", col=4)
points(filterout_edm$Nest, datout$true, col=2)
points(y, datout$true, col=3)
abline(a=0, b=1, lty=2)
legend("topleft", c("detfun0", "EDM", "obs"), pch=1, col=c(4,2,3), bty="n")

#note improvement in fit, for both filters
cor(datout$true, datout$obs)^2 #observations
cor(datout$true, filterout_det$Nest)^2 #deterministic filter
cor(datout$true, filterout_edm$Nest)^2 #EDM filter
par(op) # reset plotting parameters

## Not run:
# Commented-out code: Install BayesianTools package from GitHub if needed
#require(devtools)
#install_github("florianhartig/BayesianTools/BayesianTools")
# see BayesianTools documentation for details
require(BayesianTools)
## Run optimizers
#create priors
minvUSE<-c(-4, -4) #minimum interval for obs and proc
maxvUSE<-c(0, 0) #maximum interval for obs and proc

minvUSE_edm<-c(-4, -4) #minimum interval for obs and proc
maxvUSE_edm<-c(0, 0) #maximum interval for obs and proc

#density, sampler, and prior functions for deterministic function
density_fun_USE<-function(param) density_fun0(param = param, minv = minvUSE,
        maxv=maxvUSE)
sampler_fun_USE<-function(x) sampler_fun0(n = 1, minv = minvUSE,
        maxv=maxvUSE)
prior_USE <- createPrior(density = density_fun_USE,
        sampler = sampler_fun_USE,
        lower = minvUSE, upper = maxvUSE)

#density, sampler, and prior functions for EDM function
density_fun_USE_edm<-function(param) density_fun0(param = param,
        minv = minvUSE_edm, maxv=maxvUSE_edm)
sampler_fun_USE_edm<-function(x) sampler_fun0(n = 1, minv = minvUSE_edm,
        maxv=maxvUSE_edm)
prior_edm <- createPrior(density = density_fun_USE_edm,
        sampler = sampler_fun_USE_edm,
        lower = minvUSE_edm, upper = maxvUSE_edm)

## Run filter

```

```

niter<-5000 #number of steps for the MCMC sampler
N<-1e3 #number of particles
Euse<-Euse #number of embedding dimensions

#likelihood and bayesian set-ups for deterministic functions
likelihood_detfun0<-function(x) likelihood0(param=x, y=y,
                                              parseparam = parseparam0, detfun = detfun0_sin,
                                              procfun = procfun0, N = N)
bayesianSetup_detfun0 <- createBayesianSetup(likelihood = likelihood_detfun0,
                                              prior = prior_USE)

#likelihood and bayesian set-ups for EDM functions
likelihood_EDM<-function(x) {
  likelihood0(param = x, y=y, parseparam = parseparam0, procfun = procfun0,
              detfun = EDMfun0, edmdat = list(E=Euse, theta=tuse), N = N)
}

bayesianSetup_EDM <- createBayesianSetup(likelihood = likelihood_EDM,
                                          prior = prior_edm)
#run MCMC optimization
out_detfun0 <- runMCMC(bayesianSetup = bayesianSetup_detfun0,
                        settings = list(iterations=niter, consoleUpdates=20))
out_EDM <- runMCMC(bayesianSetup = bayesianSetup_EDM,
                     settings = list(iterations=niter, consoleUpdates=20))

#plot results, with a 1000-step burn-in
plot(out_detfun0, start = 1000, thin = 2)
plot(out_EDM, start = 1000, thin = 2)

## extract and plot parameter distributions
smp_detfun0<-getSample(out_detfun0, start = 1000, thin = 2)
smp_EDM<-getSample(out_EDM, start=1000, thin = 2)

op = par(mfrow=c(2,2))
hist(exp(smp_detfun0[,1]), xlim=c(exp(minvUSE[1]), exp(maxvUSE[1])),
     main="det. function", xlab="obs", breaks = 20)
abline(v=exp(pars_true$obs), col=2) # true value
abline(v=c(exp(minvUSE[1]), exp(maxvUSE[1])), col=1, lty=2)# Priors

hist(exp(smp_detfun0[,2]), xlim=c(exp(minvUSE[2]), exp(maxvUSE[2])),
     main="det. function", xlab="proc", breaks = 20)
abline(v=exp(pars_true$proc), col=2) # true value
abline(v=c(exp(minvUSE[2]), exp(maxvUSE[2])), col=1, lty=2)# Priors

hist(exp(smp_EDM[,1]), xlim=c(exp(minvUSE_edm[1]), exp(maxvUSE_edm[1])),
     main="EDM function", xlab="obs", breaks = 20)
abline(v=exp(pars_true$obs), col=2) # true value
abline(v=c(exp(minvUSE_edm[1]), exp(maxvUSE_edm[1])), col=1, lty=2)# Priors

hist(exp(smp_EDM[,2]), xlim=c(exp(minvUSE_edm[2]), exp(maxvUSE_edm[2])),
     main="EDM function", xlab="proc", breaks = 20)
abline(v=exp(pars_true$proc), col=2) # true value
abline(v=c(exp(minvUSE_edm[2]), exp(maxvUSE_edm[2])), col=1, lty=2)# Priors

```

```

## compare total EDM coefficient prediction error to total model error
s_full<-s_map(y, E=Euse, theta=tuse, silent = TRUE)

# over-estimation of stochastic variance by EDM
(mean(exp(smp_EDM[,1])^2 + exp(smp_EDM[,2])^2)-
  (exp(pars_true$proc)^2+exp(pars_true$obs[1])^2))
# rmse due to EDM fitting error
s_full$rmse[[1]]^2-(exp(pars_true$proc)^2+exp(pars_true$obs[1])^2)
par(op) # reset plotting parameters

## End(Not run)

```

**sampler\_fun0***Default sampler function for prior***Description**

Draws samples from a flat prior

**Usage**

```
sampler_fun0(n = 1, minv, maxv)
```

**Arguments**

- |             |                                                       |
|-------------|-------------------------------------------------------|
| <b>n</b>    | number of random draws to take from the priors        |
| <b>minv</b> | Vector of minimum values to return for each parameter |
| <b>maxv</b> | Vector of maximum values to return for each parameter |

**Value**

returns random draws from the priors

**sdproc\_abstract***calculate estimated total variance***Description**

Function for estimating stochastic variation in linear process x as a function of relative growth rate and disturbance regime standard deviation.

**Usage**

```
sdproc_abstract(sd_proc, rgr, waiting_time = 1)
```

**Arguments**

sd_proc	standard deviation of the (Gaussian) disturbance process
rgr	relative growth rate of the linear process
waiting_time	average waiting time between (random exponentially distributed through time) disturbance events

**Value**

standard deviation of stochastic variability in x

# Index

- \* **EDM**
  - EDMfun0, 5
- \* **MCMC**
  - density\_fun0, 3
  - inv\_fun0, 7
  - likelihood0, 8
  - logitnormal\_imode, 10
  - lognormal\_imode, 11
  - sampler\_fun0, 26
- \* **Taylor**
  - makedynamics\_general, 11
  - particleFilterLL, 15
- \* **colonization**
  - colfun0, 2
- \* **datasets**
  - dat, 3
- \* **deterministic**
  - detfun0, 4
  - detfun0\_sin, 5
- \* **dewdrop**
  - likelihood\_EDM\_piecewise, 9
  - particleFilterLL\_piecewise, 16
- \* **discrete-time**
  - detfun0, 4
  - detfun0\_sin, 5
- \* **error**
  - obsfun0, 13
- \* **filter**
  - indexsort, 7
  - likelihood\_EDM\_piecewise, 9
  - parseparam0, 14
  - particleFilterLL, 15
  - particleFilterLL\_piecewise, 16
- \* **law**
  - makedynamics\_general, 11
  - particleFilterLL, 15
- \* **linear**
  - sdproc\_abstract, 26
- \* **logit**
  - ilogit, 6
  - logit, 10
- \* **noise**
  - procfun0, 20
  - procfun\_ct, 21
- \* **observation**
  - obsfun0, 13
- \* **optimization**
  - density\_fun0, 3
  - inv\_fun0, 7
  - likelihood0, 8
  - logitnormal\_imode, 10
  - lognormal\_imode, 11
  - sampler\_fun0, 26
- \* **particle**
  - indexsort, 7
  - likelihood\_EDM\_piecewise, 9
  - parseparam0, 14
  - particleFilterLL, 15
  - particleFilterLL\_piecewise, 16
- \* **power**
  - makedynamics\_general, 11
  - particleFilterLL, 15
- \* **process**
  - procfun0, 20
  - procfun\_ct, 21
- \* **regression**
  - likelihood\_EDM\_piecewise, 9
  - particleFilterLL\_piecewise, 16
- \* **stability**
  - density\_fun0, 3
  - getcm, 6
  - inv\_fun0, 7
  - likelihood0, 8
  - makedynamics\_general, 11
  - parseparam0, 14
  - particleFilterLL, 15
  - sampler\_fun0, 26
- \* **stochastic**

sdproc\_abstract, 26  
\* **system**  
    sdproc\_abstract, 26  
\* **time-series**  
    density\_fun0, 3  
    detfun0, 4  
    detfun0\_sin, 5  
    getcm, 6  
    inv\_fun0, 7  
    likelihood0, 8  
    makedynamics\_general, 11  
    parseparam0, 14  
    particleFilterLL, 15  
    sampler\_fun0, 26

colfun0, 2

dat, 3  
density\_fun0, 3  
detfun0, 4  
detfun0\_sin, 5

EDMfun0, 5

getcm, 6

ilogit, 6  
indexsort, 7  
inv\_fun0, 7

likelihood0, 8  
likelihood\_EDM\_piecewise, 9  
logit, 10  
logitnormal\_imode, 10  
lognormal\_imode, 11

makedynamics\_general, 11

obsfun0, 13

parseparam0, 14  
particleFilterLL, 15  
particleFilterLL\_piecewise, 16  
process\_scof, 19  
procfun0, 20  
procfun\_ct, 21  
pttstability, 21

sampler\_fun0, 26  
sdproc\_abstract, 26