Package ‘twosamples’

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Type Package

Title Fast Permutation Based Two Sample Tests

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Description Fast randomization based two sample tests.
Testing the hypothesis that two samples come from the same distribution using randomization to create p-values. Included tests are: Kolmogorov-Smirnov, Kuiper, Cramer-von Mises, Anderson-Darling, Wasserstein, and DTS. The default test (two_sample) is based on the DTS test statistic, as it is the most powerful, and thus most useful to most users.
The DTS test statistic builds on the Wasserstein distance by using a weighting scheme like that of Anderson-Darling. See the companion paper at <arXiv:2007.01360> or <https://codowd.com/public/DTS.pdf> for details of that test statistic, and non-standard uses of the package (parallel for big N, weighted observations, one sample tests, etc). We also include the permutation scheme to make test building simple for others.

License GPL (>= 2)

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BugReports https://github.com/cdowd/twosamples/issues

NeedsCompilation yes

Repository CRAN

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R topics documented:

`ad_stat` *Anderson-Darling Test*

Description

A two-sample test based on the Anderson-Darling test statistic (`ad_stat`).

Usage

```
ad_stat(a, b, power = 2)

ad_test(a, b, nboots = 2000, p = default.p)
```

Arguments

`a`	a vector of numbers
`b`	a vector of numbers
`power`	power to raise test stat to
`nboots`	Number of bootstrap iterations
`p`	power to raise test stat to

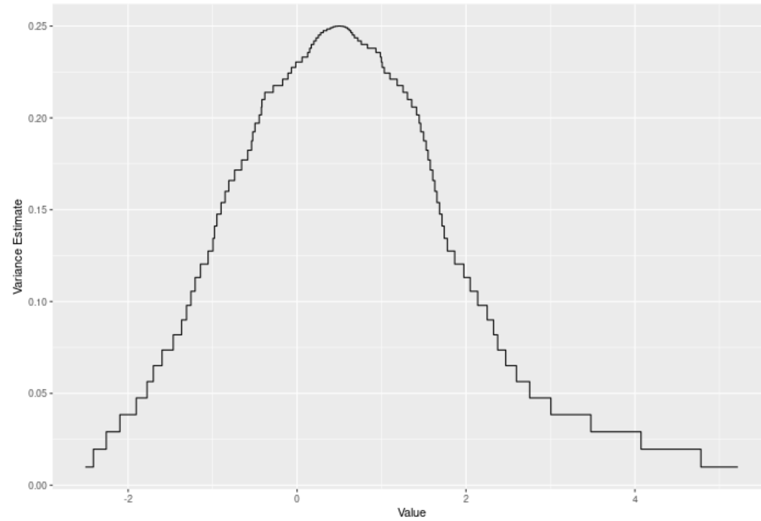
Details

The AD test compares two ECDFs by looking at the weighted sum of the squared differences between them – evaluated at each point in the joint sample. The weights are determined by the variance of the joint ECDF at that point, which peaks in the middle of the joint distribution (see figure below). Formally – if E is the ECDF of sample 1, F is the ECDF of sample 2, and G is the ECDF of the joint sample then

$$AD = \sum_{x \in k} \frac{|E(x) - F(x)|^p}{G(x)(1 - G(x))}$$

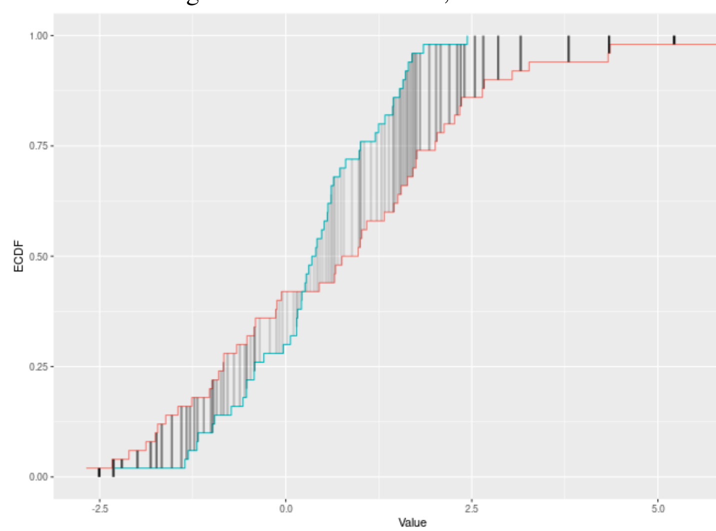
where k is the joint sample. The test p-value is calculated by randomly resampling two samples of the same size using the combined sample. Intuitively the AD test improves on the CVM test by giving lower weight to noisy observations.

In the example plot below, we see the variance of the joint ECDF over the range of the data. It clearly



peaks in the middle of the joint sample.

In the example plot below, the AD statistic is the weighted sum of the heights of the vertical lines,



where weights are represented by the shading of the lines.

Value

Output is a length 2 Vector with test stat and p-value in that order. That vector has 3 attributes – the sample sizes of each sample, and the number of bootstraps performed for the pvalue.

Functions

- `ad_stat`: Anderson-Darling Test statistic
- `ad_test`: Permutation based two sample Anderson-Darling test

See Also

dts_test() for a more powerful test statistic. See cvm_test() for the predecessor to this test statistic. See dts_test() for the natural successor to this test statistic.

Examples

```
vec1 = rnorm(20)
vec2 = rnorm(20,4)
ad_test(vec1,vec2)
```

---

cvm_stat *Cramer-von Mises Test*

Description

A two-sample test based on the Cramer-Von Mises test statistic (cvm_stat).

Usage

```
cvm_stat(a, b, power = 2)

cvm_test(a, b, nboots = 2000, p = default.p)
```

Arguments

a	a vector of numbers
b	a vector of numbers
power	power to raise test stat to
nboots	Number of bootstrap iterations
p	power to raise test stat to

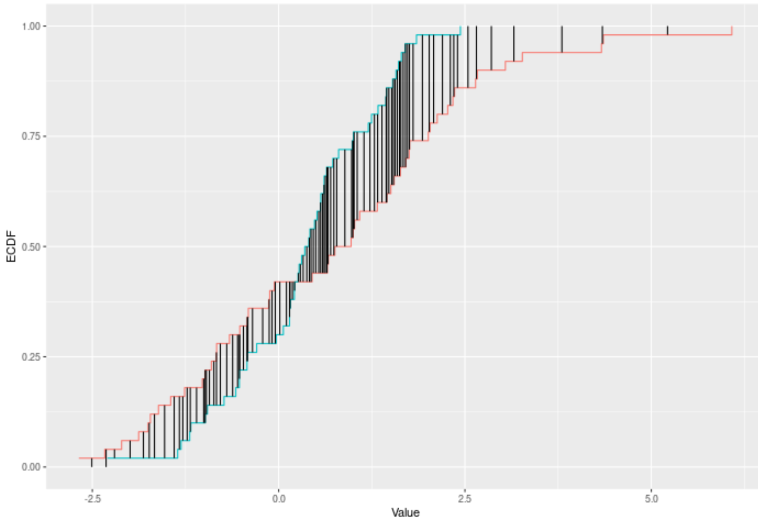
Details

The CVM test compares two ECDFs by looking at the sum of the squared differences between them – evaluated at each point in the joint sample. Formally – if E is the ECDF of sample 1 and F is the ECDF of sample 2, then

$$CVM = \sum_{x \in k} |E(x) - F(x)|^p$$

where k is the joint sample. The test p-value is calculated by randomly resampling two samples of the same size using the combined sample. Intuitively the CVM test improves on KS by using the full joint sample, rather than just the maximum distance – this gives it greater power against shifts in higher moments, like variance changes.

In the example plot below, the CVM statistic is the sum of the heights of the vertical black lines.



Value

Output is a length 2 Vector with test stat and p-value in that order. That vector has 3 attributes – the sample sizes of each sample, and the number of bootstraps performed for the pvalue.

Functions

- `cvm_stat`: Cramer-Von Mises Test statistic
- `cvm_test`: Permutation based two sample Cramer-Von Mises test

See Also

`dts_test()` for a more powerful test statistic. See `ks_test()` or `kuiper_test()` for the predecessors to this test statistic. See `wass_test()` and `ad_test()` for the successors to this test statistic.

Examples

```
vec1 = rnorm(20)
vec2 = rnorm(20,4)
cvm_test(vec1,vec2)
```

`dts_stat` *DTS Test*

Description

A two-sample test based on the DTS test statistic (`dts_stat`). This is the recommended two-sample test in this package because of its power. The DTS statistic is the reweighted integral of the distance between the two ECDFs.

Usage

```
dts_stat(a, b, power = 1)

dts_test(a, b, nboots = 2000, p = default.p)

two_sample(a, b, nboots = 2000, p = default.p)
```

Arguments

`a`	a vector of numbers
`b`	a vector of numbers
`power`	also the power to raise the test stat to
`nboots`	Number of bootstrap iterations
`p`	power to raise test stat to

Details

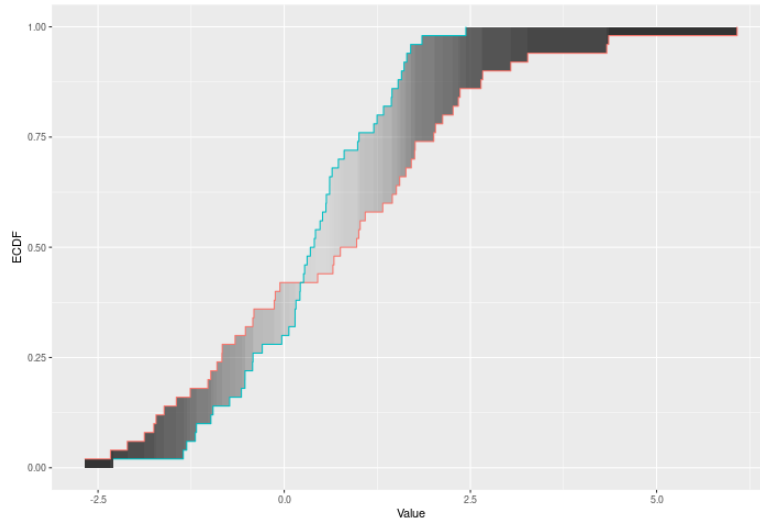
The DTS test compares two ECDFs by looking at the reweighted Wasserstein distance between the two. See the companion paper at arXiv:2007.01360 or `https://codowd.com/public/DTS.pdf` for details of this test statistic, and non-standard uses of the package (parallel for big N, weighted observations, one sample tests, etc).

If the `wass_test()` extends `cvm_test()` to interval data, then `dts_test()` extends `ad_test()` to interval data. Formally – if E is the ECDF of sample 1, F is the ECDF of sample 2, and G is the ECDF of the combined sample, then

$$DTS = \int_{x \in R} \frac{|E(x) - F(x)|^p}{G(x)(1 - G(x))}$$

for all x. The test p-value is calculated by randomly resampling two samples of the same size using the combined sample. Intuitively the DTS test improves on the AD test by allowing more extreme observations to carry more weight. At a higher level – CVM/AD/KS/etc only require ordinal data. DTS (and Wasserstein) gain power because they take advantages of the properties of interval data – i.e. the distances have some meaning. However, DTS, like Anderson-Darling (AD) also downweights noisier observations relative to Wass, thus (hopefully) giving it extra power.

In the example plot below, the DTS statistic is the shaded area between the ECDFs, weighted by the



variances – shown by the color of the shading.

Value

Output is a length 2 Vector with test stat and p-value in that order. That vector has 3 attributes – the sample sizes of each sample, and the number of bootstraps performed for the pvalue.

Functions

- `dts_stat`: Test statistic based on a weighted area between ECDFs
- `dts_test`: Permutation based two sample test
- `two_sample`: Recommended two-sample test

See Also

`wass_test()`, `ad_test()` for the predecessors of this test statistic. arXiv:2007.01360 or `https://codowd.com/public/DTS.pdf` for details of this test statistic

Examples

```
vec1 = rnorm(20)
vec2 = rnorm(20,4)
dts_stat(vec1,vec2)
dts_test(vec1,vec2)
two_sample(vec1,vec2)
```

ks_stat *Kolmogorov-Smirnov Test*

Description

A two-sample test using the Kolmogorov-Smirnov test statistic (`ks_stat`).

Usage

```
ks_stat(a, b, power = 1)

ks_test(a, b, nboots = 2000, p = default.p)
```

Arguments

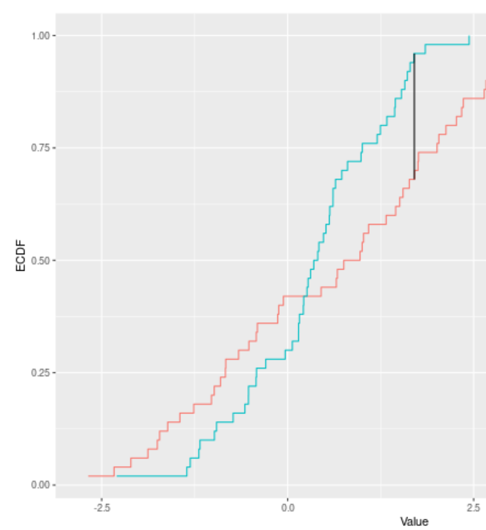
`a`	a vector of numbers
`b`	a vector of numbers
`power`	power to raise test stat to
`nboots`	Number of bootstrap iterations
`p`	power to raise test stat to

Details

The KS test compares two ECDFs by looking at the maximum difference between them. Formally – if E is the ECDF of sample 1 and F is the ECDF of sample 2, then

$$KS = max|E(x) - F(x)|^p$$

for values of x in the joint sample. The test p-value is calculated by randomly resampling two samples of the same size using the combined sample.



In the example plot below, the KS statistic is the height of the vertical black line.

Value

Output is a length 2 Vector with test stat and p-value in that order. That vector has 3 attributes – the sample sizes of each sample, and the number of bootstraps performed for the pvalue.

Functions

- `ks_stat`: Kolmogorov-Smirnov test statistic
- `ks_test`: Permutation based two sample Kolmogorov-Smirnov test

See Also

`dts_test()` for a more powerful test statistic. See `kuiper_test()` or `cvm_test()` for the natural successors to this test statistic.

Examples

```
vec1 = rnorm(20)
vec2 = rnorm(20,4)
ks_test(vec1,vec2)
```

---

kuiper_stat *Kuiper Test*

Description

A two-sample test based on the Kuiper test statistic (`kuiper_stat`).

Usage

```
kuiper_stat(a, b, power = 1)

kuiper_test(a, b, nboots = 2000, p = default.p)
```

Arguments

a	a vector of numbers
b	a vector of numbers
power	power to raise test stat to
nboots	Number of bootstrap iterations
p	power to raise test stat to

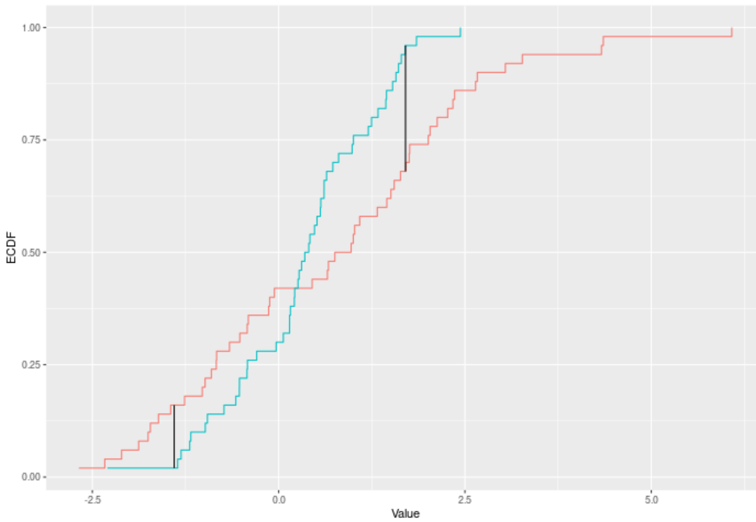
Details

The Kuiper test compares two ECDFs by looking at the maximum positive and negative difference between them. Formally – if E is the ECDF of sample 1 and F is the ECDF of sample 2, then

$$KUIPER = |max_x E(x) - F(x)|^p + |max_x F(x) - E(x)|^p$$

. The test p-value is calculated by randomly resampling two samples of the same size using the combined sample.

In the example plot below, the Kuiper statistic is the sum of the heights of the vertical black lines.



Value

Output is a length 2 Vector with test stat and p-value in that order. That vector has 3 attributes – the sample sizes of each sample, and the number of bootstraps performed for the pvalue.

Functions

- `kuiper_stat`: Kuiper Test statistic
- `kuiper_test`: Permutation based two sample Kuiper test

See Also

`dts_test()` for a more powerful test statistic. See `ks_test()` for the predecessor to this test statistic, and `cvm_test()` for its natural successor.

Examples

```
vec1 = rnorm(20)
vec2 = rnorm(20,4)
kuiper_test(vec1,vec2)
```

order_stl — *Order function in C++ using the STL*

Description

Simply finds the order of a vector in c++. Mostly for internals.

Usage

```
order_stl(x)
```

Arguments

x	numeric vector

Value

same length vector of integers representing order of input vector

Examples

```
vec = c(1,4,3,2)
order_stl(vec)
```

permutation_test_builder — *Permutation Test Builder*

Description

This function takes a simple two-sample test statistic and produces a function which performs randomization tests (sampling with replacement) using that test stat.

Usage

```
permutation_test_builder(test_stat_function, default.p = 2)
```

Arguments

test_stat_function	a function of two vectors producing a positive number, intended as the test-statistic to be used.
default.p	This allows for some introduction of defaults and parameters. Typically used to control the power functions raise something to.

Details

test_stat_function must be structured to take two separate vectors, and then a third value. i.e. (fun = function(vec1,vec2,val1) ...). See examples.

Value

This function returns a function which will perform permutation tests on given test stat.

Functions

- `permutation_test_builder`: Takes a test statistic, returns a testing function.

See Also

`two_sample()`

Examples

```
mean_stat = function(a,b,p) abs(mean(a)-mean(b))**p
myfun = permutation_test_builder(mean_stat,2.0)
vec1 = rnorm(20)
vec2 = rnorm(20,4)
myfun(vec1,vec2)
```

`wass_stat` *Wasserstein Distance Test*

Description

A two-sample test based on Wasserstein's distance (`wass_stat`).

Usage

```
wass_stat(a, b, power = 1)

wass_test(a, b, nboots = 2000, p = default.p)
```

Arguments

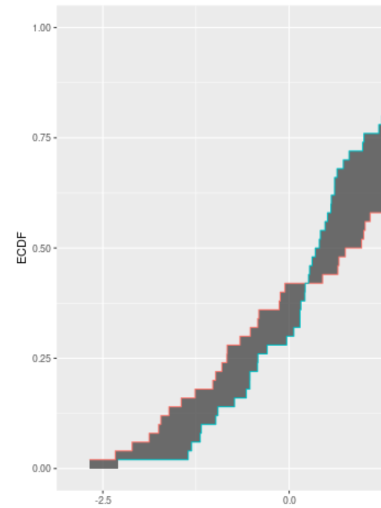
a	a vector of numbers
b	a vector of numbers
power	power to raise test stat to
nboots	Number of bootstrap iterations
p	power to raise test stat to

Details

The Wasserstein test compares two ECDFs by looking at the Wasserstein distance between the two. This is of course the area between the two ECDFs. Formally – if E is the ECDF of sample 1 and F is the ECDF of sample 2, then

$$WASS = \int_{x \in R} |E(x) - F(x)|^p$$

across all x. The test p-value is calculated by randomly resampling two samples of the same size using the combined sample. Intuitively the Wasserstein test improves on CVM by allowing more extreme observations to carry more weight. At a higher level – CVM/AD/KS/etc only require ordinal data. Wasserstein gains its power because it takes advantages of the properties of interval data – i.e. the distances have some meaning.



In the example plot below, the Wasserstein statistic is the shaded area between the ECDFs.

Value

Output is a length 2 Vector with test stat and p-value in that order. That vector has 3 attributes – the sample sizes of each sample, and the number of bootstraps performed for the pvalue.

Functions

- `wass_stat`: Wasserstein metric between two ECDFs
- `wass_test`: Permutation based two sample test using Wasserstein metric

See Also

`dts_test()` for a more powerful test statistic. See `cvm_test()` for the predecessor to this test statistic. See `dts_test()` for the natural successor of this test statistic.

Examples

```
vec1 = rnorm(20)
```

```
vec2 = rnorm(20,4)
wass_test(vec1,vec2)
```

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